

# Velocity Analysis of a Single Vertically Falling Non-Spherical Particle in Newtonian and Non-Newtonian Fluids

**Harpreeet Kaur, Neeraj Rani, B.P.Garg**

**Abstract-** An analytical investigation is applied for the velocity of a vertically falling non-spherical particle in Newtonian and non-Newtonian fluid. The velocity of vertically falling non-spherical particle can be described by the force balance equation (Basset-Boussinesq-Ossen equation). Variational Iterations Method (VIM) and Runge- Kutta 4<sup>th</sup> order method are used to solve the existing problem. The results were compared those obtained from VIM by R-K 4<sup>th</sup> order method. We obtained that VIM which was used to solve such non-linear differential equation with fractional power is simpler and more accurate than other methods. Analytical results indicate that the velocity in a falling procedure is significantly increased and more in Newtonian fluid. Also particle's velocity in Newtonian fluid reaches early at terminal velocity as compare to non-Newtonian fluid. To obtain the results for all different methods, the symbolic calculus software MATLAB is used.

**Keywords** - Newtonian fluid, non-Newtonian fluid, non-spherical particle, Terminal velocity, Variational Iteration method (VIM).

## I.INTRODUCTION

The problem of velocity of vertically falling non-spherical particles in Newtonian and non-Newtonian fluids is relevant to many situations of practical interest. In many processes it is often essential to obtain the path of particles that accelerate in the fluid region for designing or improving the process. Ganji and Gorji [1] find the solution of unsteady motion of vertically falling spherical particles in non-Newtonian fluid by Collocation Method and achieve a good results. Recently Kaur & Garg [2,3] investigate the acceleration motion of a single vertically falling non-spherical particle in incompressible Newtonian fluid by Diagonal Pade' [3/3]. Also several works have been done to study the unsteady motion of particles in Newtonian fluid [4]. Chhabra and Bagchi reported the distance traveled by accelerating spherical particles in downward vertical motion of particles in power law fluid [5,6]. Along with the same proposition, many researcher realized the physical significance of some analytical method such as the Homotopy Perturbation Method (HPH), Homotopy Analysis Method (HAM), and Variational Iteration Method (VIM) [7]. To solve the present problem Variational Iteration Method (VIM) is used and validated with Numerical Method (R-K 4<sup>th</sup> order method).

**Revised Manuscript Received on October 15, 2019.**

**Harpreeet Kaur**, Department of applied sciences, Adesh Institute of Engineering and Technology, Faridkot, Punjab (INDIA).

**Neeraj Rani**, Research scholar of I.K. Gujral Punjab Technical University, Kapurthala, Punjab (INDIA).

**B.P.Garg**, Department of applied sciences, Adesh Institute of Engineering and Technology, Faridkot, Punjab (INDIA).

E-mail id: - [harpreeet2610@yahoo.com](mailto:harpreeet2610@yahoo.com)

## II. PROBLEM STATEMENT

The Consideration of one-dimensional acceleration motion of a rigid body, non-spherical particle with diameter D, mass m and density  $\rho_s$  is vertically falling in incompressible non-Newtonian fluid of density  $\rho$  and viscosity  $\mu$ , u represents the velocity of the non-spherical particle at any instant time t, and g is the acceleration due to gravity [8]. Thus, the Basset–Boussinesq–Ossen (BBO) equation for the unsteady motion of particle in a fluid is given as:

$$m \frac{du}{dt} = mg \left(1 - \frac{\rho}{\rho_s}\right) - \frac{\pi D^2 \rho C_D}{8} u^n - \frac{\pi \rho D^3}{12} \frac{du}{dt} \quad (1)$$

where  $C_D=f(Re, n)$  drag coefficient could be obtained as

(1a)

$$C_D = \frac{24}{Re} X(n), \text{ Where } Re = \frac{\rho u^2}{K u^n D^{-n}} \text{ is the Reynolds number} \quad (1b)$$

and  $X(n) = 6^{\frac{n-1}{2}} \left(\frac{3}{n^2+n+1}\right)^{n+1}$  is a deviation factor

So by rewriting force balance Eq. (1) of motion of the particle,

$$\alpha \frac{du}{dt} + \beta(n) u^n - \gamma = 0, \quad u(0) = 0 \quad (1c)$$

In which  $\alpha = m + \frac{1}{12} \pi D^3 \rho$ ,  $\beta(n) = 3\pi KX(n)D^{2-n}$ ,

$$\gamma = mg \left(1 - \frac{\rho}{\rho_s}\right)$$

For  $\alpha = \beta = \gamma = 1$ , Eq. (1c) cab be written as follows-

$$\frac{du}{dt} + u^n - 1 = 0, \quad u(0) = 0 \quad (2)$$

## III. VARIATIONAL ITERATION METHOD (VIM)

In 1997, Jihuan He was introduced Variational Iteration Method (VIM) [9,10,11] to solve such nonlinear ordinary and partial differential equations. Slota obtained results for the Heat equation by VIM which were same as the exact solution [12]. To clarify the VIM, we consider the following differential

$$Lu(t) + Nu(t) = g(t) \quad (3)$$

Where L, N are linear and nonlinear operator respectively and  $g(t)$  is a non-homogeneous term. By using the Variational iteration method, a correction functional can be constructed as

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda \{Lu_n(\zeta) + Nu_n(\zeta) - g(\zeta)\} d\zeta \quad (3a)$$

Where  $\lambda$  is a Lagrange multiplier, which can be determined by the help of Variational theory, the subscript  $n$  means the nth approximation;  $u_n$  is restricted variation and  $\delta u_n = 0$ .

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According to VIM, firstly we will find Lagrange multiplier and after that to get the progressive iterations  $u_{n+1}$ ,  $n \geq 0$  which converge to solution. The solution is  $u = \lim_{n \rightarrow \infty} u_n$

To solve eq. (1c) using VIM

$$u_{n+1}(t) = u_n(t) + \int_0^t \Lambda \left\{ \alpha \frac{du_n(s)}{ds} + \beta u^n(s) - \gamma \right\} ds \quad (3b)$$

For  $\alpha = \beta = \gamma = 1$ , Eq.(3b) becomes

$$u_{n+1}(t) = u_n(t) + \int_0^t \Lambda \left\{ \frac{du_n(s)}{ds} + u^n(s) - 1 \right\} ds \quad (4b)$$

The stationary condition can be obtained as follows:

$$1 + \Lambda(t)_{s=t=0} = 0 \quad (4c)$$

Subsequently, the Lagrangian multiplier is obtained as:

$$\Lambda = -1 \quad (4d)$$

$$u_{n+1}(t) = u_n(t) - \int_0^t \left\{ \frac{du_n(s)}{ds} + u^n(s) - 1 \right\} ds \text{ with condition } u_0(t) = 0 \quad (4e)$$

## IV. R-K 4<sup>th</sup> ORDER METHOD

Numerical solutions have always played a significant role in properly understanding the subjective highlights and processes in various field of science. It is clear that the type of current problem is to discover the solution of differential equation of 1<sup>st</sup> order. So for this, we can apply numerical methods like Trapezoidal method, Euler's method and mid-point method etc. Thus the mid-point method (modification of Euler's method) is a suitable technique for present problem is also called R-K 4<sup>th</sup> order method (numerical method). This method is developed by C. Runge and M. W. Kutta around 1900.

$$u'(t) = 1 - u^n, \text{ with initial condition } u(0) = 0 \quad (5)$$

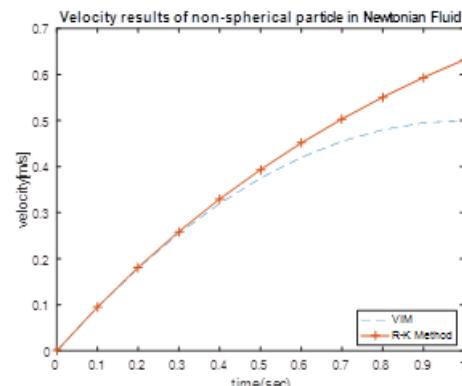
So f is a function of time and velocity

$$\text{i.e. } f(t, u) = 1 - u^n, u(0) = 0 \quad (5a)$$

## V. RESULTS AND DISCUSSION

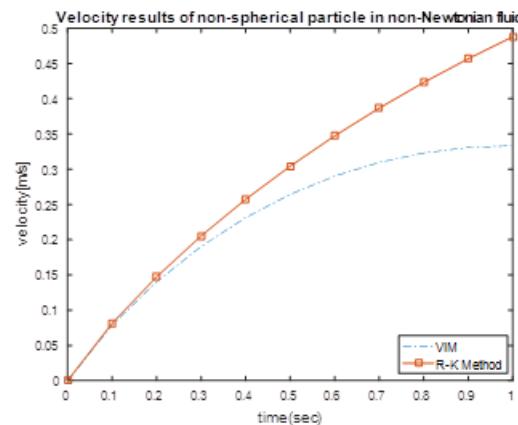
**Table.1.The velocity results of single vertically falling non-spherical particle in Newtonian and non- Newtonian Fluid for  $\alpha = \beta = \gamma = 1$**

time	Newtonian media, n=1		Non-Newtonian media, n=0.5	
In sec	VIM	R-K method	VIM	R-K method
0.0	0.0000	0.0000	0.0000	0.0000
0.1	0.0950	0.0952	0.0789	0.0813
0.2	0.1800	0.1813	0.1404	0.1474
0.3	0.2550	0.2592	0.1905	0.2054
0.4	0.3200	0.3297	0.2313	0.2573
0.5	0.3750	0.3935	0.2643	0.3043
0.6	0.4200	0.4512	0.2902	0.3472
0.7	0.4550	0.5034	0.3096	0.3866
0.8	0.4800	0.5507	0.3230	0.4230
0.9	0.4950	0.5934	0.3308	0.4566
1.0	0.5000	0.6321	0.3333	0.4879

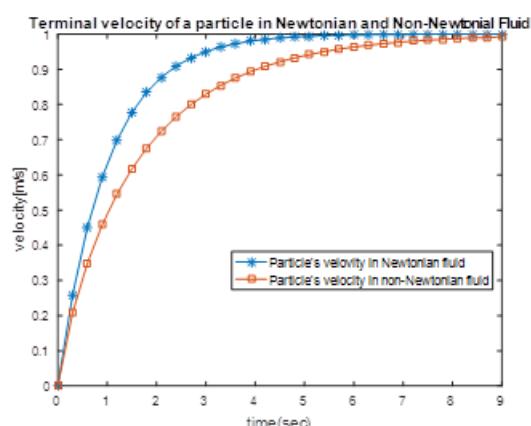


**Fig. 1. Velocity results of vertically falling non spherical particle in Newtonian fluid.**

Table.1, shows the velocity vs. time of non-spherical particle in Newtonian and non-Newtonian fluid by VIM and R-K 4<sup>th</sup> order method. Figs. 1 & 2 shows velocity results of vertically falling single non-spherical particle in Newtonian and non-Newtonian fluid. In this,  $u$ (vertically) denotes the velocity results of particle w.r.t. time  $t$  (horizontally) in seconds. Solution is obtained by VIM and R-k 4<sup>th</sup> order.



**Fig. 2. Velocity results of vertically falling non spherical particle in non-Newtonian fluid.**



**Fig. 3. Terminal Velocity of vertically falling non spherical particle in Newtonian and Non-Newtonian fluid**

These figures shows that the particle's velocity is increasing as time increasing. Fig. 3 shows Terminal velocity results of vertically falling single non-spherical particle by R-K 4<sup>th</sup> order method. It depicts that the particle in Newtonian

fluid attains its terminal velocity in short period as compare to non-Newtonian fluid. i.e. the acceleration motion of single particle which is falling in Newtonian fluid reaches early at zero (i.e. particle is not accelerating). In non-Newtonian fluid, the particle takes more time to reach at terminal velocity.

## VI. CONCLUSION

VIM is applied for the solution of a present problem without using any linearization, restrictions and transformations. From above discussion, it is clear that the VIM has a good agreement with numerical method and provides highly reliable results. Also, the current technique can be used to develop the valid solution of other nonlinear differential equations. In addition, the above discussion shows that the particle's velocity is more and the particle attains its terminal velocity in short period in Newtonian fluid. It depicts that the acceleration motion of single particle which falling in Newtonian fluid reaches early at zero (particle is not accelerating). In non-Newtonian fluid, the particle takes more time to reach at terminal velocity.

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## APPENDIX (MATLAB CODE)

### 1. NEWTONIAN FLUID, $n = 1$ (Fig. 1)

% VIM

```
t = 0: 0.1: 1;
n = length(t);
for i = 1:n;
uv1(i) = -(t(i) * (t(i) - 2))/2;
```

```
format short
end
uv1(:)
% R – K 4th order
f = @(t,u)(1-u.^n);
n = 1;
t = 0;
ur1 = 0;
h = 0.1;
t = 0:h:1;
for i = 1:(length(t) - 1);
k1 = f(t(i),ur1(i));
k2 = f(t(i) + h/2,ur1(i) + h/2 * k1);
k3 = f(t(i) + h/2,ur1(i) + h/2 * k2);
k4 = f(t(i) + h,ur1(i) + h * k3);
ur1(i + 1) = ur1(i) + (k1 + 2 * k2 + 2 * k3 + k4)
    * h/6;
format short
end
ur1(:)
Plot(t,uv1,'--',t,ur1,'-+');
```

### 2. NON-NEWTONIAN FLUID, $n = 0.5$ (Fig.2)

% n = 0.5

% VIM

```
t = 0: 0.1: 1;
n = length(t);
for i = 1:n;
uv2(i) = t(i) - (2 * t(i)^(3/2))/3;
format short
end
uv2(:)
% R – K 4th order
f = @(t,u)(1-u.^n);
n = 0.5;
t = 0;
ur2 = 0;
h = 0.1;
t = 0:h:1;
for i = 1:(length(t) - 1);
k1 = f(t(i),ur2(i));
k2 = f(t(i) + h/2,ur1(i) + h/2 * k1);
k3 = f(t(i) + h/2,ur1(i) + h/2 * k2);
k4 = f(t(i) + h,ur1(i) + h * k3);
ur1(i + 1) = ur1(i) + (k1 + 2 * k2 + 2 * k3 + k4)
    * h/6;
format short
end
ur2(:)
Plot(t,uv2,'-.',t,ur2,'-s');
```

### 3. NEWTONIAN FLUID & NON-NEWTONIAN FLUID (Fig.3)

% R – K 4th order

```
f = @(t,u)(1-u.^n);
```

n = 1;

t = 0;

ur1 = 0;

h = 0.3;

t = 0:h:9;

```
for i = 1:(length(t) - 1);
```

```
k1 = f(t(i),ur1(i));
```



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```
k2 = f(t(i) + h/2, ur1(i) + h/2 * k1);
k3 = f(t(i) + h/2, ur1(i) + h/2 * k2);
k4 = f(t(i) + h, ur1(i) + h * k3);
ur1(i + 1) = ur1(i) + (k1 + 2 * k2 + 2 * k3 + k4)
             * h/6;
format short
end
ur1(:)

% R - K 4th order
f = @(t,u)(1 - u.^n);
n = 0.5;
t = 0;
ur2 = 0;
h = 0.3;
t = 0:h:9;
for i = 1:(length(t) - 1);
    k1 = f(t(i), ur2(i));
    k2 = f(t(i) + h/2, ur1(i) + h/2 * k1);
    k3 = f(t(i) + h/2, ur1(i) + h/2 * k2);
    k4 = f(t(i) + h, ur1(i) + h * k3);
    ur1(i + 1) = ur1(i) + (k1 + 2 * k2 + 2 * k3 + k4)
                  * h/6;
format short
end
ur2(:)

Plot(t,ur1,'-*',t,ur2,'-s');
```

## AUTHOR'S PROFILE



**Dr. Harpreet Kaur**, (1985) is Assistant Professor (Mathematics) at Adesh Institute of Engineering & Technology, Faridkot affiliated to IKGPTU, Kapurthala, Punjab, India. She attained her Post graduate from GNDU, Amritsar and Doctorate degree from IKGPTU, Kapurthala. Her field of specialization is Fluid Dynamics. She has ten international and national research papers in her credit. She has been awarded 'Young Scientist Award' and 'Young Researcher Award' for achievements in the field of research.



**Neeraj Rani**, (1983) received her M.Sc. in Mathematics from LKC, Jalandhar in 2006. Presently she is research scholar in Applied Sciences Department (Mathematics) of IK Gujral Punjab Technical University, Kapurthala, Punjab (INDIA).



**Dr. BP Garg**, (1969) is Director of one of the reputed engineering colleges (Adesh Institute of Engineering & Technology, Faridkot) affiliated to IKGPTU, Kapurthala, Punjab, India. He attained his Post graduate and Doctorate degree from H.P. University, Shimla. His field of specialization is Fluid Mechanics. He has forty international and national research papers in his credit. He is life member of various academic and professional bodies. He is serving as Referee of many prestigious and internationally reputed journals. He has been awarded 'Best Educationist Award' for outstanding achievements in the field of education by International Institute of Education & Management and 'Rashtriya Vaidya Gaurav Gold Medal Award' for outstanding achievements in the field of education by Indian Solidarity Council, New Delhi