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The Unsteady MHD Flow Under the Action of Chemical Reaction and Thermal Radiation at the Stagnation Point of a Rotating Sphere

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Abstract: In the present work, we have studied the unsteady MHD flow under the action of thermal radiation and chemical reaction at the stagnation point of a rotating sphere. By using similarity transformation, the unsteady non-linear boundary layer equation obtained as a result of heat transfer and mass transfer along with momentum were changed to a set of ordinary differential equations. Through the use of MATLAB's built in solver bvp4c, the obtained differential equations were solved. Fluid velocity profile along with temperature profile and species concentration profiles are drawn for radiation parameter (R_a) ***and chemical reaction parameter*** (K_r) ***and the obtained results are discussed.***

Keywords : Unsteady flow, Rotating sphere, Thermal radiation, transfer of Heat and Mass, Chemical reaction.

I. INTRODUCTION

Transfer of heat and mass in rotating systems are widely used in many industries, in the field of engineering, in nuclear reactors, design of rotating machinery, gas turbines, shielding of rotating bodies etc. The reaction between fluid particles and a foreign mass has very good application in polymer industry, in the manufacture of glassware or ceramics and so on. Formation and dispersion of fog occurs due to this reaction. A chemical reaction may be homogeneous or heterogeneous. In homogeneous reaction all the reactants and product formed are in the same phase whereas in heterogeneous reaction the reactant and product are present in different phases. In heterogeneous system reaction occur at the interface. Howarth(1951), Nigam(1954) and Banks(1965) studied the flow of an incompressible fluid which is viscous in nature in a rotating solid sphere. Singh(1960) and Banks(1965) suggested a solution of the problem regarding the heat transfer which occurs in a viscous incompressible fluid under the influence of constantly rotating uniformly heated sphere. Kalita in 1982 also studied this problem by using a magnetic flux and the obtained equations are integrated through Karman Pohhausen procedure. Sharma et al. (2015) carried out a systematic inquiry on the impact of soret and dufour number on unsteady MHD flow of rotating sphere in porous medium.

In the present work, we have discussed the impact of electromagnetic radiation produced by thermal motions of atoms or molecules and also chemical reaction on an unsteady MHD flow of a binary mixture of a fluid in the stagnation point of a rotating sphere. The sphere surface has constant fluid temperature and has constant concentration of species present in the binary fluid. Similarity transformations are taken to change the non-linear differential equation into dimensionless, non-linear differential equation. Numerically these are solved through the use of MATLAB's built in solver bvp4c. Graphical results are given to show the impact of thermal radiation and chemical reaction parameters.

II. MATHEMATICAL FORMULATION

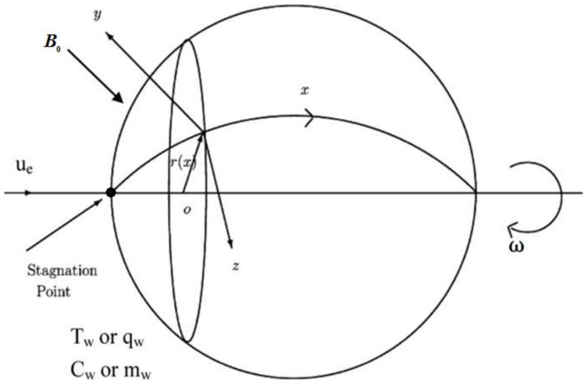

Figure 1: Diagrammatic representation of the considered problem.

Under the existence of a uniform magnetic field (B_0), the considered fluid mixture at the stagnation point of a sphere is considered. This mixture of fluid is viscous in nature, chemically reactive, can conduct electricity and can generate or absorb heat. The sphere is rotated with angular velocity $\omega(t)$, which is dependent with time. The configuration of the rotating sphere and the coordinate system are shown in the figure (1). It is considered that the properties of the fluid are constant and here we have considered 1st order chemical reaction. The velocity of the boundary layer at the edge of the boundary is assumed as u_e and it is defined as follows:

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$$u_e(x,t)=\frac{xA}{t}, x>0, A>0, t>0, \tag{1}$$

where A is the acceleration parameter.

The Boussinesq approximation for the continuity, momentum, energy and concentration equation can be expressed by following way

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial y}=0 \tag{2}$$

$$\begin{gathered}\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}+u\frac{\partial u}{\partial x}-\left[\frac{w^2}{r}\right]\frac{dr}{dx}=\frac{\partial u_e}{\partial t}+u_e\frac{\partial u_e}{\partial x}+\upsilon\frac{\partial^2 u}{\partial y^2}\\ +\left[g\beta_T(T-T_\infty)+g\beta_C(C-C_\infty)\right]-\frac{\sigma B_0^2}{\rho}(u-u_0)\end{gathered} \tag{3}$$

$$\frac{\partial w}{\partial t}+v\frac{\partial w}{\partial y}+u\frac{\partial w}{\partial x}+\left[\frac{uw}{r}\right]\frac{dr}{dx}=\upsilon\frac{\partial^2 w}{\partial y^2} \tag{4}$$

$$\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}+u\frac{\partial T}{\partial x}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{5}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k_r\left(C-C_\infty\right) \tag{6}$$

In the above equations, the velocity of the components along the x, y and z axes are denoted by u, v and w respectively. t represent the time of flow, r indicates the radial distances, T and C are the temperature and concentration of the fluid, ν indicates the kinematic viscosity, σ indicates the electrical conductivity, β implies the coefficient of thermal expansion, β_c indicates the compositional expansion coefficient, α indicates the thermal diffusivity, fluid density is denoted by ρ, B_0 implies the magnetic induction, acceleration due to gravity is denoted by g, k_r is the dimensionless parameter of chemical reaction, the latent heat of the fluid is given by C_p, κ represents the permeability of porous medium, the subscript ∞ denotes the ambient condition.

The initial as well as boundary conditions for the proposed problem are assumed as follows:

$$\left.\begin{aligned}&t=0:\\&u(x,y,t)=u_i(x,y),\ v(x,y,t)=v_i(x,y),\ w(x,y,t)=w_i(x,y),\\&T(x,y,t)=T_i(x,y),\ C(x,y,t)=C_i(x,y).\\&t>0:\\&u(x,y,t)=0,\ v(x,y,t)=V_w(x,y),\ w(x,y,t)=\omega(t)r,\\&T(x,y,t)=T_w,\ C(x,y,t)=C_w,\ at\ y=0.\\&t>0:\\&u(x,y,t)=u_e(x,t),\ w(x,y,t)=0,\ T(x,y,t)=T_\infty,\\&C(x,y,t)=C_\infty,\ as\ y\to\infty\end{aligned}\right\} \tag{7}$$

In the above conditions, V_w denotes the normal velocity, C_w and T_w are the concentration and temperature of the fluid at the wall.

The following transformations are used to dimensionless the governing equations of the problem:

$$\left.\begin{aligned}&\eta=\sqrt{\frac{2}{\upsilon t}}y,\ u_e(x,t)=\frac{xA}{t},\ u=\left(\frac{xA}{t}\right)f'(\eta),\ w=\left(\frac{Bx}{t}\right)s(\eta),\\&v=-\sqrt{\frac{2\upsilon}{t}}Af(\eta),\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty},\\&\omega(t)=\frac{B}{t},\ \lambda=\left(\frac{B}{A}\right)^2=\left(\frac{w_w}{u_e}\right)^2,\ Pr=\frac{\upsilon}{\alpha},\ Sc=\frac{\upsilon}{D},\\&\lambda_1=\frac{Gr_x}{Re_x^2},\ \lambda_2=\frac{Grc_x}{Re_x^2},\ Gr_x=\frac{g\beta(T_w-T_\infty)x^3}{\upsilon^2},\\&Grc_x=\frac{g\beta_c(C_w-C_\infty)x^3}{\upsilon^2},\ Re_x=\frac{u_e x}{\upsilon},\ M=\frac{\sigma B_0^2Ax^2}{\mu Re_x},\\&K_r=\frac{k_rAx^2}{\upsilon Re_x},\ \alpha=\frac{k}{\rho C_p},\ R_a=\frac{k_1k}{4\sigma^*T_\infty^3}.\end{aligned}\right\} \tag{8}$$

Where, the velocity gradient at the end of the boundary layer along the direction of y is denoted by B and μ represent the fluid dynamic viscosity.

In equation (5), we have used Rosseland approximation to simplify the radiative heat flux term

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{9}$$

In the equation (9), σ^* indicates the Stefan-Boltzmann constant, k^* implies the coefficient of absorption. T^4 can be represented as a linear function of the temperature by accepting the change of temperature in the flow is sufficiently small. T^4 can be written in Taylor's series in the following way:

$$T^4=T_\infty^4+4T_\infty^3\left(T-T_\infty\right)+6T_\infty^2\left(T-T_\infty\right)^2+\ldots\ldots$$

Neglecting the term apart from 1st degree in $\left(T-T_\infty\right)$ and we get

$$T^4\cong 4T_\infty^3T-3T_\infty^4 \tag{10}$$

In view of equation (9) and (10), we get

$$\frac{\partial q_r}{\partial y}=-\frac{16\sigma^*T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2} \tag{11}$$

So from equation (5), we have

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma^*T_\infty^3}{3k_1\rho C_p}\frac{\partial^2 T}{\partial y^2} \tag{12}$$

Using (8), equations $(2)-(7)$ reduce to

$$\begin{gathered}f'''+Aff''+\frac{A}{2}\left(1-f'^2+\lambda s^2+\lambda_1\theta+\lambda_2\phi\right)-\\ \frac{1}{4}\left(2-2f'-\eta f''\right)+\frac{M}{2}\left(1-f'\right)=0\end{gathered} \tag{13}$$

$$s''-A\left(f's-fs'\right)+\frac{1}{4}\left(\eta s'+s\right)=0$$

(14)

$$\frac{1}{3}(3+4R_a)\theta''+\frac{Pr}{4}(4Af\theta'+\eta\theta')=0 \qquad (15)$$

$$\phi''+\frac{Sc}{4}(4Af\phi'+\eta\phi'-2K_r\phi)=0 \qquad (16)$$

In the mentioned equation, prime indicates the differentiation of any factor with respect to η, Pr implies the Prandtl number, Sc indicates the Schmidt number, Gr_x denotes the Grashof number, Gr_{c_x} is the modified Grashof number, λ_1 and λ_2 are the buoyancy parameters, magnetic field parameter is represented by M , K_r implies the parameter of chemical reaction and R_a denotes the parameter of thermal radiation.

The transformed initial and boundary conditions are

$$\left.\begin{array}{l}\eta=0: \quad f=-f_w, f'=0, s=1, \theta=1, \phi=1,\\ \eta\to\infty: \; f'=1, s=0, \theta=0, \phi=0\end{array}\right\} \qquad (17)$$

In the equation (17), f_w can be represent as $\frac{V_w}{A\sqrt{2\upsilon/t}}$ which indicates the suction or injection parameter and here $f_w>0$ *or* $f_w<0$ depends on wall suction *or* injection.

The local skin friction is represented by C_{fx}, C_{fz} and local Nusselt number and local Sherwood number are indicated by symbol Nu and Sh.

$$C_{fx}=\frac{\left[2\mu\left(\frac{\partial u}{\partial y}\right)\right]_{y=0}}{\rho u_e^2}=\sqrt{\frac{8}{A\,Re_x}}f''(0),$$

$$C_{fz}=\frac{\left[-2\mu\left(\frac{\partial w}{\partial y}\right)\right]_{y=0}}{\rho u_e^2}=-\sqrt{\frac{8\lambda}{A\,Re_x}}s'(0),$$

$$Nu=\frac{-x}{T_w-T_\infty}\left(\frac{\partial T}{\partial y}\right)_{y=0}=-\sqrt{\frac{2\,Re_x}{A}}\theta'(0),$$

$$Sh=\frac{-x}{C_w-C_\infty}\left(\frac{\partial C}{\partial y}\right)_{y=0}=-\sqrt{\frac{2\,Re_x}{A}}\phi'(0).$$

III. Method of Solution

The ordinary differential equation from equation $(13)-(16)$ cannot be solved under the boundary conditions (17) in a closed form. Hence it is solved by implementing MATLAB built in solver bvp4c for boundary value ordinary differential equations.

IV. Numerical Analysis and Discussions

Graphical representations of the differential equations from equations $(13)-(16)$ under the boundary conditions which are given in equation (17) are given below for different values of the Schmidt number (Sc), Magnetic parameter (M), Prandtl number (Pr), Thermal radiation parameter (R_a) and Chemical reaction parameter (K_r).

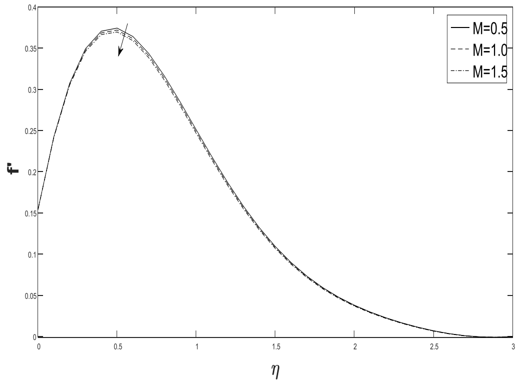

Figure 1.1(a): Influence of magnetic parameter (M) on the velocity profile.

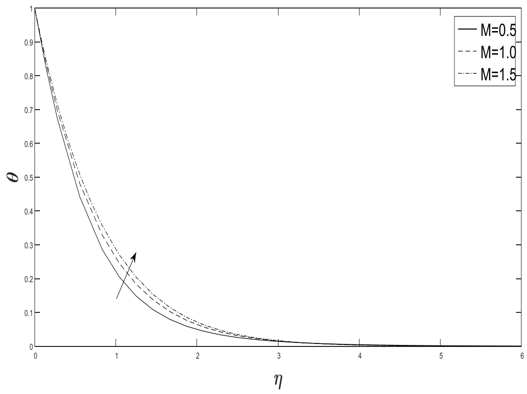

Figure 1.1(b): The impact of magnetic parameter (M) on the temperature profile.

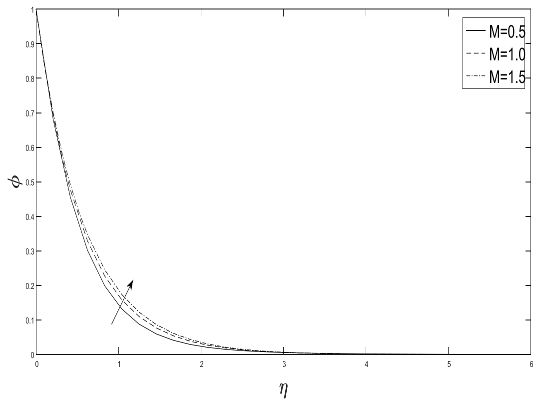

Figure 1.1(c): The impact of magnetic parameter (M) on the concentration profile.

Figures **1.1(a)** express the velocity profile for considered particular values of magnetic parameter (M). From the figure it is clear that fluid velocity falling off with raising

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value of M . There is an affinity to create Lorentz force due to the transverse magnetic field which is enforced to the normal of the flow. Hence flow velocity reduces with the raise of magnetic parameter (M). Figure **1.1(b)** and **1.1(c)** represents the temperature profile and concentration profiles for variations of magnetic parameter (M).

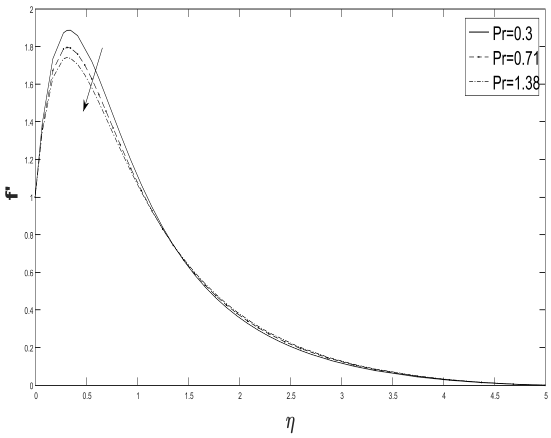

Figure 1.2(a): Influence of Prandtl number (Pr) on the velocity profile.

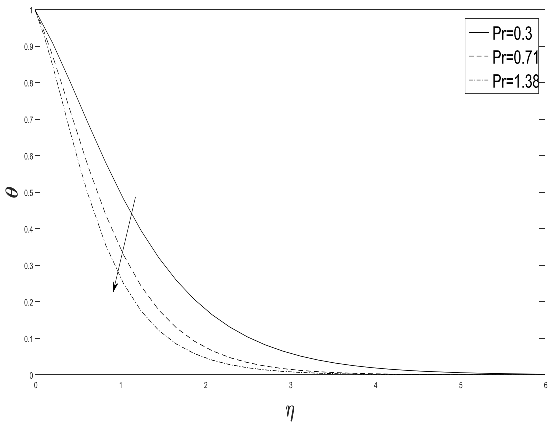

Figure 1.2(b): The impact of Prandtl number (Pr) on the temperature profile.

Figure **1.2(a)** express the velocity profiles for distinct values of Prandtl number (Pr) and figure **1.2(b)** indicates the temperature profile for different values of Prandtl number (Pr). It is noticed that velocity reduces with raising value of Prandtl number (Pr). It is also noticed that the thermal boundary layer reduces for greater values of Prandtl number (Pr) and it helps to lower the mean temperature inside the boundary layer. Smaller the value of thermal boundary layer greater is the ability to conduct heat of the considered fluid and it helps to spread the heat swiftly away from the heated surface. Greater boundary layer begins to thick due to smaller values of Prandtl number (Pr) and as a result heat transfer begins to decrease.

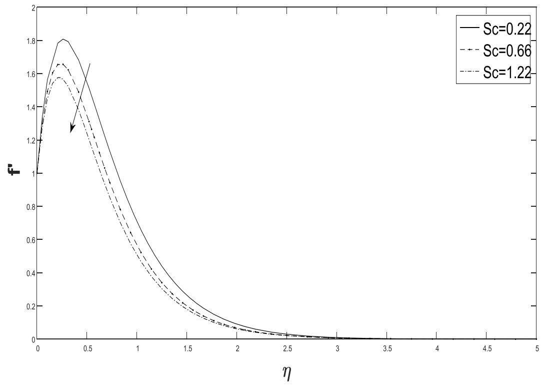

Figure 1.3(a): The impact of Schmidt number (Sc) on the velocity profile.

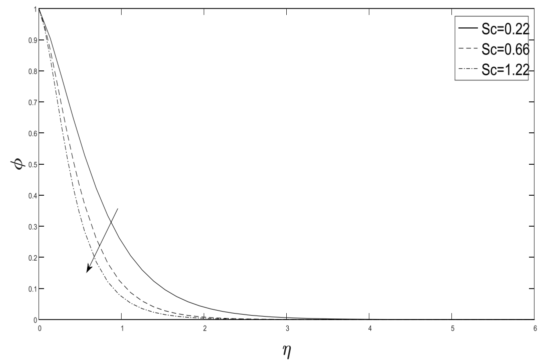

Figure 1.3(b): Influence of Schmidt number (Sc) on the concentration profile.

The figure **1.3(a)** shows the impact of Schmidt number (Sc) on the velocity and figure **1.3(b)** represent the impact of Schmidt number (Sc) in case of concentration profile. The Schmidt number is number which is dimensionless and it is obtained by dividing the momentum diffusivity by mass diffusivity. This number measures the degree up to which momentum and mass are transported by expansion in the velocity boundary layer and the species boundary layer. It is viewed that the value of concentration becomes smaller with raising the value of Schmidt number. This leads to the reduction of concentration of buoyancy effect and hence velocity of fluid begins to decrease.

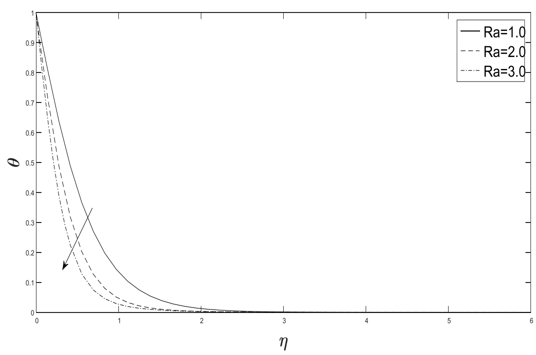

Figure 1.4(a): Influence of Radiation parameter (R_a) on the temperature profile.

Figure **1.4(a)** reflects the impact of temperature on various considered values of radiation parameter (R_a). There is a release of heat from the region of flow due to increase of R_a and it helps to decrease the temperature of the fluid. It is also noticed that thickness of boundary layers tends to reduce.

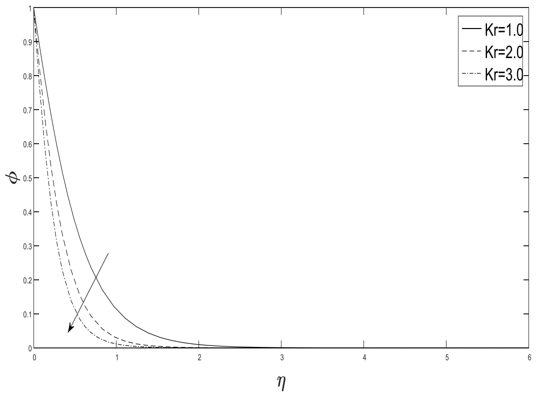

Figure 1.5(a): Influence of Chemical reaction parameter (K_r) on the concentration profile.

Figure **1.5(a)** shows the impact of chemical reaction parameter on the concentration of species. A marked effect is observed due to increase of chemical reaction parameter and it leads to falls off the species concentration within the boundary layer. The reason behind this is the destructive chemical reaction which reduces the thickness of the solutal boundary layer and it leads to the increases of mass transfer.

V. CONCLUSION

Under the existence of chemical reactions and the effect of thermal radiation, a numerical study was carried out for an Unsteady MHD Flow in the stagnation point of a rotating Sphere.

The ordinary differential equations which are non-linear are changed into the 1st order differential equation and these equations are solved by MATLAB bvp4c solver. Fluid velocity profile, temperature as well as species concentration profile is drawn for radiation parameter (R_a) and chemical reaction parameter (K_r). And the following conclusions are drawn:

(a) Concentration as well as temperature profiles are seen to increase due to increase in the magnetic field but velocity seems to decrease.

(b) The velocity and temperature decreases as the Prandtl number (Pr) is increased.

(c) The velocity and concentration decreases with increase in Schmidt number (Sc).

(d) When Radiation parameter (R_a) is increased, the temperature of the fluid will be decreased.

(e) Concentration of the binary fluid mixture will be decreases due to increase in chemical reaction parameter (K_r).

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