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# Bianchi Type V Universe and Bulk Viscous Models with Time Dependent Gravitational Constant and Cosmological Constant in General Relativity

**Reena Tandon**

***Abstract:*** ***This paper deals with Bulk Viscous Models and Bianchi type-$V$universe in the standard general relativity theory by assuming $\xi(t) = \xi_0\, \rho^m$ where $\xi_0$ and $m$ are constants, $\rho$ is the density. Einstein field equations ($EFEs$) have been solved in the presence of a variable gravitational constant as well as a variable cosmological constant together with a bulk viscous fluid. In order to find a deterministic solution of EFEs, a simple form of Hubble parameter being constant when $H(R) = a(R^{-n} + 1)$ where $a > 0$ , $n > 1$, has been considered here that shows the signature flip from early deceleration to present acceleration. Moreover, the bulk viscous coefficient is allowed to vary with the density as$\rho$. The physical and geometrical behavior are described in detail for the obtained model. The model obtained here is in good agreement with the present cosmological observations.***

***Keywords : Bulk Viscous Models, Bianchi Type -$V$ Models, Cosmological constants, Hubble constants and gravitational constant.***

## I. INTRODUCTION

Current interpretation proposes that the greatest prominent innovation of the current cosmology is that the present universe is not only extending but also accelerates. Nowadays, this behavior of the universe is established by many independent observational data like cosmic micro back ground (CMB) radiation. The cosmological constant lambda and Newtonian gravitational constant $G$ are the fundamental constants in the Einstein's theory of general relativity. One of the most important and residual problem in cosmology is of the cosmological constant. One of the most problematic problem containing fundamental particle physics theory and cosmology is the small size of the constructive cosmological constant detected currently ($\Lambda_0 \leq 10^{-56} cm^{-2}$).The objective of a variable gravitational, constant $G$ has been offered by P. A. M. Dirac [1] in the structure of general relativity. Y.-K. Lau [2] presented a variation connecting the deviation of $G$ among that of $\Lambda$ who has been functioning in the structure of general relativity. This variation permits us to use Einstein's field equations form, that is not changed as deviation in $\Lambda$ is escorted by a deviation of$G$. Using this approach, Bianchi models and Friedman-Robertson-Walker (FRW) models have been explored by numerous authors. In the Bianchi type-I model, the cosmological term is proportional to the Hubble parameter with variable $G$ and $\Lambda$. FRW model has been discovered by [3]–[13]. The codification of the FRW universe has considered by Q. Ma *et al.* [14] along with a perfect fluid in the equation of state and a cosmological constant. The matter distribution is isotropic and homogeneous because we know that universe being spherically symmetric. Gravitational constant $G$ and cosmological constant lambda plays a significant role in Einstein theory of gravitation. P. A. M. Dirac [1] was the first one who considers the probability of variable $G$ . Pauls.wesson [15] has done several amendment of general relativity to permit a variable $G$. But these assumptions were not accepted worldwide. Lambda and $G$ like coupling variables of general relativity are considered in Einstein's field equations, proposed by [2], [3], [6],[16]. The possibility of varying of probably increase of $G$ has been discussed by many other researchers [16]–[19]. The vast difference between cosmological constant and vacuum energy density has been complex problem in quantum field theory discussed by [20]–[24]. It was opinion by various researchers [25], [26]. [27] has discussed the universe had non zero cosmological constant. Time dependent $G$ and lambda were obtained by O. Bertolami [28]. Number of authors [7],[8],[13],[16],[29]–[34] has considered homogeneous isotropic cosmological model with variable $G$ and lambda in space times.

In prior stages of evolution, according to Z. Klimek [35], it was assumed that the matter treated as viscous fluid throughout neutrino decoupling. Bianchi type -$V$ viscous fluid cosmological models for a barotropic fluid distribution has been considered by A. A. Coley [36]. The Purpose of viscosity in eluding the initial big bang singularity has been considered by Charles W.Misner [37], [38]. M. H. and Z. Klimek [39] studied about the effect of bulk viscosity on the growth of the universe. T. P. and S. M. Chitre [40], A.Pradhan [41] has scrutinized with varying lambda and cosmological models with viscous fluid in LRS Bianchi Type -$V$ universe, Sistero Roberto F.[4] has investigated matter distribution for admitting anisotropic pressure and heat flow in Bianchi type-$V$ model.

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***** Correspondence Author (s)
**Reena Tandon***, Dept. of Mathematics, School of Mechanical Engineering, Lovely Professional University, Phagwara, India. reena.tandon2008@gmail.com

Bulk viscous models and Bianchi type -V universe has been studied with viscosity and heat flow by B. K. Nayak and B. K. Sahoo [42], A. Banerjee and A. K. Sanyal [43]. This paper deals in general relativity with cosmological constant and Time dependent gravitational constant with Bulk Viscous Bianchi Type –$V$ Cosmological Model considered by [44]–[48] , by assuming $\xi(t) = \xi_0 \rho^m$ where $\xi_0$, $m$ are constants, $\rho$ is the density. It has also been assumed that variation law for Hubble parameter is constant when $H(R) = a(R^{-n} + 1)$, where $a > 0, n > 1$. Models physical behavior has been also discussed.

## II. MODEL AND FIELD EQUATIONS

The form of Bianchi type-$V$ metric is considered as

$$ds^2 = -dt^2 + A^2(t)dx^2 + e^{2x}(B^2(t)dy^2 + C^2(t)\,dz^2) \qquad (1)$$

The functions of t are $A^2(t)$, $B^2(t)$, $C^2(t)$. The energy momentum ($T_i^j$ )tensor in an imperfect bulk viscous fluid is specified by $T_i^j = (\rho + \bar{p})\,\upsilon_i\upsilon_j + \bar{p}g_{i_j}$ (2)

We have relations satisfying following conditions where ρ is the energy density, the four-velocity vector of the element is $\upsilon_i$ , θ is the scalar expansion ε is coefficient of bulk viscosity, $\bar{p}$ is dissipative pressure and p is equilibrium pressure.

$$v_i\, v_j = -1 \qquad (3)$$

$$\bar{p} = p - \varepsilon\,\theta \qquad (4)$$

$$p = \omega\rho\,,\quad 0 \le \omega \le 1. \qquad (5)$$

The Einstein's field equations with cosmological constant lambda and time dependent gravitational $G$ when

$$R_{ij} - \frac{1}{2}R_1^1 g_{i_j} = -8\pi\, G(t)\,T_{i_j} + \Lambda(t)\,g_{i_j} \qquad (6)$$

Where the Ricci scalar is $R$. In commoving co-ordinate system, by substituting the value of energy momentum tensor (2) we get the field equation (6).

$$\frac{1}{A^2} - \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC} = 8\pi G\bar{p} - \Lambda \qquad (7)$$

$$\frac{1}{A^2} - \frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} - \frac{\dot{A}\dot{C}}{AC} = 8\pi G\bar{p} - \Lambda \qquad (8)$$

$$\frac{1}{A^2} - \frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} - \frac{\dot{A}\dot{B}}{AB} = 8\pi G\bar{p} - \Lambda \qquad (9)$$

$$-\frac{1}{A^2} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} = 8\pi G\rho + \Lambda \qquad (10)$$

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \qquad (11)$$

Wherever derivatives of $A, B, C$ are denoted by dots with respect to "$t$". In prospect of disappearing of the divergence of Einstein tensor, we get

$$8\pi G\left[\dot{\rho} + (\rho + \bar{p})\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right]\right] + 8\pi\rho\dot{G} + \dot{\Lambda} = 0 \qquad (12)$$

$T_{i,j}^{j} = 0$ , the usual energy conservation equation gives

$$\dot{\rho} + (\rho + \bar{p})\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right] = 0 \qquad (13)$$

$$8\pi\rho\dot{G} + \dot{\Lambda} = 0 \qquad (14)$$

From the equation (14) we conclude that for non -zero energy density, $G$ is constant and lambda is zero or constant. For Bianchi Type $-V$ space time the Average scale factor $R$ is $[ABC]^{1/3}$.

By considering the equation (7) to (9) and (11) we acquire

$$\frac{\dot{A}}{A} = \frac{\dot{R}}{R} \qquad (15)$$

$$\frac{\dot{B}}{B} = \frac{\dot{R}}{R} - \frac{K}{R^3} \qquad (16)$$

$$\frac{\dot{C}}{C} = \frac{\dot{R}}{R} - \frac{K}{R^3} \qquad (17)$$

Wherever K is constant by integrating equation (15) - (17)

$$A = K_1\, R \qquad (18)$$

$$B = K_2 R\ e^{\left[(-K\int \frac{dt}{R^3}\right]} \qquad (19)$$

$$C = K_3 R\, e^{\left[(-K\int \frac{dt}{R^3}\right]} \qquad (20)$$

Integrating constants are $K_1$, $K_2$, $K_3$., Scalar expansion is

$$\theta = 3H \qquad (21)$$

Shear tensor is $\sigma = \frac{K}{R^3}$ (22)

In this model we consider the spatial volume by taking $V = ABC$ (23)

In the model the Hubble parameter and deceleration parameter are described as

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right] \qquad (24)$$

$$q = -1 - \frac{\dot{H}}{H^2} \qquad (25)$$

Wherever $H_1 = \frac{\dot{A}}{A}$ , $H_2 = \frac{\dot{B}}{B}$, $H_3 = \frac{\dot{C}}{C}$

In direction of $x, y$ *and* $z$ here $H_1, H_2, H_3$ are the directional Hubble factors respectively.

In the Einstein's field equations from (7) to (10) shear scalar σ, deceleration parameter $q$ and Hubble parameter $H$ can also be written as

$$H^2(2q - 1) - \sigma^2 + \frac{1}{R^2} = 8\pi\, G\,\bar{p} - \Lambda \qquad (26)$$

$$3H^2 - \sigma^2 - \frac{3}{R^2} = 8\,\pi\, G\rho + \Lambda \qquad (27)$$

$$\dot{\rho} + 3(\rho + \bar{p})\,H = 0 \qquad (28)$$

It was recognized that the energy density of the universe is infinity huge and average scale factor R is near to zero while expansion of universe that is as R increases the density decreases and when R is infinity the density becomes zero.

From equation (27)

we get $\frac{\rho_v}{\rho_c} \to 1$ where

$\rho_v = \frac{\Lambda}{8\Pi G}$ , $\rho_c = \frac{3H^2}{8\pi G}$.

For $\Lambda \ge 0$, $\rho < \rho_c$

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Also, from equation (27) we get

$0 < \frac{\sigma^2}{\theta^2} < \frac{1}{3}$ and $0 < \frac{8\Pi \rho G}{\theta^2} < \frac{1}{3}$

A negative lambda will increase the anisotropy. While for $\Lambda \geq 0$ implies positive lambda, which obstructs the upper limit of anisotropy.

By taking the equation (26) and (27)

we get $0 < \frac{\sigma^2}{\theta^2} < \frac{1}{3}$ and $0 < \frac{8\Pi \rho G}{\theta^2} < \frac{1}{3}$

$$\frac{d\theta}{dt} = \Lambda + 12\pi G\theta\varepsilon - 4\pi G(\rho + 3p) - \frac{\theta^2}{3} - 2\sigma^2 \quad (29)$$

## III. SOLUTION OF THE FIELD EQUATIONS

A relation between average scale factor $R$ and Hubble parameter $H$ was assumed as

$$H = a(R^{-n} + 1) \quad (30)$$

On integrating we get

$$R^n = e^{na(t+t_0)} - 1 \quad (31)$$

By taking the big bang condition that is at $t = 0, R = 0$, we obtained $t_0 = 0$ .Thus equation (31) is

$$R^n = e^{nat} - 1 \quad (32)$$

Use equations (31) in (18)-(20)

$$A = K_1[e^{nat} - 1]^{\frac{1}{n}} \quad (33)$$

$$B = K_2[e^{nat} - 1]^{\frac{1}{n}} \exp\left(-K \int \frac{dt}{R^3}\right) \quad (34)$$

$$C = K_3[e^{nat} - 1]^{\frac{1}{n}} \exp\left(-K \int \frac{dt}{R^3}\right) \quad (35)$$

To determine the coefficient of bulk viscosity we assume

$\xi(t) = \xi_0 \rho^m$ where $\xi_0$, m are constants. (36)

by [44]–[47]

Use equation (36) in equation (28) we get

$$\rho = \frac{1[1-e^{-nat}]^{\frac{9a\epsilon_0}{n}}}{[e^{nat}-1]^{\frac{3(1+w)}{n}}} \exp\left[9a\xi_0\left[-at + \frac{1}{n(1-e^{-nat})}\right]\right] \quad (37)$$

From equation (26)-(27) we get

$$\Lambda = \frac{1}{[(1+w)(e^{nat}-1)-3a\xi_0 e^{nat}]}\left[\left[\frac{a^2 e^{nat}}{[e^{nat}-1]^2}[3(1+w)e^{nat}(e^{nat}-1) - 9a\xi_0 e^{nat} - 2n(e^{nat}-1)]\right] + \frac{K^2}{[e^{nat}-1]^{\frac{6}{n}}}[(1-w)(e^{nat}-1) + 3a\xi_0 e^{nat}] - \frac{1}{[e^{nat}-1]^{\frac{2}{n}}}[(1+3w)(e^{nat}-1) - 9a\xi_0 e^{nat}]\right] \quad (38)$$

From equation (27)

$$G = \frac{[e^{nat}-1]^{\frac{3(1+w)}{n}}}{8\pi l[(1+w)(e^{nat}-1)-3a\xi_0 e^{nat}]} \times \frac{\left[3a^2(1+w)e^{2nat} - \frac{2K^2}{[e^{nat}-1]^{\frac{6-n}{n}}} - \frac{2}{[e^{nat}-1]^{\frac{2-n}{n}}}\right]}{[1-e^{-nat}]^{\frac{9a\epsilon_0}{n}} \exp\left[9a\xi_0\left[-at + \frac{1}{n(1-e^{-nat})}\right]\right]} \quad (39)$$

## IV. SPECIFIC MODELS FOR DIFFERENT VALUES OF MODEL PARAMETER n

### A. CASE-I

If $n = 0$ then $H = 2a$ (40)

$$R = e^{2at} \quad (41)$$

$$A = K_1 e^{2at} \quad (42)$$

$$B = K_2 \exp\left[2at + \frac{k e^{-6at}}{6a}\right] \quad (43)$$

$$C = K_3 \exp\left[2at - \frac{k e^{-6at}}{6a}\right] \quad (44)$$

$$\rho = K_4 \exp[6at(3\xi_0 - (1+w)] \quad (45)$$

$$\sigma = K e^{-6at} \quad (46)$$

$$\theta = 6a \quad (47)$$

$$q = -1 \quad (48)$$

$$\Lambda = 12a^2 + \frac{K^2 e^{-12at}}{[(1+w)-6a\xi_0]}[1 - w + 6a\xi_0] - e^{-4at}\frac{[3(w-6a\xi_0)+1]}{[(1+w)-6a\xi_0]} \quad (49)$$

$$G = \frac{-[1 + K^2 e^{-8at}]}{4\pi K_4[1 + w - 6a\xi_0]\exp[2at[9\xi_0 - 1 - 3w]]} \quad (50)$$

The model does not have initial singularity. Throughout the growth of cosmos, the scalar expansion is constant. The model observes constant extension. The model is accelerating at constant rate which displays the exponential growth of the cosmos for the solution for negative value of deceleration parameter $q = -1$. Initially when $t = 0$, scale factor, shear scalar, viscosity, density, gravitational constant and cosmological constant are all constant. Throughout the evolution, the rate of expansion of the universe is constant. Also, gravitational constant and lambda becomes infinity for the huge value of "$t$". As $t \to \infty$, so $\frac{\sigma}{\theta} \to 0$ hence the model inclines to isotropy for a large value of "$t$".

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### B. CASE-II

If n=1 then

$$H = a\,(R^{-1} + 1) \tag{51}$$

$$R = e^{at} - 1 \tag{52}$$

$$A = K_1(e^{at} - 1) \tag{53}$$

$$B = K_2 e^{at}[1 - e^{-at}]^{1-\frac{K}{\alpha}} \exp\left[\frac{2\,K\,e^{-at}}{\alpha\,[1-e^{-at}]^2}\right] \tag{54}$$

$$C = K_3 e^{at}[1 - e^{-at}]^{1-\frac{K}{\alpha}} \exp\left[\frac{2\,K\,e^{-at}}{\alpha\,[1-e^{-at}]^2}\right] \tag{55}$$

$$\rho = \frac{l[1-e^{-at}]^{9a\xi_0}}{[e^{at}-1]^{3(1+w)}}\ exp\left[9a\xi_0\left(-at + \frac{1}{(1-e^{-at})}\right)\right] \tag{56}$$

$$\sigma = \frac{K}{[e^{at}-1]^3} \tag{57}$$

$$\Lambda = \frac{1}{[(1+w)(e^{at}-1)-3a\varepsilon_0 e^{at}]}\left(\left[\frac{a^2e^{at}}{[e^{at}-1]^2}\left(3(1+w)\,e^{at}(e^{at}-1) - 9a\varepsilon_0 e^{at} - 2(e^{at}-1)\right)\right] + \frac{K^2}{[e^{at}-1]^6}[(1-w)(e^{at}-1) + 3a\varepsilon_0 e^{at}] - \frac{1}{[e^{at}-1]^2}[(1+3w)(e^{at}-1) - 9a\varepsilon_0 e^{at}]\right) \tag{58}$$

$$G = \frac{[e^{at}-1]^{3(1+w)}}{8\pi l[(1+w)(e^{at}-1)-3a\xi_0\,e^{at}]} \times \frac{3a^2(1+w)e^{2at} - \frac{2K^2}{|e^{at}-1|^5} - \frac{2}{(e^{at}-1)}}{[1-e^{-at}]^{9a\varepsilon_0}\exp\left[9a\,\xi_0\left[-at + \frac{1}{(1-e^{-at})}\right]\right]} \tag{59}$$

$$ds^2 = -dt^2 + [K_1(e^{at}-1)]^2dx^2 + e^{2x}\left(\left[K_2e^{at}[1-e^{-at}]^{1-\frac{k}{\alpha}}\ \exp\left[\frac{2\,K\,e^{-at}}{\alpha\,[1-e^{-at}]^2}\right]\right]^2 dy^2 + \left[K_3e^{at}[1-e^{-at}]^{1-\frac{k}{\alpha}}\ \exp\left[\frac{2\,K\,e^{-at}}{\alpha\,[1-e^{-at}]^2}\right]\right]^2 dz^2\right) \tag{60}$$

Therefore, for model (60) it has been examined that at $t = 0$, expansion scalar $\theta$ is infinite and spatial volume $V$ is zero which proves that at $t = 0$ the universe begins growing with zero volume and has infinite rate of expansion. This model has a "point type" singularity at the initial period. The bulk viscosity, pressure, shears scalar, Hubble factor, cosmological term and energy density diverges at the initial singularity.

Since "$t$" raises, Hubble factor, bulk viscosity, pressure, shear scalar, density and gravitational constant decreases and approaches to zero asymptotically and cosmological constant is steady. The cosmic state begins from a Big-Bang at $t = 0$ and continues till $t = \infty$.

since $\frac{\sigma}{\theta} \to 0$ as $t \to \infty$, so this model moves toward isotropy for a huge value of "$t$". The model becomes conformally flat[48]. In the growth of the universe bulk viscosity has significant role.

### V. PHYSICAL AND GEOMETRIC SIGNIFICANCE

We observe that matter density $\rho$, expansion $\theta$, shear $\sigma$, cosmological constant $\Lambda$, coefficient of shear viscosity and bulk viscosity $\varepsilon$ all diverges at $t = 0$. At $t = 0$, the model starts with a big-bang from its singular state and remains to increase till $t = \infty$. Hence the model begins with a decelerating expansion. It was found that the energy density of the cosmos is infinity huge and the average scale factor $R$ is close to zero whereas on expansion of universe that is as $R$ is infinity the density becomes zero and $R$ increases the density decreases. From equation (5.27) we get

$$\frac{\rho_v}{\rho_c} \to 1,\ \text{where}\quad \rho_v = \frac{\Lambda}{8\pi G}\ \ and$$

$$\rho_c = \frac{3H^2}{8\pi G}\ \text{for}\ \Lambda \geq 0, \rho < \rho_c$$

Also, from equation (5.27), we get

$$0 < \frac{\sigma^2}{\theta^2} < \frac{1}{3}\ and\ 0 < \frac{8\pi\rho G}{\theta^2} < \frac{1}{3}$$

A negative lambda will increase the anisotropy. While for $\Lambda \geq 0$ implies positive lambda, which obstructs the upper limit of anisotropy.

In $Case\ I, n = 0$,

the model does not have initial singularity. Throughout the growth of cosmos, the scalar expansion is constant. The model observes constant extension. The model is accelerating at constant rate for the solution for negative value of deceleration parameter $q = -1$ which displays the exponential growth of the cosmos. Initially $when$ $t = 0$, scale factor, shear scalar, viscosity, density, gravitational constant and cosmological constant are all constant. The rate of expansion of the universe is constant throughout the evolution of universe. Also, gravitational constant and lambda becomes infinity for large value of "$t$". By taking the limit of large times that is

t→∞ $H = 2a, q = -1, \sigma \to 0, \theta = 6a$.

To develop a de-Sitter universe and negligible asymptotically universe it has been seen that it prohibits the model to approach to the matter density because of the existence of bulk viscosity.

For the huge values of $t$, coefficients of shear viscosity and bulk viscosity approaches to the actual constants.

For the model





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$$\frac{\sigma}{\theta} = \frac{k(1-e^{-n\alpha t})}{3a[e^{n\alpha t}-1]^{(3+6\eta_0)/n}}$$

Since at $t \to \infty, \frac{\sigma}{\theta} \to 0$ , hence the model inclines to isotropy for the large value of "$t$".
We observe that the process of isotropization accelerated the presence of shear viscosity.

In Case II, $n = 1$,

It has been examined that at $t = 0$ expansion scalar $\theta$ is infinite and spatial volume $V$ is zero which proves that at $t = 0$ the universe begins growing with zero volume and has infinite rate of expansion. This model has a "point type" singularity at the initial period. The bulk viscosity, pressure, shears scalar, Hubble factor, cosmological term and energy density diverges at the initial singularity. As per "$t$" raises, Hubble factor, bulk viscosity, pressure, shear scalar, density and gravitational constant decreases and approaches to zero asymptotically and cosmological constant is constant. Cosmic state begins from a Big-Bang at $t = 0$ and continues till $t = \infty$. As $\frac{\sigma}{\theta} \to 0$ then $t \to \infty$, *therefore* the model moves toward isotropy for a huge value of "$t$". The model does not have horizon. The model becomes conformally flat[48].

## VI. DISCUSSION AND CONCLUSION

In this paper we examined a cosmological development in the background of homogeneous, anisotropic Bianchi type $V$ space-time which offered a variation law for Hubble parameter $H$ that includes viscous fluid matter distribution. The model isotropizes asymptotically and the isotropization is accelerated by the existence of shear viscosity.

1) This paper deals in general Relativity with Cosmological constant and Time dependent gravitational constant with Bulk Viscous Bianchi Type $-V$ Cosmological Model by assuming $\xi(t) = \xi_0 p^m$ considered by [44]–[47] where $\xi_0$, m are constants. It has also been assumed a variation law for Hubble parameter is constant when $H(R) = a(R^{-n} + 1)$, where a >0, n>1.
2) Two universe models were acquired and their physical behavior has been discussed. The universe begins from singular state when $n = 1$, whereas the cosmology follows a non-singular state when $n = 0$.
3) Presences of bulk viscosity increase matter density's value. As $\frac{\sigma}{\theta} \to 0, t \to \infty$ the models approaches isotropy for large values of "$t$". When $n = 0$, at constant rate, the model is accelerating as $q = -1$.
4) A suitable depiction of the growth of cosmos could be provided by the solutions obtained in the present paper.
5) In brief , the law of variation of Hubble parameter described by [3] to Bianchi type $-V$ space-time to discover particular solutions of Einstein's field equations has been extended.

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## AUTHORS PROFILE

**Dr. Reena Tandon** earned her Ph.D. degree in Mathematics from Dravidian University, Andhra Pradesh, India. She is an Associate Professor of Mathematics in Lovely Professional University, Phagwara, India. Her research is situated in the field of Cosmology (Mathematics Physics). She presented and participated paper in the 2nd World Summit on Advances in Sciences, Indiana University – Purdue University (IUPUI), Indianapolis, Indiana, USA. She has Lifetime membership in Indian Science Congress and Astronomical Society of India. She has one of the Member cum Editor of editorial board of journal of International Journal of Advances in Engineering & Scientific Research.