International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249 – 8958, Volume-9, Issue-1, October 2019

Thermoelastic Response of a Rectangular Plate under Hyperbolic Heat Conduction Model in Differential Transform Domain

Kamini K. Chaudhari, Chandrashekhar S. Sutar

***Abstract:** In the present work, a semi-analytical solution is presented for the thermoelastic response of a finite plate of rectangular geometry considering hyperbolic heat conduction model. The solution of thermoelastic displacement, thermal stresses and temperature are obtained using differential transform method under hyperbolic, non-Fourier heat conduction theory. For special case, thermal stresses and displacement functions are determined numerically and plotted graphically to analyze the effect of the thermal relaxation time.*

***Keywords:** Differential transform, Displacement function, Hyperbolic heat conduction model, Thermal stresses*

I. INTRODUCTION

Thermoelasticity comprises the study of temperature field, stress and strain develop in a material body as an effect of thermomechanical load applied to it. Thermoelastic behaviour of a material plays a vital role in the field of design of a structural element. A study of thermal stresses is very important in the applications, which deal with ultra-rapid heating rate, short pulse duration, and extreme large gradient. Thermal stresses in the material are one of the important factors which affect the life of the material body. As an effect of heat distribution, thermal stresses occur in the body which many times cause the failure of a structural element. In a practical problem, before going to design an object, it is essential to do a thorough study of its thermal and mechanical behaviour. During the last many decades Thermoelasticity becomes a more and more attractive subject for a researcher due to its application in practical engineering problems. A vast literature is available on the application of thermoelasticity in different solids under the framework of the classical theory of heat conduction which gives the parabolic heat conduction equation. But in the last few decades, many practical problems are encountered where situation like very high-temperature gradient, enormous heat fluxes and short time interval, the speed of heat transmission in the body is finite. Thus the Classical theory of heat transport does not give the accurate result in such conditions and hence there is a need to move from parabolic to hyperbolic heat conduction model.

With the progress of science and technology the demand for the study of thermal behaviour of the solids under the hyperbolic heat conduction model increases. In contemporary engineering like nuclear engineering, cryogenic engineering, seismology etc.where in experimental result it was observed that heat is propagated with the finite speed in the solid. In the experimental result of helium solid, it was first observed that heat is propagated in the material body with the finite speed. There are many practical problems like an ultra-rapid heat source such as laser and microwave which work with very short time duration, in a dispersed phase system of nano-particles where heat is transfer in nanoscale, to heat biological tissues using radiofrequency, in thermal insulators in which heat is propagated with very low speed, pulsed laser where variation in heat flux is extremely large, where it was experimentally observed that speed of heat transfer is finite. Thus to explain such phenomenon, Cattaneo[1] and Vernotte[2] proposed a modification in the classical theory of heat conduction by bringing a new term called relaxation time for heat flux in solids, which is given by the relation

$$q + \tau_q \frac{\partial q}{\partial t} = -\lambda \nabla T \qquad (1)$$

Where τ_q is called relaxation time, q is the heat flux and λ is the thermal conductivity of the material.

Equation (1) along with energy conservation law gives the hyperbolic heat conduction equation

$$\frac{\partial T}{\partial t} + \tau_q \frac{\partial^2 T}{\partial t^2} = k\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) T \qquad (2)$$

Where $k = \frac{\lambda}{\rho c}$, ρ, c are the thermal diffusivity, mass density and specific heat capacity respectively.

In past decades a considerable research interest has been attracted by non-Fourier heat conduction, several papers on analytical as well as a numerical solution for hyperbolic heat conduction models are available in the literature. But a very few literatures on thermal responses of different solid under hyperbolic heat conduction models has appeared in the literature. Most of the research work on the thermal response of rectangular plate in the parabolic heat conduction domain is available in the literature. Chen [3] has studied the thermal stress in a rectangular plate subjected to non-uniform heat transfer in terms of power series using Lanczos-Chebyshev and discrete least square methods. Morimoto, Tanigawa and Kawamura [4] discussed the thermal buckling in a rectangular

Revised Manuscript Received October 05, 2019

* Correspondence Author

Kamini K. Chaudhari*, Applied Mathematics, Pillai College of Engineering, Navi Mumbai, India.

Chandrshekhar S. Sutar, Mathematicst, PSGVPM's A.S.C. College, Shahada, Maharashtra, India.

Retrieval Number: F90471088619/2019©BEIESP
DOI: 10.35940/ijeat.F9047.109119

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

plate subjected to uniform heat supply. Kaminski [5] have considered the physical significance of the constant τ for non-homogeneous material and determined experimentally the value of τ for some selected product. Chen and Lin [6] adopted the hybrid numerical method to solve two-dimensional hyperbolic heat conduction equations (HHCE) and also present the method for selection of the shape function. Barletta and Zanchini [7] presented the temperature distribution for the problem of HHC in a rectangular bar. Yang [8] has applied a sequential method to find the solution of direct and inverse HHCE using finite difference and the Newton Raphson method. Moosaie , Atefi and Fardad [9] have discussed the analytical solution of two-dimensional non Fourier heat conduction equations using periodic boundary condition. Wang [10] analyzed the temperature distribution and thermal stresses in one-dimensional plate and cylinder associated with HHCE for different loading and boundary conditions. Mallick, Ranjan and Sarkar [11] determined the thermal stresses in annular fin using ADM under the hyperbolic heat conduction model. Noroozi and Goodarzi[12] have studied the effect of laser heat source on a finite tissue under one-dimensional HHC model applying the ADM method. Recently, Nie and Cao [13] have proposed three different mathematical representation of HHC and solved using alternative direction implicit method.

In the available literature, various methods are available to solve different linear as well as non-linear partial differential equations. Differential transform method (DTM) is one of the methods which is used by many authors to solve partial differential equations arises in different problems and showed the result obtained is much closer to the exact solution. It is a semi-analytical method based on Taylor's series gives a reliable result. This was first proposed by Zhou [14] in 1986 for the solution of linear and nonlinear initial value problems that appear in electrical circuits. Due to its effectiveness and simplicity in the past few years, many authors have applied this method to solve different types of a partial and ordinary differential equation. Tabatabaei, Celik, and Tabatabaei [15] discussed the accuracy of DTM in the problem of heat and wave-like equations. Bagheri and Manafianheris[16] have studied the linear and non-linear Westervelt equation using DTM for three dimensional.

In the present work, we investigate the temperature distribution, thermal stress function and the thermoelastic displacement within a two- dimensional rectangular plate under the hyperbolic heat conduction model. The differential transform method has been used to obtain the solution for temperature field and thermal stress function. A comprehensive study is presented for the special case of a copper plate.

II. MATHEMATICAL MODELING FOR TEMPERATURE FIELD

Consider a finite plate of a rectangular shape whose length is considered along the horizontal axis which varies from x=0 to x=a and breadth along the vertical axis which varies from y=0 to y=b. Also assumed that there is no internal heat generated in the plate and temperature distribution in the plate follows the hyperbolic heat conduction theory which is given by (2).

To find out the temperature field, some initial and boundary conditions are imposed. Which are as follows:

$$T(x,y,0)=0 \tag{3}$$

$$\frac{\partial T(x,y,0)}{\partial t}=f(x,y) \tag{4}$$

$$T(0,y,t)=0 \tag{5}$$

$$\frac{\partial T(a,y,t)}{\partial x}=f_1(y,t) \tag{6}$$

$$T(x,0,t)=0 \tag{7}$$

$$\frac{\partial T(x,b,t)}{\partial y}=f_2(x,t) \tag{8}$$

III. SOLUTION FOR TEMPERATURE FIELD

Solving of the problem defined from (2) to (8) is done semi analytically using Differential transform method[22]. Thus, after applying DTM on (2) to (8) one will get

$$T(\zeta,\eta,\Theta+2)=\frac{1}{\tau_q(\Theta+1)(\Theta+2)}[\kappa(\zeta+1)(\zeta+2)T(\zeta+2,\eta,\Theta)+\kappa(\eta+1)(\eta+2)T(\zeta,\eta+2,\Theta)-(\zeta+1)T(\zeta,\eta,\Theta+1)] \tag{9}$$

$$T(\zeta,\eta,0)=0,T(\zeta,\eta,1)=\mathrm{F}(\zeta,\eta),T(0,\eta,\Theta)=0,\ T(\zeta,0,\Theta)=0 \tag{10}$$

Now using these transformed boundaries and initial conditions in (9), one can obtain $T(\zeta,\eta,\Theta)$ for different values of ζ,η,Θ and which is given as:

$$T(\zeta,\eta,\Theta)=\begin{cases}\dfrac{1}{\tau_q^{\frac{\Theta-1}{2}}\Theta!}\left[\sum_{n=1}^{\frac{\Theta-1}{2}}\kappa^{M}\,{}^{\left(\frac{\Theta-1}{2}+n-1\right)}C_{2n-2}S_M+\dfrac{F(\zeta,\eta)}{\tau_q^{\frac{\Theta-1}{2}}}\right], & \Theta \text{ is odd} > 1\\ \dfrac{(-1)^{\Theta+1}}{\tau_q^{\frac{\Theta}{2}}\Theta!}\left[\sum_{n=2}^{\frac{\Theta}{2}}\kappa^{N}\,{}^{\left(\frac{\Theta}{2}+n-2\right)}C_{2n-3}S_N+\dfrac{F(\zeta,\eta)}{\tau_q^{\frac{\Theta}{2}-1}}\right], & \Theta \text{ is even}\end{cases} \tag{11}$$

Where $T(\zeta,\eta,\Theta)$ is a transformed function of $T(x,y,t)$,

$$S_N=\left(\sum_{r=0}^{N}{}^{N}C_r\frac{(\zeta+2N-2r)!}{\zeta!}\frac{(\eta+2r)!}{\eta!}\frac{F(\zeta+2N-2r,\eta+2r}{\tau_q^{n-2}}\right)$$

$$S_M=\left(\sum_{r=0}^{M}{}^{M}C_r\frac{(\zeta+2M-2r)!}{\zeta!}\frac{(\eta+2r)!}{\eta!}\frac{F(\zeta+2M-2r,\eta+2r}{\tau_q^{n-1}}\right),$$

$\mathrm{M}=\left(\frac{\Theta-1}{2}-n+1\right)$ and $\mathrm{N}=\frac{\Theta}{2}-n+1$

By applying the inverse differential transform, one can obtain a series solution for $T(x,y,t)$ which is as follows:

$$T(x,y,t) = \sum_{\zeta=0}^{\infty}\sum_{\eta=0}^{\infty}\sum_{\Theta=0}^{\infty} T(\zeta,\eta,\Theta)x^{\zeta}y^{\eta}t^{\Theta} \qquad (12)$$

Using $T(\zeta,\eta,\Theta)$ for each value of ζ,η,Θ from (11) in (12) we can get the solution for temperature function.

IV. MATHEMATICAL MODELING FOR THERMAL STRESSES

Considering thermal stress function χ in rectangular coordinates system which is given by $\chi = \chi_c + \chi_p$.

Where χ_c is a complementary function and χ_p is a particular integral.

And χ_c *and* χ_p are satisfied by the equations

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2 \chi_c = 0 \qquad (13)$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right) \chi_p = -\alpha E T \qquad (14)$$

And component of stress function are given by

$$\sigma_{xx} = \frac{\partial^2 \chi}{\partial y^2} \qquad (15)$$

$$\sigma_{yy} = \frac{\partial^2 \chi}{\partial y^2} \qquad (16)$$

$$\sigma_{xy} = \frac{\partial^2 \chi}{\partial x \partial y} \qquad (17)$$

The condition $\lim_{y\to 0, t\to 0} \sigma_{yy} = 0$ and the boundary conditions

$$\sigma_{xx} = 0,\ \sigma_{xy} = 0 \text{ on } x = 0, a \qquad (18)$$

V. SOLUTION FOR STRESS FUNCTION

The complementary function χ_c can be obtained by solving (13).

Let $\chi_c = g(x,y)$ (19)

Solution of particular integral χ_p is obtained by applying the differential transform on (14). The transformed form of (14) is given by

$$(\mu+2)(\mu+1)V(\mu+2,\nu,\omega) + (\nu+2)(\nu+1)V(\mu,\nu+2,\omega) = -\alpha E \sum_{\zeta=0}^{\infty}\sum_{\eta=0}^{\infty}\sum_{\Theta=0}^{\infty} T(\zeta,\eta,\Theta)\,\delta(\mu-\zeta,\nu-\eta,\omega-\Theta) \qquad (20)$$

Where $V(\mu,\nu,\omega)$ is the transformed function of χ_p.

For each μ,ν,ω the values $V(\mu,\nu,\omega)$ by recursive method can be evaluated as:

$$V(\mu,\nu,\omega) = \begin{cases} -\dfrac{\alpha E}{\mu!}\displaystyle\sum_{n=0}^{\frac{\mu-3}{2}} A_n T^*, & \mu \text{ is odd} \\ -\dfrac{\alpha E}{\mu!}\displaystyle\sum_{n=0}^{\frac{\mu-4}{2}} A_n T^*, & \mu \text{ is even} > 2 \end{cases} \qquad (21)$$

Where,

$A_n = (\mu-2-2n)!\dfrac{(\nu+2n)!}{\nu!}(-1)^n$ and

$T^* = T(\mu-2-2n, \nu+2n, \omega)$

Now applying the definition of inverse differential transform one can obtain χ_p as:

$$\chi_p = \sum_{\mu=3}^{\infty}\sum_{\nu=3}^{\infty}\sum_{\omega=2}^{\infty} V(\mu,\nu,\omega)x^{\mu}y^{\nu}t^{\omega} \qquad (22)$$

Using (19) and (22) we will get expression for stress function χ, which is given by

$$\chi = g(x,y) + \sum_{\mu=3}^{\infty}\sum_{\nu=3}^{\infty}\sum_{\omega=2}^{\infty} V(\mu,\nu,\omega)x^{\mu}y^{\nu}t^{\omega} \qquad (23)$$

And now using this value of stress function in (15) ~(17) we will get stress component as:

$$\sigma_{xx} = g_{yy}(x,y) + \sum_{\mu=3}^{\infty}\sum_{\nu=3}^{\infty}\sum_{\omega=2}^{\infty} \nu(\nu-1)V(\mu,\nu,\omega)x^{\mu}y^{\nu-2}t^{\omega} \qquad (24)$$

$$\sigma_{yy} = g_{xx}(x,y) + \sum_{\mu=3}^{\infty}\sum_{\nu=3}^{\infty}\sum_{\omega=2}^{\infty} \mu(\mu-1)V(\mu,\nu,\omega)x^{\mu-2}y^{\nu}t^{\omega} \qquad (25)$$

$$\sigma_{xy} = g_{xy}(x,y) + \sum_{\mu=3}^{\infty}\sum_{\nu=3}^{\infty}\sum_{\omega=2}^{\infty}(\mu-1)(\nu-1)V(\mu,\nu,\omega)x^{\mu-1}y^{\nu-1}t^{\omega} \qquad (26)$$

VI. MATHEMATICAL MODELING FOR DISPLACEMENT FUNCTION

The displacement function [26] for plane stress in a rectangular coordinate is given by

$$U_x = \frac{1}{2G}\left[-\frac{\partial \chi}{\partial x} + \frac{1}{1+\nu}\frac{\partial \psi}{\partial y}\right] \qquad (27)$$

$$U_y = \frac{1}{2G}\left[-\frac{\partial \chi}{\partial y} + \frac{1}{1+\nu}\frac{\partial \psi}{\partial x}\right] \qquad (28)$$

Here G and ν are shear modulus of elasticity and poison's ratio, and ψ satisfy the following equation as

$$\sigma_{xx} + \sigma_{yy} + \alpha E T \equiv \frac{\partial^2 \psi}{\partial x \partial y} \text{ and } \frac{\partial^2}{\partial x \partial y}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2 \psi = 0 \qquad (29)$$

VII. SOLUTION FOR DISPLACEMENT FUNCTION

From (12), (24) and (25) one can obtain

$$\frac{\partial^2 \psi}{\partial x \partial y} = g_{yy}(x,y) + \sum_{\mu=3}^{\infty}\sum_{\nu=3}^{\infty}\sum_{\omega=2}^{\infty}\left(\nu(\nu-1)x^{\mu}y^{\nu-2}t^{\omega} + \mu(\mu-1)x^{\mu-2}y^{\nu}t^{\omega}\right)V(\mu,\nu,\omega) + g_{xx}(x,y) + \alpha E\sum_{\zeta=0}^{\infty}\sum_{\eta=0}^{\infty}\sum_{\Theta=0}^{\infty} T(\zeta,\eta,\Theta)x^{\zeta}y^{\eta}t^{\Theta} \qquad (30)$$

Now taking integration on both side of above (30) with respect to x and y respectively, we can get

$$\frac{\partial \psi}{\partial x} = g_y(x,y) + \sum_{\mu=3}^{\infty}\sum_{\nu=3}^{\infty}\sum_{\omega=2}^{\infty}\left(\nu x^{\mu}y^{\nu-1}t^{\omega} + \mu(\mu-1)x^{\mu-2}\frac{y^{\nu+1}}{\nu+1}t^{\omega}\right)V(\mu,\nu,\omega) + \int g_{xx}(x,y)\,dy + \alpha E\sum_{\zeta=0}^{\infty}\sum_{\eta=0}^{\infty}\sum_{\Theta=0}^{\infty} T(\zeta,\eta,\Theta)x^{\zeta}\frac{y^{\eta+1}}{\eta+1}t^{\Theta} + \phi(x) \qquad (31)$$

$$\frac{\partial\psi}{\partial y}=\int g_{yy}(x,y)\,dx+\sum_{\mu=3}^{\infty}\sum_{\nu=3}^{\infty}\sum_{\varpi=2}^{\infty}\left(\nu(\nu-1)\frac{x^{\mu+1}}{\mu+1}y^{\nu-2}t^{\varpi}+\mu x^{\mu-1}y^{\nu}t^{\varpi}\right)V(\mu,\nu,\varpi)+g_x(x,y)+\alpha E\sum_{\zeta=0}^{\infty}\sum_{\eta=0}^{\infty}\sum_{\Theta=0}^{\infty}\frac{T(\zeta,\eta,\Theta)x^{\zeta+1}}{\zeta+1}y^{\eta}t^{\Theta}+\phi(y) \tag{32}$$

Using (23), (32), (31) in (27) and (28) one can get displacement function, which is given as:

$$U_x=\frac{1}{2G}\Big[-\Big(g_x(x,y)+\sum_{\mu=3}^{\infty}\sum_{\nu=3}^{\infty}\sum_{\varpi=2}^{\infty}\mu V(\mu,\nu,\varpi)x^{\mu-1}y^{\nu}t^{\varpi}\Big)+\frac{1}{1+\nu}\Big(\int g_{yy}(x,y)\,dx+\sum_{\mu=3}^{\infty}\sum_{\nu=3}^{\infty}\sum_{\varpi=2}^{\infty}\Big(\nu(\nu-1)\frac{x^{\mu+1}}{\mu+1}y^{\nu-2}t^{\varpi}+\mu x^{\mu-1}y^{\nu}t^{\varpi}\Big)V(\mu,\nu,\varpi)+g_x(x,y)+\alpha E\sum_{\zeta=0}^{\infty}\sum_{\eta=0}^{\infty}\sum_{\Theta=0}^{\infty}\frac{T(\zeta,\eta,\Theta)x^{\zeta+1}}{\zeta+1}y^{\eta}t^{\Theta}+\phi(y)\Big)\Big] \tag{33}$$

$$U_y=\frac{1}{2G}\Big[-\Big(g_y(x,y)+\sum_{\mu=3}^{\infty}\sum_{\nu=3}^{\infty}\sum_{\varpi=2}^{\infty}\nu V(\mu,\nu,\varpi)x^{\mu}y^{\nu-1}t^{\varpi}\Big)+\frac{1}{1+\nu}\Big(g_y(x,y)+\sum_{\mu=3}^{\infty}\sum_{\nu=3}^{\infty}\sum_{\varpi=2}^{\infty}\Big(\nu x^{\mu}y^{\nu-1}t^{\varpi}+\mu(\mu-1)x^{\mu-2}\frac{y^{\nu+1}}{\nu+1}t^{\varpi}\Big)V(\mu,\nu,\varpi)+\int g_{xx}(x,y)\,dy+\alpha E\sum_{\zeta=0}^{\infty}\sum_{\eta=0}^{\infty}\sum_{\Theta=0}^{\infty}T(\zeta,\eta,\Theta)x^{\zeta}\frac{y^{\eta+1}}{\eta+1}t^{\Theta}+\phi(x)\Big)\Big] \tag{34}$$

VIII. PARTICULAR CASE

Consider $f(x,y)=xy(x^2-3a^2)(y^2-3b^2)$, $g(x,y)=\frac{c_1}{6}x^3+\frac{c_2}{2}x^2y+\frac{c_3}{2}xy^2+\frac{c_4}{6}y^3$

Applying Differential transform on $f(x,y)$ one can get

$$T(\zeta,\eta,1)=\delta(\zeta-3,\eta-3,\Theta-1)-3b^2(\zeta-3,\eta-1,\Theta-1)-3a^2(\zeta-1,\eta-3,\Theta-1)+9a^2b^2(\zeta-1,\eta-1,\Theta-1) \tag{35}$$

Now using (9) ~ (12) and (35) one can obtain the expression for temperature field which is as follow:

$$T(x,y,t)=9a^2b^2xyt-3b^2x^3yt-3a^2xy^3t+x^3y^3t+\sum_{\Theta=2}^{\infty}\Big\{\Big[(-1)^{\Theta+1}(\Theta-3)(\Theta-4)\frac{\alpha^2 2^2 3^2}{\tau_q^{\Theta-3}\Theta!}+3(-1)^{\Theta}(\Theta-2)\frac{\alpha 2.3(a^2+b^2)}{\tau_q^{\Theta-2}\Theta!}+9(-1)^{\Theta-1}\frac{\alpha^2a^2b^2}{\tau_q^{\Theta-1}\Theta!}\Big]xy+\frac{1}{\tau_q^{\Theta-2}\Theta!}\Big[(-1)^{\Theta+1}(\Theta-2)\alpha 2.3+3(-1)^{\Theta}\frac{b^2}{\tau_q}\Big]x^3y+\frac{1}{\tau_q^{\Theta-2}\Theta!}\Big[(-1)^{\Theta+1}(\Theta-2)\alpha 2.3+3(-1)^{\Theta}\frac{a^2}{\tau_q}\Big]xy^3+\frac{(-1)^{\Theta+1}}{\tau_q^{\Theta-1}\Theta!}x^3y^3\Big\}t^{\Theta} \tag{36}$$

Using the assumed $g(x,y)$ and the temperature function obtained in (37) and applying the differential transform to find particular integral of stress function one will get the expression for stress function as:

$$\chi=\frac{c_1}{6}x^3+\frac{c_2}{2}x^2y+\frac{c_3}{2}xy^2+\frac{c_4}{6}y^3-\sum_{\Theta=1}^{\infty}\Big(\frac{T(1,1,\Theta)}{2.3}x^3y+\frac{T(1,3,\Theta)}{2.3}x^3y^3+\frac{T(3,1,\Theta)}{4.5}x^5y-\frac{T(1,3,\Theta)}{4.5}x^5y+\frac{T(3,3,\Theta)}{4.5}x^5y^3\Big)t^{\Theta} \tag{37}$$

Using this value in (15), (16) and (17) one will get the value of stress components as:

$$\sigma_{xx}=c_3x+c_4y-\alpha E\sum_{\Theta=1}^{\infty}\Big(T(1,3,\Theta)x^3y+\frac{6T(3,3,\Theta)}{4.5}x^5y\Big)t^{\Theta} \tag{38}$$

$$\sigma_{yy}=c_1x+c_2y-\alpha E\sum_{\Theta=1}^{\infty}\{T(1,1,\Theta)xy+T(1,3,\Theta)xy^3+(T(3,1,\Theta)-T(1,3,\Theta))x^3y+T(3,3,\Theta)x^3y^3\}t^{\Theta} \tag{39}$$

$$\sigma_{xy}=c_2x+c_3y-\alpha E\sum_{\Theta=1}^{\infty}\Big\{\frac{T(1,1,\Theta)}{2}x^2+\frac{3T(1,3,\Theta)}{2}x^2y^2+(T(3,1,\Theta)-T(1,3,\Theta))\frac{x^4}{4}+\frac{3T(3,3,\Theta)}{4}\frac{x^4y^2}{4}\Big\}t^{\Theta} \tag{40}$$

Now applying the boundary condition defined in (18) we will get

$$c_1=c_3=c_4=0$$

$$c_2=\alpha E\sum_{\Theta=1}^{\infty}\Big\{\frac{T(1,1,\Theta)}{2}a+(T(3,1,\Theta)-T(1,3,\Theta))\frac{a^3}{4}\Big\}t^{\Theta}$$

Putting these value of constant in (38), (39) & (40) one can obtain the expression for thermal stress components as:

$$\sigma_{xx}=-\alpha E\Big[\Big(-3a^2t+\frac{3a^2}{\tau_q 2!}t^2\Big)x^3y+\frac{6}{4.5}\Big(t-\frac{t^2}{\tau_q 2!}\Big)x^5y+\sum_{\Theta=3}^{\infty}\Big(T(1,3,\Theta)x^3y+\frac{6T(3,3,\Theta)}{4.5}x^5y\Big)t^{\Theta}\Big] \tag{41}$$

$$\sigma_{yy}=\alpha E\Big[\Big(9a^2b^2t-\frac{9a^2b^2}{\tau_q 2!}t^2\Big)\Big(\frac{ay}{2}-xy\Big)-3a^2t+\frac{3a^2}{\tau_q 2!}t^2\Big)\Big(\frac{a^3y}{4}-x^3y+xy^3\Big)+\Big(-3b^2t+\frac{3b^2}{\tau_q 2!}t^2\Big)\Big(\frac{a^3y}{4}-x^3y\Big)-\Big(t-\frac{t^2}{\tau_q 2!}\Big)x^3y^3+\sum_{\Theta=3}^{\infty}\Big\{T(1,1,\Theta)\Big(\frac{ay}{2}-xy\Big)-T(1,3,\Theta)\Big(\frac{a^3y}{4}-x^3y+xy^3\Big)+T(3,1,\Theta)\Big(\frac{a^3y}{4}-x^3y\Big)-T(3,3,\Theta)x^3y^3\Big\}t^{\Theta}\Big] \tag{42}$$

$$\sigma_{xy}=\alpha E\Big[\Big(9a^2b^2t-\frac{9a^2b^2}{\tau_q 2!}t^2\Big)\Big(\frac{ay}{2}-\frac{x^2}{2}\Big)-3a^2t+\frac{3a^2}{\tau_q 2!}t^2\Big)\Big(\frac{a^3x}{4}+\frac{3x^2y^2}{2}-\frac{x^4}{4}\Big)+\Big(-3b^2t+\frac{3b^2}{\tau_q 2!}t^2\Big)\Big(\frac{a^3x}{4}-\frac{x^4}{4}\Big)-\Big(t-\frac{t^2}{\tau_q 2!}\Big)\frac{3x^4y^2}{4}+\sum_{\Theta=1}^{\infty}\Big\{T(1,1,\Theta)\Big(\frac{ay}{2}-\frac{x^2}{2}\Big)+T(3,1,\Theta)\Big(\frac{a^3x}{4}-\frac{x^4}{4}\Big)-T(1,3,\Theta)\Big(\frac{a^3x}{4}+\frac{3x^2y^2}{2}-\frac{x^4}{4}\Big)-T(3,3,\Theta)\frac{3x^4y^2}{4}\Big\}t^{\Theta}\Big] \tag{43}$$

To calculate displacement function given by (27)&(28) ,firstly one must calculate $\frac{\partial\psi}{\partial x}$ & $\frac{\partial\psi}{\partial y}$ which can be determined by taking integration on eq.(29) with respect to x&y respectively, Therefore

$$\frac{\partial\psi}{\partial x}\equiv-\alpha E\sum_{\Theta=1}^{\infty}\Big\{\frac{3T(3,3,\Theta)}{4.5}x^5y^2-\frac{T(1,1,\Theta)}{4}ay^2-\frac{T(3,1,\Theta)}{8}a^3y^2+\frac{T(1,3,\Theta)}{8}a^3y^2\Big\}t^{\Theta}+\emptyset(x)$$

$$\frac{\partial\psi}{\partial y}\equiv-\alpha E\sum_{\Theta=1}^{\infty}\Big\{\frac{T(3,3,\Theta)}{4.5}x^6y-\frac{T(1,1,\Theta)}{2}axy-\frac{T(3,1,\Theta)}{4}a^3xy+\frac{T(1,3,\Theta)}{4}a^3xy\Big\}t^{\Theta}+\emptyset(y)$$

Using the above values in (27) &(28) and neglecting the rigid deformation ,one can get the displacement function as:

$$U_x = \frac{-\alpha E}{2G}\Bigg[\left(9a^2b^2t - \frac{9a^2b^2}{\tau_q 2!}t^2\right)\left(\frac{av}{v+1}\frac{xy}{2} - \frac{x^2y}{2}\right) - \left(3b^2t - \frac{3b^2}{\tau_q 2!}t^2\right)\left(\frac{a^3v}{v+1}\frac{xy}{4} - \frac{x^4y}{4}\right) + \left(3a^2t - \frac{3a^2}{\tau_q 2!}t^2\right)\left(\frac{a^3v}{v+1}\frac{xy}{4} + \frac{x^2y^3}{2} - \frac{x^4y}{4}\right) + \left(t - \frac{t^2}{\tau_q 2!}\right)\left(\frac{1}{v+1}\frac{x^6y}{20} - \frac{x^4y^3}{4}\right) + \sum_{\Theta=3}^{\infty}\left\{T(1,1,\Theta)\left(\frac{av}{v+1}\frac{xy}{2} - \frac{x^2y}{2}\right) + T(3,1,\Theta)\left(\frac{a^3v}{v+1}\frac{xy}{4} - \frac{x^4y}{4}\right) - T(1,3,\Theta)\left(\frac{a^3v}{v+1}\frac{xy}{4} + \frac{x^2y^3}{2} - \frac{x^4y}{4}\right) + T(3,3,\Theta)\left(\frac{1}{v+1}\frac{x^6y}{20} - \frac{x^4y^3}{4}\right)\right\}t^{\Theta}\Bigg] \tag{44}$$

$$U_y = \frac{-\alpha E}{2G}\Bigg[\left(9a^2b^2t - \frac{9a^2b^2}{\tau_q 2!}t^2\right)\left(\frac{x^2a}{4} - \frac{1}{v+1}ay^2 + \frac{x^3}{6}\right) - \left(3b^2t - \frac{3b^2}{\tau_q 2!}t^2\right)\left(\frac{x^3a^3}{8} - \frac{a^3}{v+1}\frac{y^2}{8} + \frac{x^5}{20}\right) + \left(3a^2t - \frac{3a^2}{\tau_q 2!}t^2\right)\left(\frac{x^2a^3}{8} - \frac{a^3}{v+1}\frac{y^2}{8} + \frac{x^5}{20} - \frac{x^3y^2}{2}\right) + \left(t - \frac{t^2}{\tau_q 2!}\right)\left(\frac{v+2}{v+1}\frac{3x^5y^2}{20}\right) + \sum_{\Theta=3}^{\infty}\left\{T(1,1,\Theta)\left(\frac{x^2a}{4} - \frac{1}{v+1}ay^2 + \frac{x^3}{6}\right) + T(3,1,\Theta)\left(\frac{x^3a^3}{8} - \frac{a^3}{v+1}\frac{y^2}{8} + \frac{x^5}{20}\right) - T(1,3,\Theta)\left(\frac{x^2a^3}{8} - \frac{a^3}{v+1}\frac{y^2}{8} + \frac{x^5}{20} - \frac{x^3y^2}{2}\right) + T(3,3,\Theta)\left(\frac{v+2}{v+1}\frac{3x^5y^2}{20}\right)\right\}t^{\Theta}\Bigg] \tag{45}$$

Where value of T for $\Theta \geq 3$ is given by

$$T(1,1,\Theta) = (-1)^{\Theta+1}(\Theta-3)(\Theta-4)\frac{k^2 2^2 3^2}{\tau_q^{\Theta-3}\Theta!} + (-1)^{\Theta}(\Theta-2)\frac{18k(a^2+b^2)}{\tau_q^{\Theta-2}\Theta!} + (-1)^{\Theta-1}\frac{9a^2b^2}{\tau_q^{\Theta-1}\Theta!}$$

$$T(3,1,\Theta) = (-1)^{\Theta+1}(\Theta-2)\frac{6k}{\tau_q^{\Theta-2}\Theta!} + (-1)^{\Theta}\frac{3b^2}{\tau_q^{\Theta-1}\Theta!}$$

$$T(1,3,\Theta) = (-1)^{\Theta+1}(\Theta-2)\frac{6k}{\tau_q^{\Theta-2}\Theta!} + (-1)^{\Theta}\frac{3a^2}{\tau_q^{\Theta-1}\Theta!}$$

$$T(3,3,\Theta) = (-1)^{\Theta+1}\frac{1}{\tau_q^{\Theta-1}\Theta!}$$

Table- I: Properties of Copper

α	$17 \times 10^{-6}(\,^{\circ}C)^{-1}$
τ_q	$0.02s$
ν	0.33
G	4.5×10^{10}
E	$1.19 \times 10^{11} N/m^2$
k	$1.1283 \times 10^{-4} m^2/s$

IX. NUMERICAL RESULT AND GRAPHICAL DISCUSSION

To obtain the numerical results, the copper plate is used for which material parameter are given in Table- I [18]. The stress and displacement variation along the spatial direction is determined and plotted. All the values are calculated taking a=1cm, b=2cm at $t = 1s$.

Fig. 1 & 2 shows the thermal stress variation occurred in mid-point of the plate corresponding to different values of x and y respectively. It seems that stress is initially zero in both the cases but decreasing exponentially as moving along the x-direction and decreases linearly in multiples of 10^{-3} as we move along the y-direction. The figure shows that compressive stress occurs along x-direction.

Fig. 3 &4 show the graph of sigmayy vs x and sigmayy vs y. As we increase the value of x the compressive stress along y-direction occurs in a plate. The compressive stress occurs in a range of 10^{-14}. But the tensile stress occurs along y-direction as the value of y increases.

Fig. 5& 6 shows the variation of sigma xy for increasing values of x and y respectively. Initially, sigma xy increases along x-direction and from mid of the plate, it starts decreasing as touch at the end of the plate. Figure 6 shows that initially there is some stress in a plate and as we move in y-direction towards the end of the plate the stress increases and reach to a maximum level.

Fig. 7&8 shows the displacement along horizontal direction for increasing values of x and y in two different graphs. As increasing the value of x the displacement in a plate decreases in a negative direction. But as increasing the value of y the displacement in a plate increases in a positive direction. The value of displacement along x-direction in both the cases is occurring in multiples of 10^{-14}.

Fig. 9 shows the displacement in a plate along with the y-direction versus x and fig. 10 shows the displacement along y-direction versus y. In both cases, we get negative displacement. In fig. 9 it seems that displacement rapidly decreases as we move in horizontal direction. The displacements values are observed in terms of10^{-13}. In fig.10 it is noted that initially, displacement is decreased slowly as it reaches to mid of the plate the value of displacement decreases rapidly. Here the values of displacement are in terms of 10^{-15}.

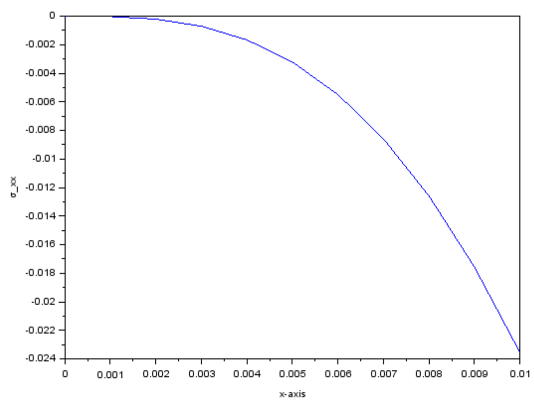

Fig.1 Stress distribution σ_{xx} against x.

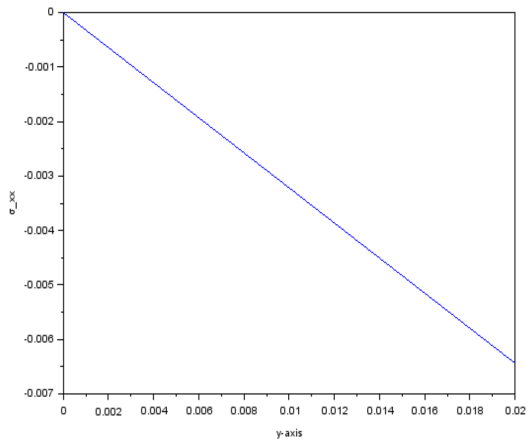

Fig.2 Stress distribution σ_{xx} against y.

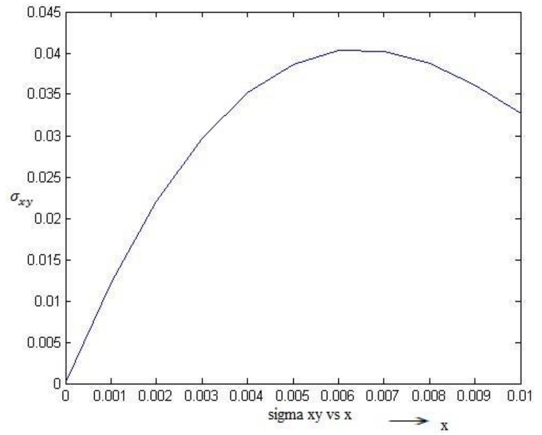

Fig.5 Stress distribution σ_{xy} against x

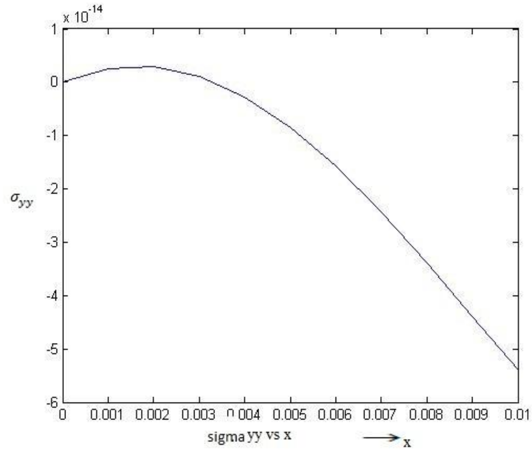

Fig.3 Stress distribution σ_{yy} against x.

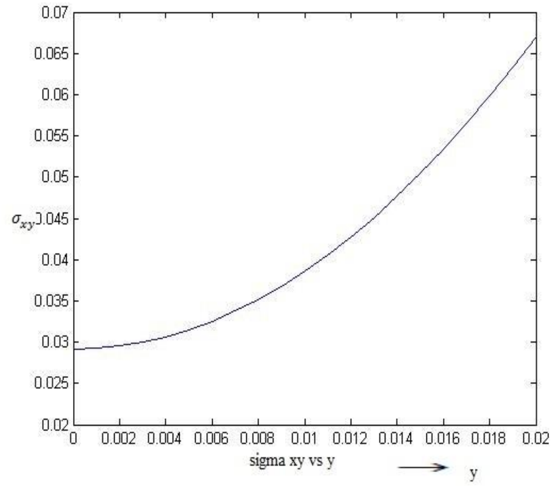

Fig.6 Stress distribution σ_{xy} against y.

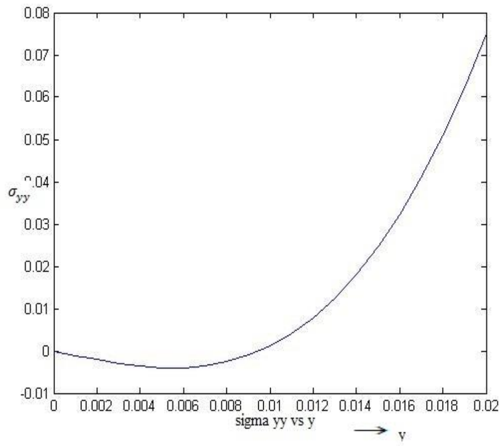

Fig.4 stress distribution σ_{yy} against y.

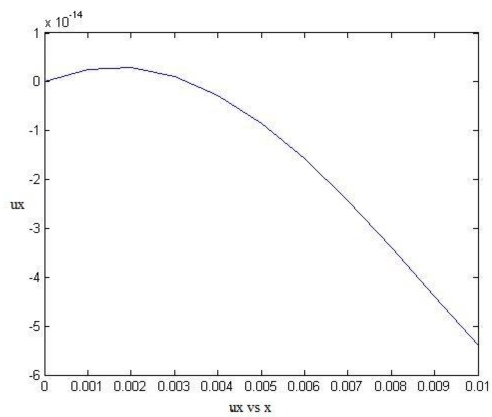

Fig.7 Displacement U_x along x-direction.

Retrieval Number: F90471088619/2019©BEIESP
DOI: 10.35940/ijeat.F9047.109119

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

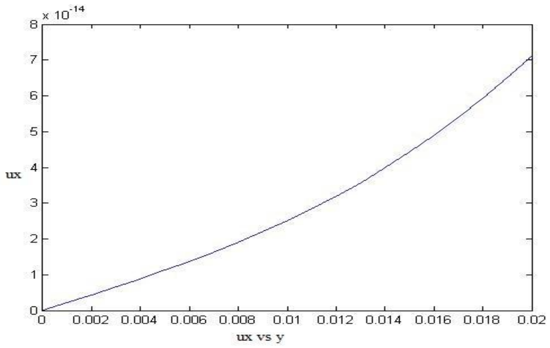

Fig.8 Displacement U_x along y-direction.

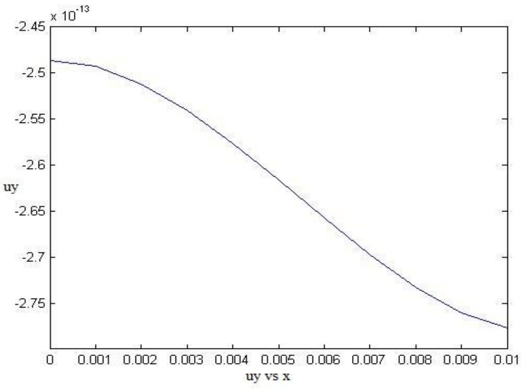

Fig.9 Displacement U_y along x-direction

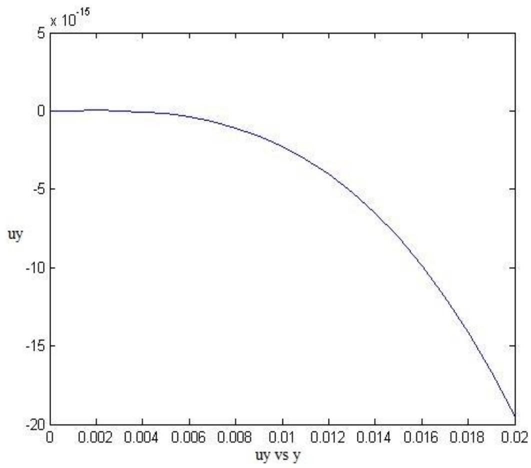

Fig.10 Displacement U_y along y-direction.

X. CONCLUSION

In the present work, a thermoelastic response of a finite rectangular plate with no internal heat generation is modeled. In the problem, the homogeneous body is considered. Thermal behaviour of the plate is studied under the transient hyperbolic heat conduction model. The boundary condition is taken in such a manner so that one end in both the direction is kept insulated. Also, the slope of temperature field is considered as a function of spatial coordinates.

The governing heat conduction problem is solved using differential transform method which gives the approximate analytical solution in infinite series form. In this method firstly the PDE is transformed into an algebraic equation and then solve using recursive method and finally, inverting the solution of the algebraic equation to get an approximate result in the form of infinite series. This method is reliable and simple for solving such heat conduction equations.

Thermal stresses and displacement under the effect of relaxation time, developed in the plate is also calculated by differential transform method for the general loading. Using this one can obtain the special case of interest. Numerical results for the special case are also determined and represent graphically. The result also shows the effect of relaxation time on thermally induced stresses and displacement. From the numerical result and discussion, it is observed that

(i) Stress component σ_{xx} shows the exponential behaviour along horizontal direction and behave linearly along vertical direction due to the effect of phase lag in heat flux.

(ii) Stress component σ_{yy} shows the compressive and tensile stress along x and y direction respectively.

(iii) Stress component σ_{xy} fluctuates up and down along x-axis and continuously increases along y-direction due to the traction- free boundary condition.

(iv) Displacement U_x is in the opposite direction as we move along the x-direction and it is in the same direction as we move along the y-direction.

(v) Displacement U_y decreases in both the direction, but the rate of decreasing are different.

REFERENCES

1. Cattaneo, C., "A Form of Heat Conduction Equation Which Eliminates the Paradox of Instantaneous Propagation", *CompteRendus*, **247**, pp. 431-433 (1958).
2. Vernotte, M.P., "Les paradoxes de la theorie continue del equation de la chaleur", *Comptes Hebdomadaires Des Seances De Academie Des Science*, **246**, pp. 3154-3155 (1958).
3. Chen, P. Y. P., "Transient thermal stresses in rectangular plate due to nonuniform heat transfer coefficient", *Journal of Thermal Stresses*, **11:2**, pp. 115-125(1988).
4. Morimoto,T. , Tanigawa,Y. and Kawamura,R. "Thermal bukling analysis of inhomogeneous rectangular plate due to uniform heat supply",*Journal of Thermal Stresses*,**26:11-12**,1151-1170(2003).
5. Kaminski, W. "Hyperbolic Heat Conduction Equation for Materials with a Nonhomogeneous Inner Structure", *Journal of Heat Transfer*, **112**, 555-560(1990).
6. Chen,H.T. and lin,J.Y., "Analysis of two-dimensional hyperbolic heat conduction problems", *International journal of heat mass transfer*", **37**, No. I, pp. 153-164(1994).
7. Barletta, A. and Zanchini, E., "Three dimensional propagation of in a solid bar with rectangular cross section hyperbolic thermal waves", *International Journal of Heat and Mass Transfer*, **42**,219-229(1999).
8. Yang,C.Y., ''Direct and inverse solution of Hyperbolic heat conduction problem'',*Journal of Thermophysics and heat transfer*,**19**,no.2, 2005.
9. Moosaie,A., Atefi,G., Fardad,A.A., ''Two-dimensionalnon-Fourier heat conduction with arbitrary initial and periodic boundary conditions'', *Forsch Ingenieurwes*,**72:** ,pp. 67–76,DOI 10.1007/s10010-008-0068-x(2008).
10. Wang, B., "One-dimensional Temperature and Stress Distributions Associated With Hyperbolic Heat Conduction'', *Applied Mechanics and Materials*, **249-250** ,pp. 962-967(2013).
11. Mallick,A., Ranjan,R., Sarkar, P.K., "Effect of heat transfer on thermal stresses in an annular hyperbolic fin: an approximate analytical solution", *Journal of theoretical and applied mechanics*, **54**, 2, pp. 437-448, Warsaw (2016), DOI: 10.15632/jtam-pl.54.2.437.
12. Noroozi,M.J. and Goodarzi,M., ''Nonlinear analysis of a non-Fourier heat conduction problem in a living tissue heated by laser source''**,** *International Journal of Biomathematics*,**10**, No. 8,1750107, DOI:

10.1142/S1793524517501078(2017).

13. Nie, B.D., Cao, B.Y., ''Three mathematical representations and an improved ADI method for hyperbolic heat conduction'', *International Journal of Heat and Mass Transfer*, **135**, pp. 974–984(2019).
14. Zhou, J.K, "Differential Transformation and Its Applications for Electrical Circuits" (in Chinese), *Huazhong Univ. Press*, Wuhan, China (1986).
15. TABATABAEI, K. , CELIK,E., and TABATABAEI,R., " The differential transform method for solving heat-like and wave like equations with variable coefficients" ,*Turk J Phys*, **36** ,pp.87-98(2012).
16. Bagheri, M. and Manafianheris, J., "Deferential Transform Method for Solving the Linear and Nonlinear Westervelt Equation'', *Journal of Mathematical Extension*, **6**, No. 3, pp. 81-91(2012).
17. Noda, N., Hetnarski, R.B., Tanigawa, Y.,*Thermalstresses*,Second Edition,Taylor &Francis,London,UK,pp.190-192(2002).
18. Sherief,H.H. & Anwar,M.N., ''Two-dimensional generalized thermoelasticity problem for an infinitely long cylinder'', *Journal of Thermal Stresses*, **17:2**, 213-227, DOI: 10.1080/01495739408946256(1994)

AUTHORS PROFILE

Kamini K Chaudhari obtained her Bachelor degree in electronics in 2001 from Jiwaji University,Gwalior,M.P. she completed her Post graduation in Mathematics from Jiwaji university, Gwalior,Madhya Pradesh (India). She is qualified SET (Maharashtra)in december 2013.She is pursing her doctoral studies at KBC North Maharashtra University Jalgaon, Maharashtra (India).She has 13 years of teaching experience in engineering colleges. Currently, she is working as a assistant professor in Pillai College of engineering,New Panvel,Navi Mumbai ,Maharashtra (India). She has presented papers in 3 international conferences and published one research paper in international journal. She is a life member of ISTE.

Dr.Chandrashekhar S Sutar obtained his Ph.D. degree from KBCNMU, Jalgaon Maharashtra. He has 11 years of experience in engineering and science colleges. Currently, working as a assistant professor in PSGPVM's A.S.C. College, Shahada, Maharashtra (India) .He is recognized research supervisior at KBCNMU,Jalgaon,Maharashtra (India).Currently three students are pursuing Ph.D. under his supervision. His research interest includes thermoelasticity and integral transform. He has published eight papers in international journal and actively participated in national and international conferences.