

# Fractal Geometry and Its Application to Antenna Designs

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**Abstract:** This paper primarily focuses on various fractal geometries and their applications to antenna designs. Several natures inspired and human inspired fractal geometries are presented one by one. Their importance and design procedure are also briefly discussed. The dimensions of such fractal geometries are found using their mathematical modeling. Considering modeling and their corresponding shapes various low profiles, low cost, small size and, light weight antenna designs for various wireless applications are described. The broadband, wideband, and multiband nature of the design due to fractal application are discussed. Finally advantages, disadvantages, major applications, and future scope of such fractal geometries are mentioned.

**Keywords:** Fractal geometries, Sierpinski carpet, Sierpinski gasket, IFS, Koch Curve, Hilbert Curve.

## I. INTRODUCTION

Now a day, the wireless communication devices become portable due to implementation of the recent technologies. Antenna is a major element used in communication devices. Hence, antenna minimization is the latest research topic for many researchers. Antenna minimization depends on the relationship between the physical sizes of the antenna with its operational wavelength. This relationship is a major and most essential parameter in the area of antenna design. The physical size of antenna element is inversely proportional to its operational frequency. But when the physical size of antenna is reduced then its electrical size also reduces as there is no change in the operational frequency. Again the electrical size of antenna is expressed in terms of the operational wavelength ( $\lambda$ ). The physical size of an antenna is normally considered as the half or quarter of its operational wavelength. The antenna operates satisfactorily over the range of frequencies called as the bandwidth which is generally 10-40% of the center wavelength. But when the dimensions of the antenna become much smaller than its operating wavelength then it reduces the radiation resistance, S11 parameters, bandwidth radiation performance and efficiency of the antenna. Some common examples of antennas with the quarter-wavelength of the electrical size are monopole antennas, helical antennas and planar inverted-F antennas (PIFAs) [1].

The fractal geometries have wide applications in the field of biology, geography and engineering. In the field of engineering, fractal geometries have been used in the process of antenna designs, frequency selective surface designs, image processing and bio medical signal processing. The concept of fractal antenna theory is a relatively new research area in the field of antenna design. But due to various attracting features the Fractal antennas and the corresponding superset fractal electrodynamics is a major attraction of current research activity [2]. Fractal geometries are considered as the complex geometric shapes with self-similarity, self-scaling and space-filling properties. These properties make them a suitable candidate in miniaturized antenna designs. The Space-filling property results in electrically large size features. Self-similar property enables use of iteration function system with similar shapes. Self-scaling property allows iteration function system to use similar shapes of multiple scales. These features enable them to be efficiently packed, thus easily represented into small areas. The antenna miniaturization process can be achieved through the implementation of self scaling, space filling and self similarity properties of fractals that produces the curves which are electrically very long with a compact structured physical space [3]. Due to self-similarity, self scaling and space filling properties, fractal geometries are widely used in Fractal antenna designs. When Fractal antennas are compared with the conventional antenna, then it is found that the fractal antennas have much greater bandwidth with very compact antenna size. By using the fractal antennas multiple resonant frequencies can be achieved which are multiband but are not harmonics in nature [4]. Hence, antenna designs based on fractal geometries are suitable for various wireless applications.

The theoretical and conceptual foundations of antennas were laid on famous Maxwell's equations. The Scottish scientist James Clark Maxwell observed the theories of electricity and magnetism in 1873 and eventually represented their relationship through a set of mathematical equations called as Maxwell's Equations. And in 1886 German scientist Heinrich Rudolph Hertz verified the Maxwell's Equations and invented that the electrical disturbances could be detected with a secondary circuit of particular dimensions for resonance and contains an air gap for occurrence of sparks [5]. The Italian scientist Guglielmo Marconi designed a microwave device of parabolic cylindrical shape at a particular wavelength of 25 cm for his original code transmission and further worked at larger wavelengths for improvement in the communication range. Hence the Marconi is regarded as the "father of amateur radio". In the early years the antenna developments were limited by the availability of signal generators.

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But in 1920, the resonant length antennas were invented when the De Forest triode tube was introduced to produce the continuous wave signals ranging up to 1MHz [6].

The term 'fractal' means wrecked or broken or irregular segments. This term 'fractal' was invented by the French scientist Dr. B Mandelbrot in 1975. The term 'fractal' was first derived from the word frangee (i.e. a Greek word) and the word fractus (i.e. a Latin word) which means wrecked or broken or irregular segments. Dr. B Mandelbrot investigated that there exists a fundamental relationship between the fractal dimensions and the nature patterns that exist in nature [7].

In 1988, the first type of the fractal antenna was designed by Canadian scientist Dr. Nathan Cohen. And he has suggested that the Fractal antenna is a new type of antenna which is simple and easy to manufacture, follows self-similar and self-repetitive characteristic, thus could be suitable in military as well as commercial applications. Dr. Nathan Cohen has introduced the new concept on fractalization of various geometries of a dipole or loop antenna initially. This concept suggests in bending of a wire in such a fractal way that the entire length of the particular antenna remains the same but the antenna size is reduced with the addition process of continuous iterations. When this fractal concept is properly implemented then an efficient technique of miniaturized antenna design is possible. Dr. Nathan Cohen compared the perimeter of a particular Euclidean antenna with a fractal shaped antenna and stated that the fractal antenna follows a perimeter which is not directly proportional to the antenna area. Also he has concluded that in multi-iteration fractal geometry the antenna area is smaller than an Euclidean shaped antenna [8-9].

The concept of fractal dimension is very old. Several fractal geometries are inspired from nature, human. These nature and human inspired fractal geometries are widely used in various science and engineering fields. Fractal geometries are characterized in terms of their corresponding dimensions. The fractal dimensions are determined using their mathematical modeling. The mathematical modeling are specified in the form of mathematical expressions. Various shapes based on these mathematical expressions are used to design small size, low profile, low cost, and light weight antenna designs. Fractal geometries provide multiband, wideband, and broadband nature in the antenna designs. Fractal geometries are currently major cause of interest for various researchers in the field of science and engineering due to their key advantages. Hence, these make fractal geometries a suitable candidate in wireless communication for various major applications.

Our research goal is to discuss the concept and various applications of fractal geometry to antenna structures. The revolution of fractal geometry is shown in section 2. Section 3 represents the mathematical modeling of various fractal geometries. The detailed literature review is done in section 4. Section 5 contains the overall discussion of major features of fractal geometry. The conclusion of paper is presented in the section 6. Eventually, the future work of this work is presented in section 7.

## II. REVOLUTION OF FRACTAL GEOMETRY

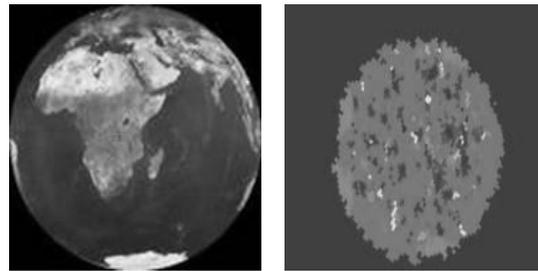
The term "fractal" may be described as any geometric object i.e. it may be a line or a rectangle or a circle which is

irregular or rough in terms of length or size. So, it looks like a 'broken up' shape in a systematic and thorough way. In fractal geometry, the original object is sub-divided into several individual parts where each part is very similar to the original one. This property is called as self-similar property which is occurring at various stages of magnification. In fractal geometry, the original object is scaled with different dimensions which are called as scaling property. So the natural objects are usually self-similar that makes fractal structures suitable in field of antenna design [10].

### 2.1 Fractal Geometry: Inspiration through nature to human

Various fractal structures are inspired from the nature. Following are some typical example of fractal structures that are inspired through nature to human body.

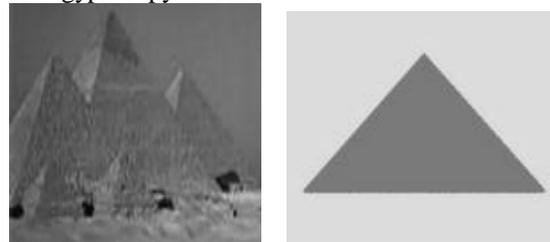
We know that the Earth is the only one planet of the solar system where the Life is possible. There are four multiple layers present inside the earth. Here the fractal shown in the following figure represents the super formula which is near  $c=0$  with 215 iterations.



**Fig.1: The Earth and its corresponding fractal shape.**

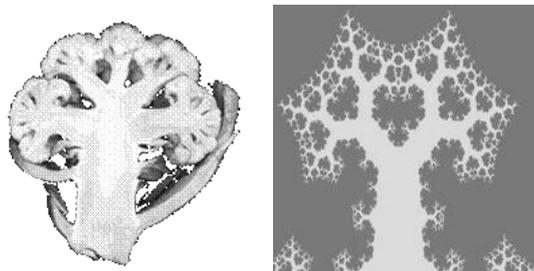
The Egyptian Pyramids were constructed following the images of the star's positions in the sky. Hence there lies a correlation between the earth and the sky.

So we may assume that the pyramid shapes were found as the first similarity with the fractal structures. And the Sierpinski gasket fractal antenna structure is very similar to the Egyptian pyramids.



**Fig.2: The Pyramid and its corresponding fractal shape.**

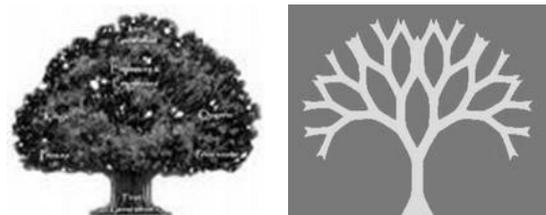
There are some vegetables like Cauliflowers and broccoli that possess like a fractal tree-shaped typical structure. Here the fractal geometry is designed by using an if and else equation and the individual branches on each stems are originated by using the power terms of the factors like  $(z_n + 1)$  or  $(z_n - 1)$ .



**Fig.3: The Fungus and its corresponding fractal shape**

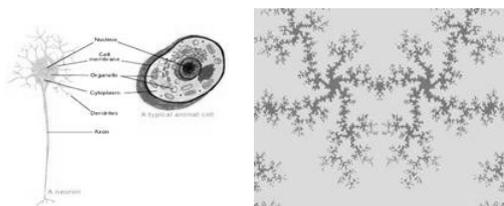
We know that a tree structure is the simplest example of fractal dimension in the living world of the nature from the biology. Here a mathematical formula is stated below that generates this fractal form. And the iteration numbers are related to the branch numbers.

$$z_{n+1} = (z_n + 1)/c \text{ or } z_{n+1} = (z_n - 1)/c \text{ for } x > 0 \text{ and } x < 0 \text{ respectively}$$



**Fig.4: The Tree and its corresponding fractal shape**

We know that the nerve cells or Neurons are the cells those are electrically excitable belonging to nervous system used for the processing of the transmitting information. And the neurons are mainly combination of a cell body called as soma which is a dendrite tree with an axon. Here the used fractal formula follows the form of if and else formula which is the combination of the terms like  $\sin(n)-1$  and  $\sin(n)+1$ .



**Fig.5: The Neurons and its corresponding fractal shape**

Continuous scale-invariance is a property containing un particles but not particles those are used to interpret by fractal dimensions of iteration of various complex functions. Koch Curve is a such type of fractal structure which is a case of discrete scaled invariance property as it remains the same when multiplied with a constant number.



**Fig.6: The un particles and its corresponding fractal shape**

## 2.2 Types of fractal geometry commonly used in the field of mathematics and sciences

### a) Barnsley Fern as Fractal geometry

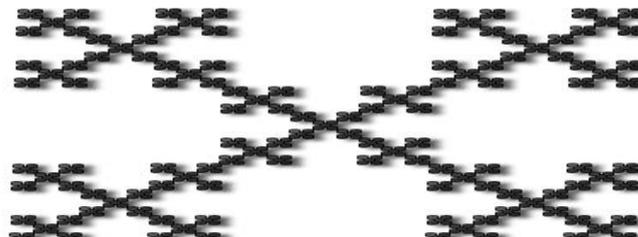
Barnsley Fern fractal geometry is shown in the following figure and it follows the self similarity pattern up to a large extent.



**Fig.7: Example of Fractal as of Barnsley Fern**

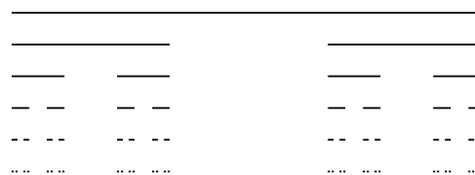
### b) The famous Box as Fractal geometry

The famous Box as fractal geometry follows box shapes as shown in the following figure.



### c) The Cantor Set as Fractal geometry

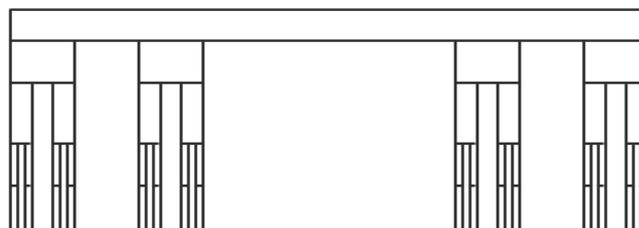
The Cantor Set as fractal geometry is generated by cutting a single line from its centre repeatedly as shown in the following figure.



**Fig.9: Example of fractal geometry made up by the Cantor Set**

### d) The Cantor comb as Fractal geometry

The Cantor Comb as fractal geometry follows a comb structure as shown in the following figure.

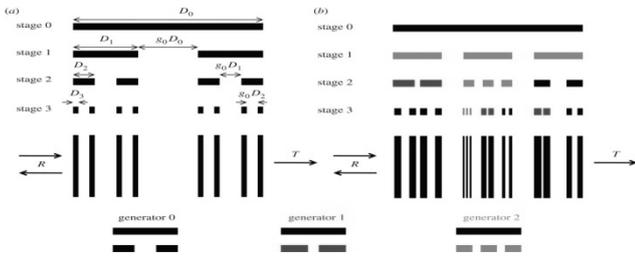


**Fig.10: Example of fractal geometry made up by the Cantor Comb**

### e) The Cantor Curtains as Fractal geometry

The Cantor Curtains as fractal geometry is generated by making a gap along the single line from its centre and the process is repeatedly for the multiple iterations as shown in the following figure.

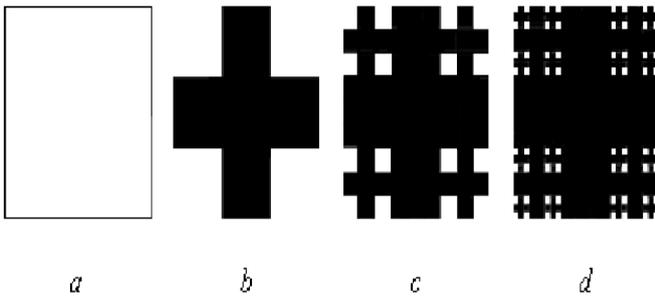
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**Fig.11: Example of fractal geometry made up by the Cantor Curtains**

## f) The Cantor Square as Fractal geometry

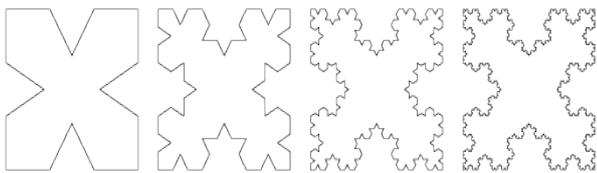
The Cantor Square as fractal geometry is generated by taking a Plus shaped structure as the basic shape. Again four numbers of plus shaped structure is created in second iteration and sixteen numbers of squares are created and this procedure continues for further iterations as shown in the following figure.



**Fig.12: Example of fractal geometry made up by the Cantor Square**

## g) The Cesaro Sweep as Fractal geometry

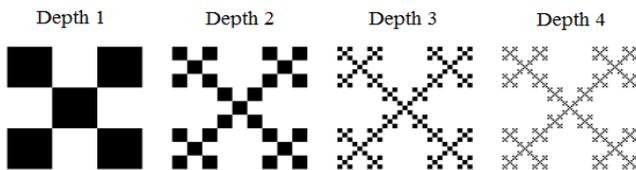
The Cantor Square as fractal geometry is generated by making four triangular slots of a square along midpoint of each side. Then the same process is repeated for multiple iterations as shown in the following figure.



**Fig.13: Example of fractal geometry made up by the Cesaro Sweep**

## h) Vicsek snowflake-box fractal

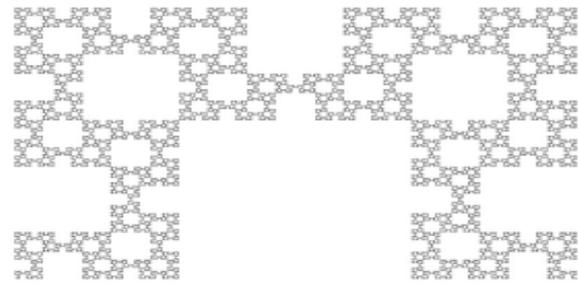
The Vicsek snowflake-box fractal as fractal geometry is generated by taking a square as the basic shape. In the first iteration four numbers of squares are placed at each corners of the square and the process is repeated for multiple iterations as shown in the following figure.



**Fig.14: Example of fractal made up by the Vicsek snowflake-box fractal**

## i) Hausdorff Dimension Fibonacci Fractals

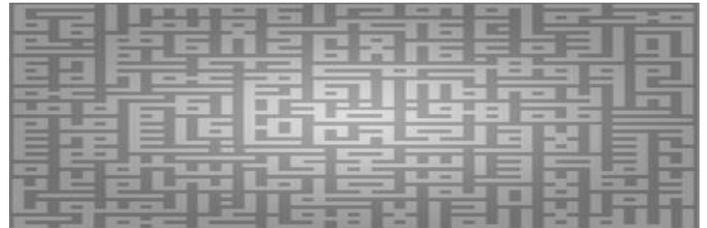
The Hausdorff Dimension Fibonacci Fractals as fractal geometry is generated by taking a triangle as the basic shape as shown in the following figure.



**Fig.15: Example of fractal made up by the Hausdorff Dimension Fibonacci Fractals**

## j) The Hilbert Curve as Fractal geometry

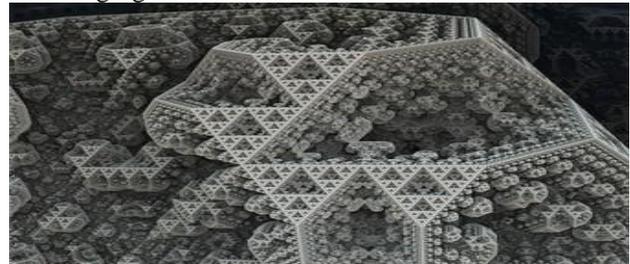
The Hilbert curve as fractal geometry follows a Hilbert curve structure as shown in the following figure.



**Fig.16: Example of fractal geometry made up by the Hilbert Curve**

## k) The Sierpinski Pyramid as Fractal geometry

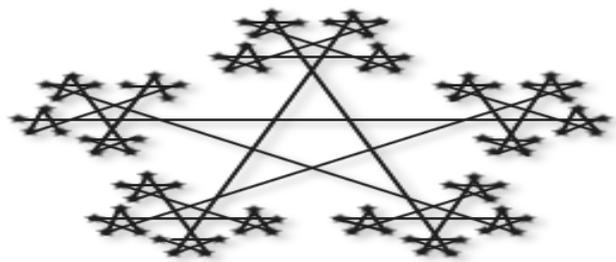
The Sierpinski Pyramid as fractal geometry follows a triangular shape and generates a three dimensional structural space of a form of pyramidal shape as shown in the following figure.



**Fig.17: Example of fractal made up by the Sierpinski Pyramid**

## l) The Star Fractal as Fractal geometry

The star fractal as fractal geometry follows a star shape structure with self similarity property in a particular direction.



**Fig.18: Example of fractal made up by the Star Fractal**

## 2.3 Iterated Function System Fractals

Fractal geometries are very complex in nature which is generated from a single formula using multiple iterations. Here, one formula is generally repeated again and again with a little different value.



This repetition process results in generation of similar structures from the previous iteration. Fractal geometries using the Iterated Function System (IFS) method are generated based on the simple analytical methods of plane transformations like up scaling method, down scaling method, Dislocation of objects and the plane of axes rotation methods. The IFS fractal geometry creation consists of the following steps [11] as follows.

**Step 1:** Defines a set of planes that can be used for transformations.

**Step 2:** Draws any initial geometry pattern on the plane.

**Step 3:** The transformation of the initial geometry pattern using the transformation process stated in the first step.

**Step 4:** The transformation of the newly created picture which is the combination method of initial and transformed patterns by using the same method and same set of transformations.

**Step 5:** Repetition of the fourth step more and more as many times as possible i.e. theoretically infinite number of times.

Some of commonly used IFS fractals are the Koch curve, Sierpinski Carpet, Sierpinski Gasket, Minkowski curve and the Koch Snowflake etc.

### III. CALCULATION OF FRACTAL DIMENSION

#### 3.1 Fractal Dimensions

We know that the term Dimension can be used as the Topologic Dimension where a point is known for the dimension 0, a *line* is known for the dimension 1, a *surface* is known for the dimension 2 and a cube is known for dimension 3. So the dimension of an object can be represented by a number of parameters or coordinates [12].

Fractal dimension has a very unique feature. There are various types of representations used for the fractal dimensions like Hausdorf dimension, topological dimension, self-similarity Dimension and Box counting method dimension etc. And the self-similarity property is the major parameter which is used for the analysis and characterization of the different fractal geometries [13].

The self-similarity parameter of the fractal geometry is defined as

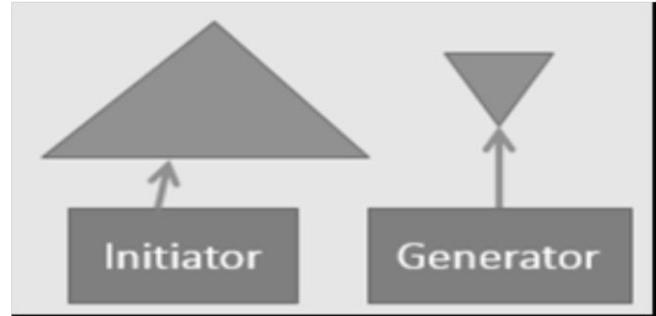
$$D_s = \frac{\text{Log}(N)}{\text{Log}\left(\frac{1}{s}\right)}$$

where  $s$  is the scaling factor and  $N$  is the number of self-similar structure copies.

Fractal design has commonly two dimensions and components as

A) The Initiator (0th stage) which is the basic structure of the fractal geometry.

B) The Generator which is the structure which is repeated in a sequential pattern on the initiator in the consequent stages with the different dimensions.

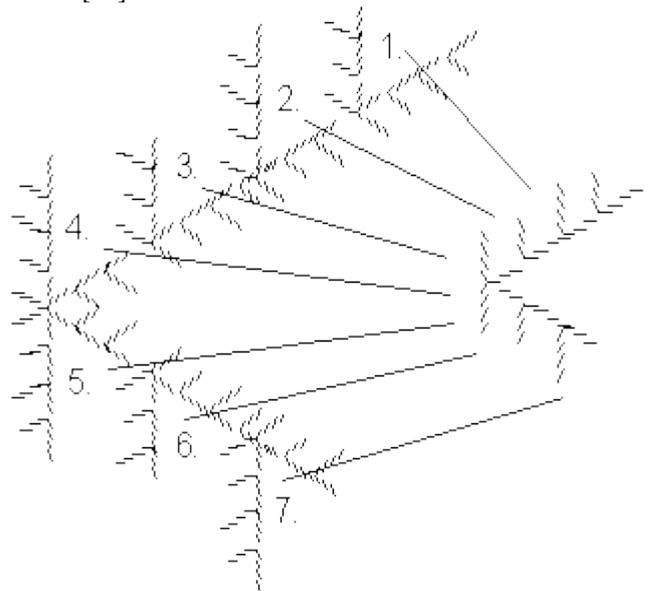


**Fig.19: Fractal generator structure**

The Fractal generator can be used to unite or subtract the original area of the initiator  $n$  from each subsequent iteration stage in a regular pattern generating the required fractal structure.

#### 3.2 Calculating Dimensions of Self-Similar Fractals

Generally linear fractals are used for calculation of the fractal geometry of the self-similar shapes. This is a mathematical approach and is limited to the mathematically predetermined fractal geometry structures producing precise values. [14].



**Fig. 20: The self-similar fractal dimension and geometry.**

We know that the fundamental and basic equation for calculation of any dimension is  $n = \frac{1}{s^D}$

And here the dimension  $D$  has an integer value.

Here we have to perform some numerical analysis and calculations for dissimilar values of  $n$  and  $s$ , where  $D$  is a partial or fractional dimension (not an integer dimension).

We know that the standard equation is  $n = \frac{1}{s^D}$

Now by taking logarithm of above equation, we'll have

$$\text{Log } n = \text{Log} \left( \frac{1}{s^D} \right)$$

Then factorizing the exponent  $D$  out of the scale factor

$$\text{Log } n = D \text{Log} \left( \frac{1}{s} \right)$$

By dividing with  $\text{Log} \left( \frac{1}{s} \right)$  to calculate the value of  $D$  we'll have

$$D = \frac{\text{Log } n}{\text{Log} \left( \frac{1}{s} \right)}$$

In practical fractal mathematics the replacement of the line segments with the seeds is a lengthy process.

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So in a general case it is written as  $n^l = \frac{1}{s^D}$ , where  $l$  is called as the level where the fractal seed is replicated.

$$\text{So it results } D = \frac{\log nl}{\log(\frac{1}{s})}$$

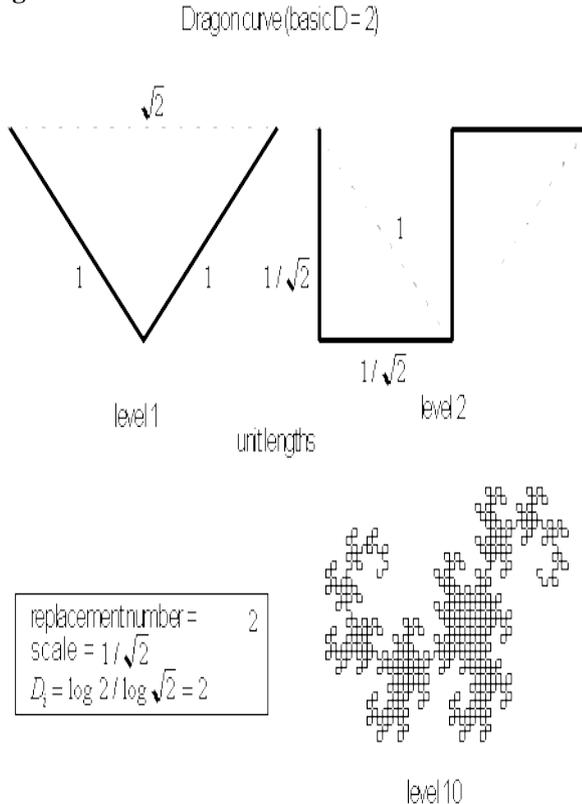
Here as the term  $l$  exists in both numerator and the denominator part of the above equation the term can be factored out and the equation becomes as  $D = \frac{\log n}{\log(\frac{1}{s})}$  where

the term  $l$  is factored out.

So the formal equation for fractal dimension is

$$\lim_{s \rightarrow 0} \frac{\log n}{\log(\frac{1}{s})}$$

### 3.3 Calculation of the Dimensions of the Varying Dragon Curves-



**Fig.21: Component length curves and the completed Dragon curve.**

The Fractal geometry can be obtained by varying the dimension curve of the dragon curve as the basic construction structure. Here the two seed segments are of unit length and replaces a single line structure segment of length  $\sqrt{2}$ . This process generates in a reduction of scale by a factor of  $\frac{1}{\sqrt{2}}$ .

So the dimension is calculated as  $D = \frac{\log 2}{\log(\sqrt{2})}$ .

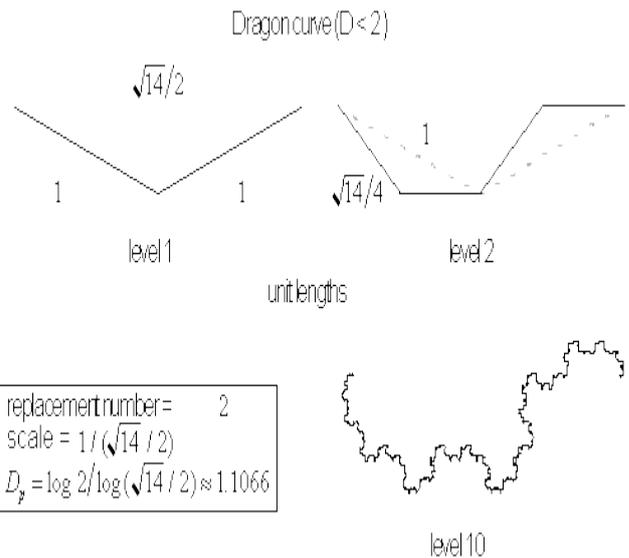
Here the curve is enclosed and occupies a region of the restrained area as shown in the above figure.

The dragon curve exists in different forms with varying dimensions [15].

### 3.4 Lower dimensions, its area disappears

When for a basic dragon curve the line segments are reduced the fractal dimension of the seed is lowered and cannot cover an area. And it is done reducing the seed segments while the initial and finish points are kept constant simultaneously or by enhancing the distance between the start and finish points [16].

Varying of the dragon curves with suitable dimensions less than dimension 2 is shown in the following figure.

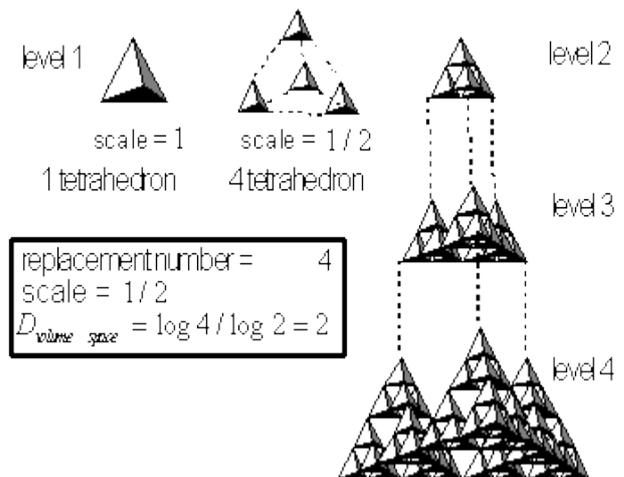


**Fig.22: Dragon curves with dimension less than 2.**

### 3.5 Calculating a Sierpinski pyramid (tetrahedron) dimension

Here it is studied the varying of dragon curve with component lengths i.e. the generator are enhanced related to its overall length.

Calculating dimension for the Sierpinski tetrahedron



**Fig.23: Calculation of a Sierpinski pyramid or tetrahedron dimension.**

Here the Sierpinski pyramid or tetrahedron contains a primary pyramid shaped structure that can be replaced by four smaller pyramids having half scaling of the original scale.

So the calculated dimension is represented as  $D = \frac{\log 4}{\log 2}$ .

Here the Sierpinski pyramid represents as a basic example of a two dimensional structure without any area. When such a Sierpinski pyramid exists that'll exist in a volume space i.e. its physical structure does not follow any other volume spaces [17].

### 3.6 Richardson's Dimension Equation

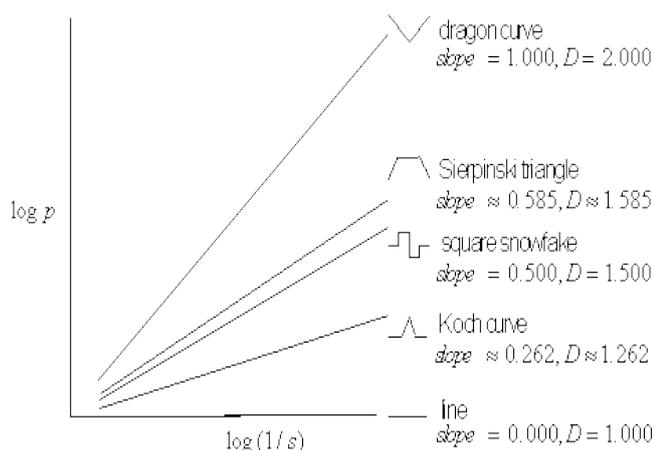
This method is used for calculation of fractal dimensions with the varying measurement lengths.

Here for the calculation of a dimension, first a establishment of a logarithmic relationship is required between the total measured length of the object and the length of the ruler used that is used to measure it. And this is done by taking graphs of the various values of  $\text{Log } p$ , where  $p$  is the length of the structure perimeter, against the corresponding values of  $\text{Log} (\frac{1}{s})$ , where  $s$  is called as scaling factor or the rulers length which is used for the perimeter measurement.

Now the resulted slope of the graph  $\frac{\text{Log } p}{\text{Log} (\frac{1}{s})}$  is used to relate the fractal dimension and structure of the measured object.

For a linear or smooth object, the smaller rulers are used to measure similar lengths similar to the larger ones, thus the slope is horizontal. And the graph with a horizontal slope used here corresponds to the dimension of 1. With the increase in the fractal dimension of any object, the level of the slope also increases [18].

Fractal's measured slope dimension



**Fig.24: Various slopes of  $\frac{\text{Log } p}{\text{Log} (\frac{1}{s})}$  corresponding to various fractal dimensions.**

The formula used for the Richardson slope method is represented as  $D_s = 1+d$ , where  $d$  is the slope dimension used for the  $\frac{\text{Log } p}{\text{Log} (\frac{1}{s})}$  graph which is added along the dimension of a line having a value of 1 and where  $D_s$  is the standard dimension.

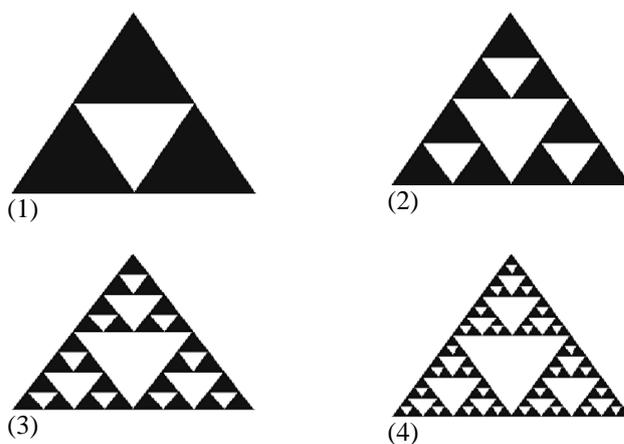
### 3.7 Generation of some common fractal elements

The Fractal antenna is used to resolve the limitations of the conventional antenna like

1. The electrical size of the antenna affects the antenna performance i.e. affecting the antenna performance parameters like  $S_{11}$  parameter, VSWR, gain, input and output impedance and radiation patterns etc.
2. Antenna size minimization is achieved by increasing the effective length of fractal antenna.

#### a) Calculation of the Dimension of the Sierpinski Gasket

This is a type of fractal geometry where a triangle is used as the basic geometry. The first iteration is taken by taking the midpoints of each side of an equilateral triangle and then connecting them to form another equilateral triangle which is slotted out from the original one. The higher iterations are achieved by repeating this same process into infinite number of times. Here the multiple iteration steps of Sierpinski gasket are shown in the following figure [19].



**Fig.25: The Multiple iteration steps of Sierpinski Gasket**

Also we can prove that the fractal dimension is not an integer by using Sierpinski Gasket fractal geometry as an example.

Now we'll determine how the size of any object behaves with the increase in its linear dimension. And in the one dimensional case, we can take a line segment for the consideration. When the linear dimension of the line segment is doubled, then it can be observed that the characteristic size i.e. length of the line is also doubled [20].

In the two dimensional case, if the linear dimensions of any shape like a square is doubled then it can be observed that the characteristic size i.e. the area is increased by a factor of 4.

In the three dimensional case, if the linear dimension of any shape like a box is doubled then it can be observed that the characteristic size i.e. the volume is increased by a factor of 8.

This relationship between linear scaling  $L$ , the dimension  $D$ , and the result of size increasing  $S$  can be represented as  $S = L \cdot D$

Rearranging the above formula and taking logarithm we'll obtain as

$$D = \frac{\text{Log } S}{\text{Log } L}$$

The above equation shows an expression for the dimension which is used to change the size as a function of the linear scaling.

Here the value of  $D$  is any integer  $\square$  like 1, 2, 3  $\square \dots n$  and it depends on the dimension of the geometry. And this above relationship holds good for all the Euclidean geometries and shapes. But now the question is how about fractals?

But a different concept is used for fractals geometries and shapes.

From the above figure it is clear that in the first step of building the Sierpinski gasket, if the linear dimension of the basic geometry triangle ( $L$ ) is doubled, then it can be observed that the area of total fractal geometry increases by a factor of 3 ( $S$ ).

Now by using this pattern as stated above, the dimension for fractals geometry based on the Sierpinski gasket is calculated as

$$D = \frac{\text{Log } (3)}{\text{Log } (2)} = 1.585$$

And this calculation result proves regarding non-integer fractal dimension.

## Fractal Geometry and Its Application to Antenna Designs

### b) Calculation of the Dimension of the Koch Curve

Now we can use the formula which is derived earlier to generate the Koch curve. The construction of this curve is uniform and we can calculate its dimension can be calculated easily comparative to other dimensions.

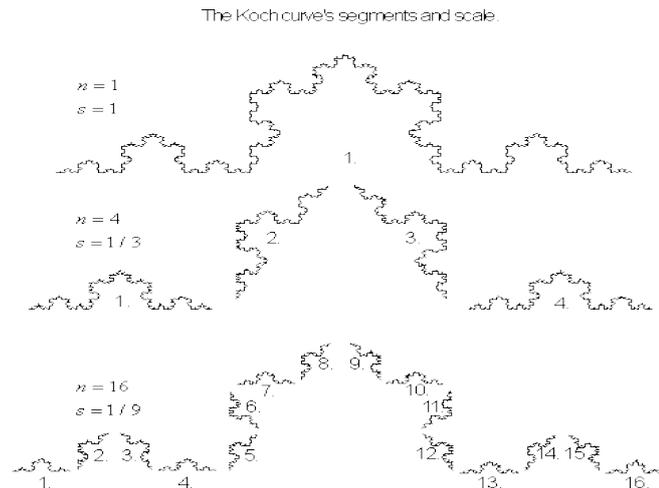


Fig.26 Koch curve fractal geometry showing 3 levels for n and s values.

We know that the number of line segments in the Koch curve dimension is 4. And here each line segment is replaced by a replica of the original and reduced in scale by  $\frac{1}{3}$ .

The dimension of the Koch curve can be calculated as follows

- I. The basic equation for dimension calculation is  $n = \frac{1}{s^D}$
- II. Now n can be replaced with 4 for the number of unit line segments and s with  $\frac{1}{3}$  for the scale factor. So the equation becomes  $4 = \frac{1}{(\frac{1}{3})^D}$  i.e. simply  $4 = 3^D$ .
- III. Then D is calculated by taking the logarithm of both sides. And by simplifying we get  $D = \frac{\text{Log } 4}{\text{Log } 3} = 1.26$ .

### c) Calculation of the Dimension of the Koch Snowflake

The fractal dimension based on Koch Snowflake can be constructed by using triangle as the basic curve. We can start the construction by using an equilateral triangle with each side of length L. A new triangle is created by connecting the middle of each side and the new triangle is of one-third size of that of the original. The same process is repeated for an infinite number of iterations.

So here the length of the boundary is  $3 \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \dots \infty$ . But the area becomes less than the area of a circle drawn around the original triangle i.e. an infinitely long line surrounds a finite area. And the construction of a Koch Snowflake looks very similar to the coastline of a sea-shoreline as shown in the following figure [21].

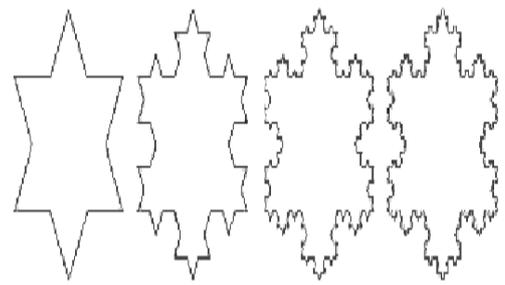


Fig.27: Multiple iterations steps for Koch Snowflake fractal

## IV. LITERATURE REVIEW

Here we have presented a detailed literature survey on various fractal geometries and their dimensions suitable for particular applications. These are categorized as follows

### 4.1 Based on Sierpinski gasket fractal geometry

The fractal antenna based on the Sierpinski gasket geometry is an antenna where a triangle is used as the basic structure and it is named according to the mathematician W. Sierpinski. Here we can start the construction by using an equilateral triangle as the patch with sides of any length L. The middle point of each side is taken and connected them to make a new triangle and this new triangle is taken out of the triangle shaped patch. This step is called as the first iteration. And the same process is repeated for the multiple numbers of iterations [22].



Fig.28: Sierpinski Gasket Fractal Antenna

C. Puente *et al.* [23] (1996) proposed a fractal monopole antenna that is designed using fractal geometry. Here the authors suggested that by using the Sierpinski gasket fractal geometry that follows the self-similarity property enables the proposed antenna to behave as the multi-resonating antenna. Here the proposed antenna is designed, simulated and validated using FDTD technique and it resonates up to five bands self-similarity property of the antenna.

C. Punete *et al.* [24] (1998) presented a Microstrip patch antenna based on the Sierpinski fractal geometry suitable for multiband applications and compared it with a single band bow-tie antenna. The proposed antenna showing self-similarity properties is simulated and fabricated. And the author concluded that better performance parameters like return loss, VSWR, multiband operation and radiation patterns are reflected by the experimental and simulated results.

The designed antenna exhibits a multi-frequency behavior and an efficient current distribution, but changes at all resonant modes. This efficient current distribution change provides various radiation characteristics resonating at higher frequencies showing radiation pattern with increased number of grating lobes.

**C. T. P. Song et al. [25] (1999)** presented a Sierpinski gasket fractal antenna that uses triangular shape as basic structure and is suitable for the different applications like WLAN, Wi-Fi, DECT and GSM. The proposed antenna is fed with a 50Ω strip line feeding technique and is designed and simulated by using HFSS software and it is analyzed by using FDTD method. Here the authors have suggested that there is improvement in fundamental frequency and also improvement in antenna optimization and ground plane results.

**C. P. Baliarda et al. [26] (2000)** presents a Sierpinski fractal antenna designed with iterative transmission line model for multiband applications. Authors have used various flare angles ( $\alpha$ ) and predicted that the proposed antenna behaves efficiently at  $\alpha = 30^\circ$ .

**C. Puente et al. [27] (2000)** presented an improved monopole antenna based on Sierpinski gasket fractal geometry suitable for dual band operated wireless devices. Here antenna element is designed on the dielectric laminate area that is extended beyond the circuitry of the device.

Here authors have suggested that the proposed antenna is operational at frequency range that can be used for allocation of 2.4 / 5.2 GHz ISM bands using only a single microstrip feed and also without using any matching network. Authors have also presented different modification techniques allowing different bands allocation without using similarity factor i.e without changing flare angle of triangles.

**C.T.P. Song et al. [28] (2000)** presented a modified design of monopole antenna based on the Sierpinski gasket fractal geometry. Here the presented antenna is a monopole antenna whose size ratio is 2 of the outer 3 loops with disk feed monopole. The designed antenna provides comparatively better results like good return loss achieves efficient radiation pattern control. This proposed antenna is suitable for various applications like DECT, GSM, and HIPERLAN.

**D. H. Werner et al. [29] (2001)** presented a sierpinski gasket fractal monopole antenna on the portable devices wireless in nature. Here the height of the sierpinski fractal monopole type antenna is reduced the control of first 2 bands spacing. And use of wide flare angles and property of affine transformation are used to get bands spacing. But this antenna structure suffers one disadvantage like decreased bandwidth and shallow resonances.

**C.T.P. Song et al. [30] (2004)** presented a modified monopole fractal antenna. Here multiband property for a perturbed type Sierpinski gasket fractal structure and a perturbed Parany shaped monopole antenna are used respectively for the two antenna structures. These two structures are used for antenna matching without using any matching circuits.

**W. J. Krzysztofik et al. [31] (2006)** proposed a modified monopole antenna by using Sierpinski gasket geometry. Here the presented antenna provides compact patch size, very high efficiency and can be operational at ISM-bands (both the 2.4 and 5.2 GHz). Authors have suggested that as the input impedance parameter characteristics of the upper bands of Sierpinski gasket geometry maintain symmetry thus the modified proposed antenna provides the multiband behavior. Due to different modification techniques the proposed antenna makes the monopole very flexible in fine-

tuning and suitable band allocation. Here the proposed antenna is analyzed by using the MoM and FDTD. **Raj Kumar et al. [32] (2007)** presented a rectangular shaped proximity coupled feed miniaturized fractal antenna based on sierpinski gasket geometry. Here the presented antenna is constructed with dielectric constant of substrate  $\epsilon_r = 4.3$ , having the height of substrate  $h = 1.53\text{mm}$  and of rectangle patch dimension of ( 36.08 X 29.6)  $\text{mm}^2$ . Here the authors have suggested that the patch size reduction of the proposed patch antenna is achieved by 2nd iteration and the bandwidth enhancement is possible by implementing an air-gap of 0.8mm in the patch.

**Mehdipour et al. [33] (2010)** presents a multiband Sierpinski Fractal antennas with single wall carbon nanotube. Here the presented antenna is designed and simulated by using CST Microwave Studio software and fabricated by using high-precision milling machine by printing carbon nanotube both sides of the substrate and again the desired shape is truncated. Here the resin infiltration technique is applied to strengthen the carbon nanotube material. Here the proposed antenna provides better gain and radiation patterns and it can be used for various applications like Bluetooth, WLAN and UHF RFID. The authors have suggested that the proposed antenna provides the 0.17 dB/cm and 0.22 dB/cm loss at the resonant frequency of 2.4GHz and 5.8GHz respectively. And here the gain of the presented antenna can be controlled by suitably varying the microstrip length. The authors have concluded that the presented antenna is simulated and fabricated; hence it is found that simulated and experimental results are matching in good agreement with each other.

**J. Malik et. al. [34] (2011)** presented a miniaturized dual band Sierpinski Gasket fractal antenna which is a small sized antenna as compared to the Euclidean-type. Here the proposed antenna uses two different patch structures which are electromagnetically coupled with stacked structure and operational at two resonant frequencies like 2.4 GHz and 5.8 GHz. So the proposed antenna can be useful for dual band WLAN applications.

**S. Jagadeesha et. al. [35] (2012)** presented a multiband H shaped fractal antenna using sierpinski gasket geometry. Here the authors have suggested that the proposed antenna uses two numbers of substrate layer of FR4 material with dielectric constant of  $\epsilon_r=4.4$ . Here these two substrate layers are of thickness of 1.6mm and are placed vertically one over another. Here the authors have suggested that due to these two stage substrate layers this proposed antenna behaves as multiband antenna.

**Abdelati Reha et. al. [36] (2018)** presented a CPW fed fractal antenna based on sierpinski geometry. Here the authors suggest that the inter-relationship between the number of iterations and the resonant frequencies i.e. when the number of iteration increases the resonant frequency decreases with a constant ratio. Here the relative permittivity of the substrate material is increased; the proposed antenna shows improved performances.

From the above survey, it is clear that the Sierpinski gasket geometry maintains symmetry, so provides multiband behavior. It shows an efficient current distribution, thus improving radiation patterns. It also provides large broadband performance, so useful in broadband antenna design.

## Fractal Geometry and Its Application to Antenna Designs

### 4.2 Based on frequency independent fractal geometry

In this section, we have discussed on frequency independent antenna based on the fractal geometry. Here some major features of frequency independent antennas like LPDA antenna structures, antenna structures using EBG materials, reconfigurable antenna structures, defective ground structure antennas, and multi layered fractal structured antennas are discussed.

**D.H. Werner et al. [37] (1996)** proposed a log periodic dipole array (LPDA) antenna where its directive gain is demonstrated as a log periodic function behavior with the frequency. The proposed antenna is designed using the fractal geometry that is frequency independent possessing the self-similarity property. Here authors suggested that the fractal geometrical structures could be useful for development of linear array radiators showing multiband action.

**Y. Wang et al. [38] (2007)** presented a microstrip antenna using the EBG materials which is constructed by using dielectric rods those have fractal cross sections. The proposed antenna can be useful for multiband applications and stop-band enhancement. Authors have demonstrated that this Fractal dimension and geometry is successfully implemented the on uni-planar PBG structures. Again this fractal dimension and geometry is applied on a PVEBG (Pad and Via EBG) structure and can be used to achieve multi-frequency band gap application.

**W.J. Krzysztofik et al. [39] (2008)** focused on the plane-filling nature of fractal geometries. Here authors have suggested how antenna minimization can be possible. Authors have concluded that an antenna is called as small when its size of the larger dimension is always less than twice the radius of the radian sphere i.e.  $\lambda < 2\pi$ .

**J. Pourahmadazar et al. [40] (2011)** presented a modified Pythagorean tree fractal (MPTF)-based antenna based on multifractal technique. Here authors have suggested that the MPTF provides good miniaturization technique as it is constructed by using self-similar properties without altering the efficiency and the bandwidth of the antenna. Thus the proposed antenna can be useful for UWB application.

**A. Kumar et al. [41] (2014)** proposed a construction of multi-frequency reconfigurable fractal antenna. Here the reconfigurability is achieved by varying the feed line by using flexible coaxial cable attached to the feed line and the coaxial cable is moved automatically to the various locations by using a microcontroller. The authors have suggested that the feed line is moved to 144 numbers of various positions to obtain optimized results and analyzing the results concluded that that the feed position at 34.08 mm provides the triple band. The authors have concluded that the proposed antenna is simulated and fabricated; hence it is found that simulated and experimental results are matching in good agreement with each other.

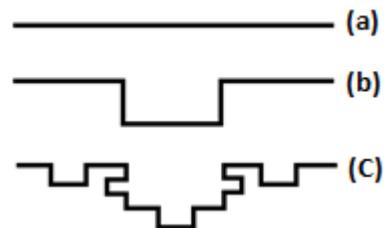
**R. Kubacki et al. [42] (2018)** presented microstrip antennas using fractal layer designs. The proposed antenna uses planar structure periodic geometries thus achieving improved characteristics. Here the fractal structure uses a single-fractal layer design and on both the top and bottom layers of antenna. The proposed antenna is a double fractal layer antenna which is optimized for the gain and bandwidth enhancement. The authors have concluded that the proposed antenna provides improved antenna performance significantly like an ultra-wide bandwidth that ranges in 4.1

GHz to 19.4 GHz, high gain of 6 dBi, radiation capacity situated in the horizontal plane direction of the antenna.

From the above survey, it is observed that fractal geometries using self-similar property are used in frequency independent antennas suitable to transmit and receive over a large range of frequencies. The frequency independent antennas shows steady performance over large frequency range provides condensed mutual coupling. These fractal geometries are also suitable for design of fractal array antennas.

### 4.3 Based on Minkowski island fractal geometry

In this section, we have discussed on fractal antenna structures using Minkowski fractal geometry. The Minkowski fractal geometry uses a generation curve as the basic structure and it is named according to the mathematician H. Minkowski. Here we can start the construction by using a square shape geometry in which each of four straight sides of square is being replaced with the generator curve and then iteration width is applied by a scaling of 1/3 to each straight sides of the square at the each iteration. Here the depth is adjusted to an optimization in order to achieve accurate result steps for construction of minkowski geometry as shown in figure [43].



**Fig. 29: (a) Initiator geometry, (b) Generator geometry and (c) Proposed Minkowski curve**

**S. R. Best et al. [44] (2002)** presented a Minkowski island based fractal antenna. Here authors suggested that by adjusting fractal iteration levels we can modify resonant frequencies. Here authors implement unit cell geometry with Minkowski island first iteration in the design of proposed antenna. But with increase in fractal iteration steps the proposed antenna shows decrease in broadband characteristics and also worst electrical properties are provided when fractal iterations increase.

**S. N. Shafie et al. [45] (2010)** presented a Minkowski fractal antenna. This fractal structure resonates at the multiple resonant frequencies and the modified ground structure plane is suitable for bandwidth enhancement. The authors have suggested that the arm length can be reduced and consequently the size of proposed antenna could be reduced by using this modified Minkowski fractal structure.

Authors have concluded that the presented antenna can operate at 2.3 Ghz, 2.45 GHz and 5.2 GHz, thus can be useful for applications like WLAN, WiMAX, and HiperLAN.

**Moraes et al. [46] (2011)** presented a multi band Minkowski fractal antenna. Here the proposed antenna uses the fractal structures based on Minkowski squares. The proposed antenna operates at multiple (four) resonant frequencies and suitable for the WLAN/WiMAX systems.

**S. Behera et al. [47] (2012)** proposed a fractal antenna using the Minkowski geometry.



Here the authors have used the multiport network method to validate the performance of the patch antenna. Here the Minkowski geometry is used to replace the patch side opposite to the position of feed arm for microstrip patch square ring antenna. Here by using the Minkowski geometry the Dual frequency band operation is achieved. Here the resonant characteristics are controlled by increasing the width of the sides. Authors have controlled that the impedance matrix (multiport network model) of the proposed antenna is analyzed and simplified using the self-similarity property of the fractal geometry with the greater precision and high accuracy with the less analysis time.

**N. Abdullah et. al. [48] (2015)** presented a miniaturized dual band Minkowski fractal antenna useful in wireless communication. The proposed antenna resonates at two frequencies at 1800 MHz and 1.5 GHz respectively with maximum return loss of -20.62dB. Here the proposed antenna provides improved parameters like reflection coefficient, bandwidth, VSWR, beam width and radiation pattern. Authors have concluded that the presented antenna could be useful for various wireless applications like GSM and GPS.

**S. Dhar et. al. [49] (2015)** presented a CPW-fed multiband miniaturized fractal antenna using Minkowski geometry. The presented antenna is a CPW- fed slot antenna which is burdened with the dielectric resonator i.e. here slot loop is used as both feed mechanism and an antenna. Here the Dielectric load can be used for the impedance bandwidth improvement at the higher frequency band and enhancement of the overall gain. Here the proposed antenna is designed based on an equivalent model consisting of lumped resonators, impedance transformers and distributed resonators of the fractal slotted antenna which is dielectric loaded and exhibits an insight into the proposed antenna functioning. The authors have concluded that the proposed antenna resonates at seven frequency bands and provides better return loss of -30dB and high gain of 3.1 dB.

**A. Sarkar et. al. [50] (2017)** presented a modified compact fractal antenna based on Minkowski geometry. Here the presented antenna is a right/left-handed composite high gain leaky-wave antenna CRLH based frequency tunable antenna which is compact in size and of high gain leaky-wave antenna operating at the Ku-band. Here the proposed antenna uses novel Minkowski fractal geometry. And by using suitably placed varactor diodes and the frequency tunable leaky-wave antenna structure the antenna size miniaturization is achieved. Here the authors have suggested that the operating frequency is changed from 15-16.2 GHz to 14-15 GHz by altering the varactor diodes position with the beam scanning range of  $32^{\circ}$  and  $30^{\circ}$  respectively with maximum gain of 16.7 dBi.

From the above survey, it is achieved that this type of fractal geometry is suitable for broadband antenna designs in the field of wireless applications.

#### 4.4 Based on Koch curve/island fractal geometry

In this section, we have discussed on fractal antenna structures using Koch curve fractal geometry. The fractal antenna based on the Koch Curve geometry is an antenna where a generation curve is used as the basic structure and it is named according to the mathematician H. V. Koch. Here we can start the construction by using a straight line and then this straight line is sub divided into three equal parts. And the middle segment is substituted with two other segments which are of approximately same length. It is the

first iteration step of Koch Curve fractal antenna. And the same process is repeated for the multiple numbers of iterations [51].



**Fig.30: Koch Curve structures**

**C. Borja et al. [52] (2003)** presented a loop antenna using the Koch island for increase in input resistance parameter. Here the proposed antenna is designed following the generator consisting of only four segments which is of equal length similar to the Minkowski fractal loop that uses five segments of two different scales. Authors have also used a circular loop with equal radius that circumscribes the fractal loop for the comparison purpose.

**Anjam Riaz et al. [53] (2006)** presented a design of fractal antenna based on koch curve geometry and compared its performance with the performance of a standard dipole antenna. Here the presented antenna uses self-symmetry and scaling property and provides improved performance parameters like compact patch size, broad bandwidth, high efficiency suitable for multi-band operation. Here the proposed antenna is designed and simulated by using the MMANA software and is compared with a normal dipole antenna.

Authors have concluded that the proposed Koch curve dipole antenna provides superior performance compared to the standard dipole antenna.

**T.P. Wong et al. [54] (2007)** designed a vertical patch antenna (VPA) based on dual-Koch loop geometry for wideband applications. Authors have suggested that the proposed antenna is constructed by using fractal dimension and geometry. An impedance bandwidth of 42 % and a realized gain of 8 dBi at the center frequency is achieved.

**D. D. Krishna et al. [55] (2008)** presented a modified fractal antenna based on Koch geometry for dual band and wide-band applications. In this case authors have suggested that by using Koch iteration technique the operating frequency of a triangular slot antenna can be lowered. Here size of the proposed antenna with the ground and substrate plane is compact and provides wide operational bandwidth. Here the proposed antenna results an impedance bandwidth of 40 % from 2.38-3.95 GHz and 20 % from 4.95-6.05 GHz and covers efficiently the 2.4/5.2/5.8 GHz WLAN bands and the 2.5/3.5/5.5 GHz WiMAX bands.

**B. Mirzapour et al. [56] (2008)** presented a Koch snowflake fractal antenna for Wideband applications. Here authors have suggested that by using Koch snowflake structure an impedance bandwidth of 49 % can be achieved. Authors have also suggested that, a 70 % reduction in patch surface size in comparison to an ordinary Koch snowflake is achieved by using an air filled substrate and capacitive feed.

**Fazal et al. [57] (2012)** presented the partial Koch triangular fractal antenna. Here the authors have suggested that the proposed antenna provides improved parameters like return loss and gain compared to conventional triangular shaped patch antennas.

## Fractal Geometry and Its Application to Antenna Designs

The authors have concluded that presented antenna could be useful in applications like GSM booster units and indoor LANs etc.

**Manisha Gupta et al. [58] (2017)** presented a modified microstrip feed microstrip fractal antenna based on various fractal geometries suitable for different wireless applications. Here the authors have implemented various fractal geometries like Koch, Sierpinski and Minkowski fractal geometries and suggested that Koch fractal geometries provides better performance parameters like good return loss, good VSWR, enhanced bandwidth, good gain and good axial ratio bandwidth.

**A. S. Gvozdarev et al. [59] (2018)** presented a multiband non-planar steerable fractal dipole antenna based on Koch geometry. Here the proposed antenna uses fractal iterations, base angle and angle of steering through its parts providing very high directivity.

From the above survey, it is clear that this is the most fundamental fractal geometry that provides improved performance parameters particularly good input impedance matching and high efficiency. Enclosure area of this fractal geometry is comparatively less than Hilbert curve geometry. Matching circuitry is also not required for Koch curve geometry.

### 4.5 Based on Hilbert curve fractal geometry

In this section, we have discussed on fractal antenna structures based on Hilbert curve fractal geometry. The fractal antenna based on the Hilbert Curve geometry is an antenna where a generation curve is used as the basic structure and it is named according to the mathematician D. Hilbert. This fractal antenna uses the simplest geometry as it covers the area it occupies. And as here the line segments are not intersecting to each other, thus property reduces the complexity in the antenna structure [60].

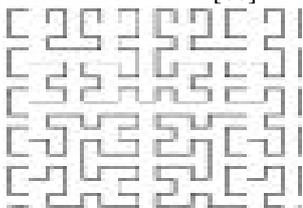


Fig.31: Hilbert Curve

**M. Z. Zaad et al. [61] (2005)** presented a Hilbert curve fractal antenna. Here authors have investigated that the Hilbert curve and Peano fractals based on the space-filling properties can be used as suitable method for the compact sized resonant antennas designing.

**R. Azaro et al. [62] (2009)** presents a novel monopole multiband fractal antenna using Hilbert geometry. Here the presented antenna is optimized by using the PSO technique and almost 39% length is reduced. The authors have concluded that the proposed antenna is simulated and fabricated and it is found that simulated and experimental results are matching in good agreement with each other.

**J. T. Huang et al. [63] (2010)** proposed a Miniaturized inverted-F antennas (IFAs) by using Hilbert geometry. Here authors have suggested that the overall patch size reduction is achieved up to 77% when compared to a simple rectangular patch antenna. Here the authors have suggested that the presented antenna can be useful in wireless sensor network (WSN).

**K. Jaroniek et al. [64] (2013)** presented a new dual band PIFA using Hilbert fractal meander suitable for handset applications. Here authors have suggested that the use of the

Hilbert fractal meander profile increases efficiently the current flow path length to achieve a miniaturized antenna. Here two numbers of branches of radiating element are used to obtain dual-band performance.

**Tao Donga et al. [65] (2018)** presented a Electrically very small Hilbert fractal antenna. Here the authors have demonstrated that better impedance matching can be achieved by using near-field resonant parasitic (NFRP) antennas that uses Hilbert fractal geometry curves acting as the NFRP resonators and small vertical monopoles acting as the driven elements.

The authors have suggested the advantages of this proposed NFRP antennas based on fractal geometry that there is no need of additional matching network and provides high radiation properties, while maintaining a small electrical size.

From the above survey, it is observed that these types of fractal structures provide improved performance parameters particularly compact sized antenna structure. Hilbert curve provides a mapping algorithms, so it is largely used in the field of computer science.

### 4.6 Based on sierpinski carpet fractal geometry

In this section, we have discussed on fractal antenna structures based on sierpinski carpet fractal geometry. The fractal antenna based on the Sierpinski carpet geometry is an antenna where a rectangle is used as the basic structure and it is named according to the mathematician W. Sierpinski. Here the length, width and the total area are calculated using geometric progressions. We can start the construction by a rectangle as the patch with the dimension of any length and width (LXW). Then a new rectangle is created by considering a central slot in the patch. And the central slot size is considered as  $\frac{1}{n}$ , where n is any integer. Here the same scaling is property is used in higher iterations also. And the same process is repeated for the multiple numbers of iterations. This is the simplest fractal antenna and it is most widely used fractal antenna as it follows multiband operations [66].

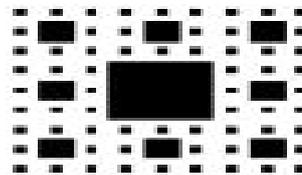


Fig. 32: A third iterated Sierpinski carpet fractal antenna

From this paper we have concluded that Sierpinski carpet geometry is widely used as using this geometry in a single antenna can be suitable for multiple frequency operation.

**Yoonjae Lee et.al. [67] (2002)** presented a modified conformal Sierpinski fractal antenna which is multiband in nature operating through triple band frequencies. Here the proposed antenna is a printed antenna using substrate dielectric constant of  $\epsilon_r=4.3$  and provides improved performances like impedance matching, radiation patterns and multiband applications.

**J. Anguera et al. [68] (2005)** presented a modified Sierpinski-based microstrip fractal antenna. The proposed antenna structure provides miniature design, wide bandwidth, better radiation and improved efficiency.

This modified antenna consists of a parasitic patch and an active patch. Authors suggested that when the parasitic patch is added, there is improve in performance parameters like the bandwidth (BW) is increased through a factor of 15 and it is compared to the single active element, a bandwidth ( $BW$ ) = 2.7% which is at  $SWR=2:1$  and the radiation parameter efficiency of 84%.

**S.N. Sachendra et al. [69] (2007)** presented a sierpinski carpet fractal antenna using self-affine fractal geometry. Here the proposed antenna is designed by applying scaling property to a square through a factor of 3 in the horizontal direction and through a factor of 2 in the vertical direction, providing nine rectangles and now out of these rectangles the central rectangle is removed. This process is called as the first iteration step. The same process is repeated for the remaining rectangles which are called as the second iteration step and the process can be continued upto infinite steps. This process is called as the iterated function system (IFS) and is a very useful tool in the fractal antenna designing.

**W. L. Chen et al. [70] (2008)** presented a modified Sierpinski carpet fractal antenna. Here authors have used a novel technique by applying etching process to the patch edges and implements koch curve geometry as inductive loading. Also the Sierpinski carpet geometry shaped slots are made into the patch as slot loading for the microstrip patch antenna size minimization. Also authors have experimentally suggested that with the increase in iteration order steps of the fractal dimensions the resonant frequency of the microstrip patch can be adequately lowered.

**R. Guo et al. [71] (2009)** presented a microstrip planar monopole antenna which is based on the binary tree sierpinski carpet fractal geometry. Here the proposed antenna is optimized using GA method with the full-wave electromagnetic simulation and cluster parallel computation to achieve wide impedance bandwidth. Here the presented antenna achieves promising impedance bandwidth of 42.8 % (2.46-3.80 GHz). The authors have concluded that the proposed antenna is simulated and fabricated and it is found that simulated and experimental results are matching in good agreement with each other.

**Sagne D. S. et al. [72] (2013)** presented a microstrip feed fractal antenna based on Sierpinski carpet geometry which is designed up to the 3rd iteration. Here size reduction of patch is achieved as 33.9% when compared with the conventional microstrip antenna. Here the presented antenna can be used for different applications like Bluetooth, WiMAX , Wi-Fi (5.1 GHz - 5.825 GHz), military and meteorological satellite systems (8 GHz to 12.5 GHz), radar and navigation services, Point To Point communication in US military (6 GHz), broadband fixed wireless access services, Medium and High Capacity Point To Point communication (7725 MHz -8275 MHz), UMTS (1920 MHz - 2170 MHz) applications etc .

**Manas Ranjan Jena et. al. [73] (2013)** presented a compact multiband fractal antenna based on Sierpinski carpet geometry for fixed microwave and aviation applications. Here the presented antenna is simulated by using the CST Microwave Studio EM simulation software. Here the presented antenna resonates at four resonant frequencies at 0.85 GHz, 1.83 GHz, 2.13 GHz and 2.68 GHz and provides efficient return loss (-15 dB, good VSWR (1.3), better directivity (6dBi) and high gain (6dB).

**Manas Ranjan Jena et. al. [74] (2014)** presented a multiband fractal antenna based on Sierpinski carpet suitable for satellite communication. Here the authors have suggested that after the theoretical design procedure, CST EM software is used for the numerical simulation up to 2nd iteration to obtain design parameters like patch size and feeding location. Authors have suggested that when the number of iteration steps increases the antenna bandwidth increases and in the second iteration the antenna starts showing the multiband behavior. The proposed antenna shows improvement in different performance parameters like better return loss, low VSWR, large bandwidth, high directivity and gain.

**F. D. Nicola et. al. [75] (2018)** presented a Multiband plasmonic fractal antenna based on sierpinski carpet geometry. The presented antenna is constructed by using deterministic fractal antennas. Here the antenna structure shows the enhanced localized electromagnetic fields in the infrared range with a hierarchical spatial distribution. The proposed antenna is a modified plasmonic multiband antenna based on metamaterial design which is based on the space-filling sierpinski carpet fractal with an independent tunable and polarization optical response exhibiting multiple resonances from the visible to mid-infrared rage. The proposed antenna uses gold SCS that is fabricated by electron beam lithography used on CAF2 and Si/Sio2 substrates allowing resonances originating from the diffraction mediated localized surface plasmons which can be tailored in the deterministic fashion by tuning the size, shape and position of the fractal elements. Authors have suggested that SCs with high order of complexity provides a strong and electromagnetic near-field of plasmonic modesthus distributed hierarchically and provides a technique for the realization of compact active devices with a broadband and strong spectral response operating in the visible/mid-infrared range. The proposed antenna provides adavantages of surface enhanced raman spectroscopy (SERS) used on the brilliant Cresyl Blue molecules deposited onto plasmonic SCs. The authors have concluded that the proposed antenna can achieve a broadband SERS enhancement factor up to  $10^4$  , thus can be used for the chemical diagnostics applications.

From the above survey, it is observed that these types of fractal structures provides improved performance parameters particularly good impedance matching, impendence bandwidth, wideband/multiband, frequency independent, high radiation patterns. With the advantages of the condensed mutual coupling it is also suitable in design of fractal array antennas.

#### 4.7 Based on hybrid fractal geometry

In this section, we have discussed on fractal antenna structures based on some hybrid fractal geometry. Hybrid fractal antennas are combinational structured antennas those use a combination method of different antenna structures. And it is an effective technique for improving the fractal antenna performance.

The Hybrid fractal antennas are designed by using the following possible combinations [76].

- ntra Fractal shape with itself like Sierpinski-Sierpinski , Minkowski - Minkowski , Koch-Koch etc.
- Koch-Sierpinski
- Koch-Meander

## Fractal Geometry and Its Application to Antenna Designs

□ □ Sierpinski-Giuseppe peano

There are some other combinations of fractal geometries as shown in the figure.

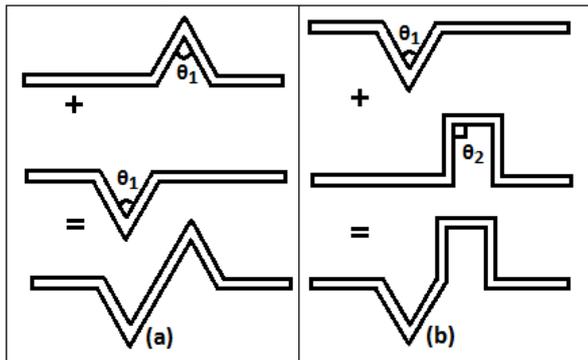


Fig. 33: Hybrid fractal slot - (a) Koch- Koch geometry and (b) Koch- Minkowski geometry

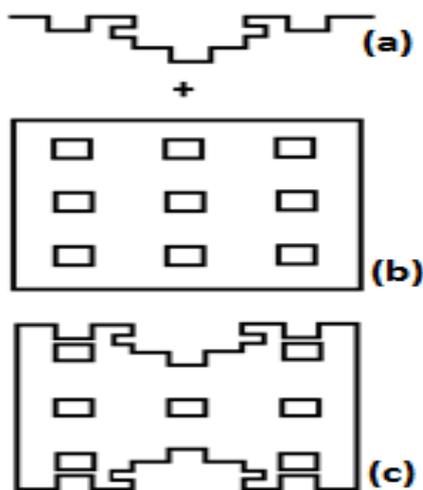


Fig. 34: (a) Minkowski Curve geometry, (b) Sierpinski Carpet geometry and (c) Minkowski- Sierpinski Hybrid fractal structure

**Q. Luo et al. [77] (2009)** presented a printed multiband monopole antenna based on fractal geometry used for WLAN-USB dongle applications.

Here the presented antenna is combinational antenna which is the combination of fractal geometry and meander line. The proposed antenna may be used for various WLAN applications covering the frequency bands in the range of 2.22 to 2.52 GHz and 5.03 to 5.84 GHz. The proposed antenna resonates at the lower frequency band with constant realized gain of 1.8 dBi and of radiation efficiency 95% and at the upper band it provides gain of 2.4 dBi and better radiation efficiency of 94%. Authors have concluded that the proposed antenna is simple in shape and easily fabricated.

**R. Azaro et al. [78] (2009)** presented a modified multiband hybrid fractal antenna. The proposed antenna is a hybrid antenna which is designed by using Sierpinski fractal geometry and Meander shapes. Here a Customized PSO technique is used for improvement of the performance parameters of the proposed antenna. And the optimal value of VSWR is obtained as 1.3 for GSM (Upper frequency band), 2.4 for GSM (Lower frequency band) and 3.2 for Wi-Fi frequency band. Authors have concluded that the presented antenna could be useful for various wireless

applications like GSM (925 MHz and 1850 MHz) and Wi-Fi (2.44 GHz).

**A. Jamil et al. [79] (2011)** presents a hybrid shaped fractal antenna which is hybrid combination of both Koch geometry and Meander structure. Here the presented antenna is designed and simulated by using CST microwave studio simulator and it provides dual bands of bandwidth (2.2909 GHz - 2.553 GHz) and (5.1406 -GHz-5.8737 GHz). Here the authors have suggested that the proposed antenna provides the minimum value of S11 is -28.9 dB at a low frequency band (2.41 GHz) and maximum S11 is -20.8 dB at high frequency band (5.36 GHz). Authors have suggested that the presented antenna can be suitable for WLAN USB dongle application.

**R. A. Kumar et al. [80] (2012)** presented a microstrip line feed compact hybrid fractal antenna. The proposed antenna is compact in size of 30×25 mm<sup>2</sup> with a semi-elliptical ground plane. Here the presented antenna is constructed by combining Giuseppe Peano and Sierpinski Carpet fractal geometries. Here the proposed antenna provides omnidirectional radiation pattern with high gain. Authors have concluded that the presented antenna is useful for various UWB applications.

**Y. K. Choukiker et. al. [81] (2014)** proposed a hybrid planar monopole fractal antenna suitable for wireless communications particularly in MIMO structures. The proposed antenna is a multi frequency antenna uses hybrid structure that uses a combination of Minkowski and Koch curve which are edge to edge separated at 1.75 GHz. Here a T-shaped stripline is inserted into the patch structure and rectangular slots are made towards upper side of the ground plane layer for improved impedance matching parameter and the isolation of antenna. Here the measured impedance matching with the fractional bandwidths as observed as 14% and 80% for band 1 (1.65 GHz to 1.9 GHz) and band 2 (2.68 GHz to 6.25 GHz) respectively. The authors have concluded that the proposed antenna is simulated and fabricated; hence it is found that simulated and experimental results are matching in good agreement with each other and the presented antenna could be used for different handheld mobile devices.

**S. Singh et al. [82] (2015)** presented a compact Multiband hybrid fractal antenna. The presented antenna is a hybrid structure of both Koch and Minkowski geometry curve altogether. Hence the presented antenna provides miniaturized patch structure, good return loss, better VSWR, omni-directional radiation pattern, High gain and multiband characteristics. The presented antenna is constructed up to 2nd iteration by using HFSS and MATLAB. Authors have concluded that the presented antenna could be useful for various wireless applications like Bluetooth, GPS, ISM band, WLAN, Mobile communication, aeronautical navigation system and fixed satellite system.

**Y. Kumar et. al. [83] (2015)** presented a Hybrid multiband fractal antenna. The proposed antenna is a hybrid structure of the Meander and Koch curve geometry. Here an Iterative Function System method is used to achieve a miniaturized antenna using software like HFSS and MATLAB. The proposed antenna resonates at four frequencies like 2.53 GHz, 4.07 GHz, 5.38 GHz and 7.3 GHz respectively.

Authors have concluded that the presented antenna could be useful for different types of wireless applications like Bluetooth and WLAN.

**M. Sowmya et. al. [84] (2017)** presented a modified low profile multiband fractal antenna. Here the presented antenna is constructed by using RT/Duroid as substrate of  $\epsilon_r = 2.2$  and height of substrate as 1.575 mm and two fractal geometries like minkowski and sierpinski carpet geometry are applied on patch antenna and design structure is iterated upto 3<sup>rd</sup> iteration. The authors have concluded that the presented antenna is suitable for entire S-band and C-band applications.

**M. Tarbouch et. al. [85] (2018)** presented a hybrid miniaturized H-tree shaped multiband fractal antenna. Here the proposed antenna uses a combination concept of two widely used miniaturization techniques. Here the first technique is a ground plane modification technique uses the making slots on the ground plane placed on the back side of the patch structure and second technique is a trial technique that uses H-tree shaped fractal slots in the created ground plane. Here the radiating patch element is a small patch with the dimension of (20X20) mm<sup>2</sup> and substrate dimension of (47 X 47 X 1.6) mm<sup>3</sup>.

The authors have concluded that the presented antenna provides better impedance matching and high gain and can be useful for PCS, LTE, RFID, WLAN, and WIMAX.

**Narinder Sharma et. al. [86] (2018)** presented a modified hybrid fractal antenna which is combination of Minkowski curve and Sierpinski Carpet antenna structure for various wireless applications like Wi-Max, C band application, point-to-point hi speed wireless, X-band applications. The proposed antenna is a compact multiband antenna that resonates at different frequencies like 3.43GHz, 4.78GHz, 6.32GHz, 8.34GHz and 9.64GHz with better performance parameters like return loss, VSWR, gain. Here the dimensions of the proposed antenna are (45 × 38.92 × 1.6) mm<sup>3</sup> and further research can be done on the antenna minimization.

From the above survey, it is clear that these types of fractal structures provides improved performance parameters particularly good impedance matching, enhanced bandwidth, high radiation efficiency, high gain, omnidirectional radiation patterns, multiband properties.

#### 4.9 Based on various fractal geometry slots

In this section, we have discussed on fractal antenna structures designed with some fractal geometry slots. Various types of fractal shaped slots like kuch curve, sierpinski carpet, sierpinski gasket, miskowski curve, Hilbert curve are used for improved performances.

**W.L. Chen et al. [87] (2009)** presented a microstrip line fed printed fractal slot antenna with enhanced bandwidth. Authors have suggested that the impedance bandwidth of 56.2 % is obtained at the center frequency of 3.825 GHz the impedance bandwidth of 56.2 % is achieved at the center frequency of 3.825 GHz by using etching the wide slot as fractal shapes.

**V. V. Reddy et al. [88] (2013)** presents a multiband probe feed circular polarized design of microstrip patch antenna using fractal slots. Here authors have suggested that multiband property is achieved by using Koch curve geometry at boundaries of rectangular slot and square patch. The proposed antenna provides 3dB axial ratio bandwidths for the fractal slots like 3.2%, 1.6%, and 3.0% resonating at frequencies 2.45 GHz, 3.4 GHz, and 5.8 GHz respectively.

The authors have concluded that the presented antenna can be used for WLAN and Wi-MAX applications.

**M. Kaur et. al. [89] (2016)** presented a modified construction of a multiband Plus Slotted Fractal Antenna Array structure. Here the proposed antenna is the combination of the fractal antenna structure and the antenna array. The presented antenna is designed up to 2nd iteration and resonates at five different frequencies and a high gain of 10.26 dB at 6.9 GHz. The authors have concluded that the proposed antenna is suitable for major bands like S, C and X band applications.

**Kedar Trivedi et. al. [90] (2017)** presented an ultra wide band (UWB) stacked fractal antenna using triangular shaped fractal slots. Here the proposed antenna uses a tetrahedron shaped dielectric resonator and stacking of two different types of DR materials are considered along with use of small parasitic type conducting microstrip patches. Authors have suggested that the proposed antenna achieves UWB performance by using the combination of stacking of materials, fractal design and use of parasitic patches. The presented antenna gives a high impedance bandwidth within the frequency range of 3.5 GHz - 13.3 GHz, high gain of 7 dBi and radiation efficiency of 98.8 %. The authors have concluded that the proposed antenna is suitable for entire c-band and X-band applications.

**Yaqeen Sabah Mezaal et. al. [91] (2018)** presented printed novel slot antenna based on Sierpinski carpet geometry. Here authors have used a quasi-fractal type device with FR4 Substrate material specified by dielectric constant of 4.4, thickness 1.6 mm and loss tangent of 0.02 suitable for metrological satellite and WiMax applications. The proposed antenna provides dual-band compact structure with enhanced parameters like bandwidth and gain.

**Arpan Desai et. al. [92] (2018)** presented a microstrip patch antenna fractal geometry and defected ground structure for various wireless applications like cordless phone, wireless devices, Wi-Fi and wireless sensor networks. Here authors have demonstrated that the use of defected ground structure enables enhanced bandwidth in the microstrip patch antenna. The proposed antenna is of the dimension of (45×40×1.6) mm<sup>3</sup>, provides two resonance frequencies operating at 3.79GHz and 5.5 GHz with wide band ranging in 1.31GHz to 6.81 GHz i.e. a bandwidth enhancement of 135% is achieved with high gain of 6.5 dBi. The authors have concluded that the proposed antenna is simulated and fabricated and it is found that simulated and experimental results are matching in good agreement with each other.

From the above survey, it is observed that these types of fractal structures provide improved performance parameters particularly antenna minimization, high impedance bandwidth, high radiation efficiency, high gain, high radiation patterns and multiband/wideband properties.

## V. DISCUSSION ON FEATURES OF FRACTAL GEOMETRY

Fractal geometries shows various major advantages, hence these are widely used now a days for different applications. Some major advantages, disadvantages and applications are listed as follows.

# Fractal Geometry and Its Application to Antenna Designs

## 5.1 Advantages and disadvantages of using fractal geometry

### Advantages:

- I. Smaller in size, Lighter in weight and less in cost
- II. Use of fewer components
- III. Provides space filling, self similarity and scaling properties
- IV. Provides Increased bandwidth
- V. Provides Enhanced gain and directivity
- VI. Used for good input impedance matching
- VII. Provides multi-frequency performance
- VIII. Uses the essential combination of inductance and capacitance providing multiple resonances for the similar structures.
- IX. Using fractal compression technique the image can be enlarged, so there is no need for pixelisation.

### Disadvantage:

- I. Complexity in modeling the antenna
- II. Loss of gain in some cases
- III. Numerical analysis limitations
- IV. Not so beneficial after first few iterations

## 5.2 Major applications of fractal geometry

- I. Wideband antenna designs
- II. In design of frequency selective surfaces(FSS)
- III. Biological Sciences i.e to analyze biomedical phenomenon like bacteria growth pattern, pattern of nerve dendrites etc.
- IV. In fluid mechanics
- V. In computer graphics
- VI. Analyze seismic patterns
- VII. Fractal image rendering and image compression schemes

## VI. CONCLUSION

Several fractal geometries and their applications to various antenna designs are properly presented. This expands the area of research in wireless communication. Most of the nature inspired and human inspired fractal geometries are briefly summarized. The role of the mathematical modeling, the implementation procedure corresponding to the mathematical expressions is successfully explained. Low profile, low cost, and light weight antennas being popular are also discussed considering various fractal geometries. Broadband, wideband, and multiband being the most desirable characteristics of all the antennas are properly discussed considering the implementation of fractal geometry. Advantages, disadvantages, major applications, and future scope are properly presented to highlight the importance of the fractal geometry.

## VII. FUTURE SCOPE

- I. The detailed study the biosensor attractions can be done using fractal geometry.
- II. Fractal geometry mesh generation can also be used to reduce memory size requirements and analysis of finite element for CPU of the vibration problems by using space filling nature of fractal geometries.
- III. In computer applications, the fractal image compression and Fractal image rendering schemes can

also be used for the memory size reduction and processing time reduction.

- IV. Other modifications like bending or folding the antenna, introducing fractal geometry of the radiator, variation of length of ground plane, varying substrate thickness can be considered to study effects on the radiation properties.
- V. Using fractal geometries 3D polymer printing and metallization could be an alternative and effective method for low-cost rapid prototyping and testing of new and complex antenna systems.

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