International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249 – 8958, Volume-9 Issue-1, October 2019

Sparse Finite Impulse Response Low Pass Filter Design using Improved Firefly Algorithm

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Abstract: ***In this work, optimal sparse linear phase Finite impulse response filters are designed using swarm intelligence-based Firefly optimization algorithm. Filters are designed to meet the desired specification with fixed and variable sparsity. The objective function is formulated consisting of three parameters, i. e., maximum passband ripple, maximum stopband ripple and stopband attenuation. The effectiveness of the proposed method is evaluated in two stages. In first stage, the designed filters have been compared with non- sparse in terms of deviation in their specification. The Comparative analysis depicts that the proposed approach of sparse linear phase FIR filter design method performs better than the conventional methods without significantly deviating from the desired specification. The proposed designed filter is then implemented on xilinx ISE14.7(Vertex7) design environment and their performance is compared in terms of time delay, resource utilizaion and frequency of operation. In the second stage, designed sparse FIR filters are compared with earlier state of art sparse FIR filters design techniques.***

Keywords : FIR Filters, Global Optimization, Firefly algorithm, Sparse.

I. INTRODUCTION

Finite impulse response(FIR) digital filter have continually been a source of attractions for researcher due to their highly desirable attributes such as high stability, linear phase and ease of implementation. These qualities have been resulted in wide variety of applications of signal processing[1]. It has played major role in diverse field such as Biomedical, Radar and Digital communication [2]. However design of FIR digital filter is still challenging because of its multi-requirements related to implementation complexity, hardware cost, power minimization, filter order reduction, lower pass band and stop band ripple of FIR filter [3]. In this regard different techniques reported includes differential evolution[3], seeker optimization[4], Opposition based harmony search[5] and Artificial Bee colony [6]. Designing of sparse FIR filter is more challenging in addition to meeting the desired specification similar to FIR filter.

The advantage of using sparse FIR filter is that it reduces the implementation complexity by omitting the multiplier and adder corresponding to zero valued coefficients[7,8]. However the optimization of Sparse FIR filter based on the specific frequency domain constraint is highly non-convex, which is unable to give efficiently the global optimal solution in the polynomial time[9]. Because of its non-convexity l_1 norm has been used as a good alternation to the l_0 norm. The techniques in [10], use linear programming to obtain sparse filters while in [11] sparsity is treated as a specification and tries to minimize least square error through l_1 norm minimization. An iterative design methodology is recommended in [12] to reduce reweighted *'l'* norm of the coefficients and another iterative method 'second arrange cone programming' is discussed in [13] , these algorithms showed the enhancement over [10] and [11]. In [14], the orthogonal Matching Pursuit (OMP) is utilized in designing linear phase sparse FIR filters. The OMP algorithm is a greedy method. A greedy algorithm is introduced in [14] that consecutively turns the impulse response of an FIR filter to zeros, until the desired specification of the filters are violated. Two heuristic methodologies are presented in [15] to design sparse FIR filters. The first one is the smallest-coefficient method and another one is a minimum-increase method. As its name suggest, the first method select the coefficients with smallest magnitude, on the other hand the second method sets a coefficient that will give a minimum increase of approximation error. A genetic algorithm based method [16] which encodes possible index of zero value coefficients as the gene on chromosome. The results shows that the genetic algorithm based method acquire better results than algorithm method in [13]. However, the genetic algorithm has a limitation that computational complexity is relatively high. In [17] binary particle swarm optimization (BPSO) technique is used to successively thin the filter coefficient until no sparse solution is obtained. Thus showed the enhancement over [13] and [15]. In [18] non-convex problem is designed as combinational optimization problem which is then solved by simulated annealing(SA) and showed the improvement over [13] and [15] .

Here, sparse FIR filter is designed by using Firefly Optimization (FFO) technique. The FFO is an impactful meta - heuristic method. It is designed by Yang in 2007 to solve the optimization problems. FFO has easier implementation concept and simpler design technique. The many researcher therefore studied the FFO and variants were described to solve various optimization problem, for example combinatorial, continuous, dynamic and multi objective. However, FFO is a very effective algorithm but it lacks in proper balancing between the exploitation and exploration capabilities. Sometimes, the algorithm also stuck in local optimal value. So, improved FFO algorithm is proposed to overcome the issues in FFO.

IFFO is used to determine the optimal coefficients of sparse FIR low pass filter and compared with exiting algorithms. The result obtained by IFFO shows the major improvement in passband ripple (PBR), stopband ripple (SBR) and stopband attenuation (SBA) as compared to the reported values. In signal processing, speed of the processor depends on speed of the multiplier. So, area and delay efficient digital

Revised Manuscript Received on October 15, 2019

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processor are extremely important in digital signal processing application. Thus, the designed FIR filter circuit (sparse and non-sparse) is simulated on Xilinx system generator with MATLAB2013 and compared in terms of in terms of number of slice register, number of LUTS, time delay, maximum frequency.

The manuscript is divided into 4 sections. The second section provides an information about the formulation of sparse FIR filter design as an optimization problem. Third section describes the IFFO. Fourth Section present result analysis of designed sparse FIR filters using the proposed IFFO. Last section concludes the manuscript.

II. FORMULATION OF SPARSE FIR FILER DESIGN AS AN OPIMIZATION PROBLEM

Formulation of linear phase sparse FIR filter design as an optimization problem has been discussed here. The coefficients of symmetric FIR filter of order N where N=2M can be given as

$$s_n = h_{M+n} = h_{M-n} \qquad n=1\ldots..M \tag{1}$$

and the amplitude response is given below

$$H_a(\omega) = \sum_{n=0}^{\infty} s_n = s_0 + \sum_{n=1}^{M} 2s_n \cos n\omega = a^T s \quad \text{for } s_n \in R \tag{2}$$

where ,

$$a(w) = \left[1, 2\cos w, \ldots\ldots\ldots\ldots, 2\cos\frac{M}{2}w\right]^T$$

(3)

$$s = [s_0, s_1, \ldots\ldots\ldots\ldots s_M]^T \tag{4}$$

The sparse FIR filter coefficients minimization for an order N is denoted as $[s_0, s_1 \ldots s_M]$. The sparsity of an FIR filter is calculated by finding number of non-zero filter coefficients in set s[16, 17]. The problem formulation of sparse FIR filter design can be written as

$$\min_{s0,s1,..,s_M} : \quad \|s\|_0 \tag{5}$$

$$\text{s.t} : 1-\delta_p \le |H_a(\omega) - H_d(\omega)| \le 1+\delta_p \quad \text{for } \omega\varepsilon[0,\omega_p]$$

$$-\delta_s \le H_a(w) - H_d(w) \le \delta_s \quad \text{for } \omega\varepsilon[\omega_s,\pi] \tag{6}$$

Where δ_p and δ_s are PBR and SBR, $\omega_{p,}\omega_s$ are Passband edge and Sideband edge. $H_d(\omega)$ is the desired frequency response. The main aim here is to optimize the coefficients, i. e., s_n such that the value of maximal absolute error

$E(\omega) = H_a(\omega) - H_d(\omega)$ is minimal over $0 \le \omega \le \pi$. (7)

Now by utilizing a uniform dense grid over PB and SB, the problem formulation of optimal filter design can be written as

$$\min_{s_n,\delta} : \quad \delta \tag{8}$$

$$\text{S.t} : \quad W(\omega_i)[H_a(\omega) - H_d(\omega)] \tag{9}$$

where $W(\omega_i)$ is weight factor, $\omega_i \epsilon[0,\pi], i = 1, \ldots\ldots\ldots\ldots L$. Now by expanding the absolute value term and plugging the error expansion[18]. Design problem is formulated into a standard LPP that is written as

$$\text{Min:} \quad a^T s \tag{10}$$

$$\text{S.t} : \quad Qs \le c \tag{11}$$

where

$$a = [0\ 0 \ldots\ldots 0\ 1]^T \in R^{2+n}$$

(12)

$$c = \begin{bmatrix} d \\ -d \end{bmatrix} \in R^{2L}$$

(13)

$$d = [H_d(\omega 1) \ldots \ldots H_d(\omega 2) \ldots H_d(\omega k)] \in R^L \tag{13}$$

(14)

$s = [h_{NZ}^T \quad \delta] \in R^{2+n}$ where h_{NZ}^T vector contains the filter coefficients of NZ. (14)

(15)

$$Q = \begin{bmatrix} C & -v \\ -C & -v \end{bmatrix} \in R^{(n+2)*2L} \tag{15}$$

(16)

where (16)

$$v = \left[\frac{1}{W(\omega_i)} \cdots \frac{1}{W(\omega_i)}\right]^T \in R^L$$

(17)

and (17)

$$C = \left[\begin{pmatrix} 1 & 2\cos(\omega_1) & \cdots & 2\cos(M\omega_1) \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 2\cos(\omega_1) & \ldots & 2\cos(M\omega_1 \end{pmatrix}\right] \in R^{(n+1)*L}$$

(18)

So by solving (9) using LPF, optimal solutions is obtained. However the l_0 norm measure the number of non-zero value coefficient. Nevertheless, l_0 norm is a non-convex minimization problem and is difficult to find computationally efficient global optimal solution. So in the next section, we consider a FFO algorithm that has potential to solve the objective function in order determine the optimum filter coefficients.

III. FIREFLY OPTIMIZATION ALGORITHM

Xin-Sheng Yang at Cambridge University developed FFO in 2007 year. This algorithm is based on flashing style of fireflies[19]. The flashing behavior of fireflies are produced by bioluminescence process.

The main aim of flashing light of fireflies is to attract the potential prey and potential mate. Fireflies have their own pattern of flashing. FFO has three basics idealized rules which are based on the characteristic of the flies[19,20]. These rules are as follow:

1. Fireflies are unisex. They are attracted to each other irrespective of their gender.
2. Attractiveness and brightness of two fireflies are correlated. So less attractive firefly moved toward brighter firefly.

3. The brightness of firefly depends on the objective function (11).

Mathematical formulation of FFO

FFO is based on two key: first, light intensity variation and second, attractiveness formulation[21]. For easiness, suppose that firefly's attractiveness is assessed by its brightness, which is linked with the objective function. The intensity of light is inversely proportional with distance r from the source of light. In this way, the light intensity reduces as the distance increases[20]. Mathematically it is presented as follows:

$$I = I_0\, e^{-\gamma x^2} \qquad (18)$$

Where, I_0 = initial intensity of light, I = intensity of light, x= distance between firefly i and j, γ = absorption coefficient of light. As stated earlier, intensity of light is proportional to attractiveness. Therefore, the attractiveness is given by the equation

$$\beta = \beta_0\, e^{-\gamma x^2} \qquad (19)$$

where,

β = Attractiveness

β_0 = attractiveness at x = 0

The distance between two fireflies is expressed by using Cartesian distance formula:

$$x_{ij} = |s_i - s_j| = \sqrt{\sum_{k=1}^{d}(s_{i,k} - s_{j,k})^2} \qquad (20)$$

Firefly are attracted toward the more alluring fireflies from position i to j. So the updated position is expressed as:

$$\Delta s_i = s_i + \beta_0\left({s^t}_j - {s^t}_i\right) + \alpha\varepsilon_i \qquad (21)$$

Where $\gamma = [0,\infty]$. Here the second component of the equation represents the attractiveness and the third component is randomization parameter. The parameter γ play important role in showing the behavior of algorithm and determine its convergence speed. The value of γ is varies from 0 to 10 according to applications.

Improved FFO algorithm

After the simulation of firefly algorithm, it has been observed that the fireflies are gradually moving around the global maximal value point or local maximal value point with the reduction in distance between fireflies iteratively [22]. But the attractiveness between the fireflies is increased gradually. Thus, the fireflies are not able to locate the optimal solution in the global best position. Therefore, to enhance the firefly optimization accuracy the performance of the firefly algorithm is to be improving by adding inertia weight into the position updating of firefly algorithm. As the inertia weight linearly decreases with respect to time. So, in initial stages the inertia weight linearly increases which enhances the global exploration process and in the last stage inertia weight decreases which enhance the local exploration process. It also influences the fireflies to move fast and accurately toward the best maximal value point iteratively. The improved updated position is expressed as

$$\Delta s_i = w * s_i + \beta_0\left({s^t}_j - {s^t}_i\right) + \alpha\varepsilon_i \qquad (22)$$

where,

$$w = w_{max} - \frac{w_{max} - w_{min}}{itr_{max}} * itr \qquad (23)$$

Where w_{min} and w_{max} is the initial and final weight.

Convergence of IFFO algorithm

In this subsection, karush kuhn tucker optimal condition has been used to prove the convergence of improved firefly algorithm analytically.

minimize: $f(s)$ (24)

subject to : $(s - u) \leq 0$ and $(-s + l) \leq 0$ (25)

where u and l are upper bound limit and lower bound limit. The global optimal karush kuhn tucker condition for first order at point $s \in f$ are

$\Delta f = \Lambda_1 + \Lambda_2 = 0$ (26)

$\Lambda_1 (c - l) \leq 0$ *and* $\Lambda_2 (-c + l) \leq 0$ (27)

where, Λ_1 and Λ_2 $|\Lambda_1 \Lambda_2 \in R^n|$ (28)

The other karush kuhn condition are also given as below:

1. if $m_i < s_i < l_i$, which implies that $(\nabla f(s))_i = 0$.
2. if $s_i = l_i$ and $m_i < s_i$, then $(\Lambda_2)_i = 0$, which implies that $(\nabla f(s))_i = (\Lambda_1)_i \cong (\nabla f(s))_i \leq 0$.
3. if $s_i = m_i$ and $s_i < l_i$, then $(\Lambda_1)_i = 0$, which implies that $(\nabla f(s))_i = (\Lambda_2)_i \cong (\nabla f(s))_i \leq 0$.

The taylor series expansion of a function $f(s)$ at point s_k is given as

$$f(s) = f(s_k) + (s - s_k) + \Delta f(s_k) + \frac{(s - s_k)}{2!} H(s - s_k)^T + \cdots$$
(29)

where H is hessian matrix.

The first two term are expanded in

$$\Delta f(s) = \Delta f(s_k) + (s - s_k) \qquad (30)$$

$$\Delta f(s) = \Delta f(s_k) + (s - s_k)\mathrm{H} \qquad (31)$$

If $f(s)$is the optimal solution of (s^*) in the range m < (s^*) < l, then according to karush kuhn tacker condition, ($\Delta f(s)$=0)

$$\Delta f(s_k) + (s - s_k)\mathrm{H} = 0 \qquad (32)$$

$$s = s_k - H^{-1}(s - s_k) \qquad (33)$$

For initial point (s_k), the next point is calculated by equation:

$$s_{k+1} = s_k + \mu_k H_k \Delta f(s_k) \qquad (34)$$

where μ_k is the step size of iteration k.

H_k is the hessian matrix approximation at point k. When karush kuhn tucker condition is satisfied by Δf = 0. In IFFA, the fireflies position is calculated by using the equation (11). The fitness function corresponding to it is given by

$$f\left(v_{ij}(x)\right) = f\left(s_{ij}(x)\right) + H \times \Delta f \qquad (35)$$

where H is the hessian approximation matrix , which produces disruption $H \times \Delta f$ in the fitness $f\left(s_{ij}(x)\right)$. In each iteration process, there is some improvement in fitness function which assure the convergence of fitness function of the improved _rey algorithm to the optimum point.

IV. RESULT AND DISCUSSION

In this section, effectiveness of the proposed sparse FIR Low pass filter design is discussed. The design procedure for sparse FIR filter elaborated in the previous section has been implemented using MATLAB. The efficacy of the proposed method has been shown by the following parameter: maximum PBR, maximum SBR, maximum SBA , HD and number of multipliers. The control parameters of

algorithms used are set as iterations= 1000 and upper bound and lower bound= [10, {10], population size =100. The Filter has been designed in MATLAB 14.1 and further it is implemented on Virtex-7 with Xilinx ISE14.7. Hamming Distance is assimilate in objective function to minimize the transition
between successive filter coefficients. While hamming distance is calculated by representing the filter coefficients in IEEE 754-32 bit floating-point. The filter design parameters are given in Table 1.

Table I. Filter Specification

Filter order	20
Passband edge frequency	0.45
Stopband edge frequency	0.55
Passband ripple	0.1
Stopband ripple	0.01

The proposed sparse FIR filter design using IFFO has been evaluated in two stages. In the first stage, the performance of sparse FIR filter design using the proposed IFFO algorithm has been compared in terms of passband ripples, stopband ripples, HD, number of multipliers with standard FFO and other reported algorithms i.e. paricle swarm optimization (PSO), differential evolution (DE), cat swarm optimization (CSO), seeker optimization algorithm (SOA). Further, the designed filter has been simulated and analysed in Xilinx ISE14.7 design envi- ronment. The filter coefficients obtained from the proposed algorithm are shown Table II.

Table II. Sparse FIR Filter coefficients

Coefficients	FFO	IFFO
h(1) = h(21)	0.00000000000000	0.00000000000000
h(2) = h(20)	-0.0148405375379	0.00981137559560
h(3) = h(19)	0.00000000000000	0.00000000000000
h(4) = h(18)	0.01362711839655	-0.00681110568138
h(5) = h(17)	0.00000000000000	0.00000000000000
h(6) = h(16)	-0.0386736899125	0.03629280255804
h(7) = h(15)	-0.0807307008100	0.07808941388578
h(8) = h(14)	-0.0377264508368	0.03927230699237
h(9) = h(13)	0.11455415544510	-0.11101734904306
h(10)=h(12)	0.29487371715936	-0.29479698695048
h(11)	0.37840275260219	-0.37470380822630

It can be seen from the Table II that at three positions, filter coefficients are zero, which means FIR filter is designed with 25 % of sparsity. Here, filter order is 20. Therefore, the length of the filter is 21. Fig.a. depicts the magnitude response of the sparse FIR Low pass filter.

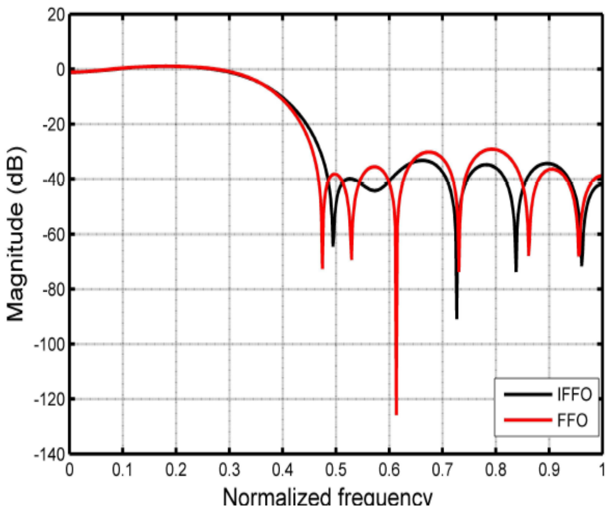

Fig. a. Magnitude response of sparse FIR filter designed using FFO and IFFO.

The parameter of magnitude response of sparse FIR filter has been used to compare the IFFO and standard FFO in terms of Passband edge frequency (f_p), passband ripple (δ_P), stopband attenuation(As), Passband edge frequency (f_s) is depicted in Table III.

Table III. Parameter of Sparse FIR Filter

Algo	parameter	Min.	Max.	Mean	Std.
FFO	f_p	0.267	0.269	0.268	0.001
	δ_p	0.121	0.131	0.127	0.005
	A_s	29.04	30.35	29.59	0.680
	f_s	0.470	0.494	0.479	0.013
	δ_s	0.030	0.035	0.033	0.025
IFFO	f_p	0.251	0.273	0.264	0.011
	δ_p	0.091	0.130	0.116	0.021
	A_s	29.09	33.200.476	31.48	2.133
	f_s	0.472	0.035	0.473	0.002
	δ_s	0.021		0.026	0.007

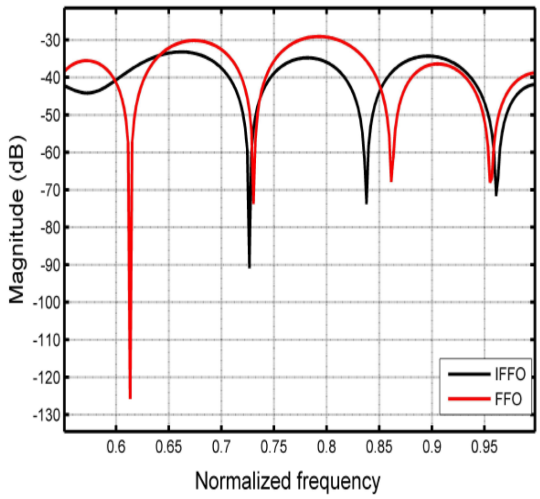

Fig. b. Stopband response of sparse FIR filter designed using FFO and IFFO

It is evident from the comparative study which is shown Table IV that IFFO algorithm gives better performance in terms of stopband ripples and high Stopband attenuation where as standard FFO algorihm has better performance in passband ripples in the design of sparse FIR Low pass filter in comparison to non-sparse FIR filter design using other existing algorithms. It also be seen that due to the trade off between the filter specification (passband and stopband), desired filter are not able to meet them simultaneously. Fig.b. depict the magnitude plots of Stopband of sparse FIR Low pass filter.

Table IV. Comparison of FIR filter parameters obtained using FFA and IFFA with existing algorithms

Algos	PBR normalized	SBR normalized	HD	No. of Multiplier
Kaiser Window	0.0800	0.0880	9.430	21
PM [5]	0.0664	0.0665	8.718	21
RGA [5]	0.1142	0.0495	9.031	21
PSO [5]	0.1230	0.0397	8.500	21
DE [3]	0.1500	0.0390	-	21
Hybrid PSO &DE [7]	0.2570	0.2590	-	21
CSO [6]	0.1640	0.0198	8.937	21
SOA [4]	0.1380	0.0243	-	21
FFO	0.1218	0.0353	7.625	15
IFFO	0.1280	0.0218	6.875	15

Table V shows the comparative analysis of sparse FIR Low pass filter design with non-sparse FIR Low pass filter design in terms of percentage change in MPR, SBA, HD and number of multipliers. It is observed from comparative analysis that low complexity FIR filter can be designed with insignificant loss in filter specifications.

Table V. Percentage improvement in filter specification using IFFA algorithm with respect to other existing algorithms

Algorithm	PBR	SBR	HD	No. of Multiplier
Kaiser window	40.000	84.0000	27.0943	28.5714
PM [5]	16.666	79.1150	21.1474	28.5714
RGA [5]	97.180	70.1265	23.8758	28.5714
PSO [5]	21.730	60.2693	19.1176	28.5714
DE [3]	44.000	59.3103	-	28.5714
Hybrid PSO & DE[7]	82.165	95.2610	-	28.5714
CSO[6]	56.250	-20.4081	23.0769	28.5714
SOA[4]	26.315	17.4825	-	28.5714
FFO	28.440	53.3596	9.8360	0.0000
Overall improvement	45.860	55.7157	20.6913	25.3968

The FIR filter circuit (sparse and non-sparse) using proposed algorithm is implemented on Virtex-7 with Xilinx ISE14.7 design environment and comparison is made in terms of number of slice register, number of LUTS, time delay, maximum frequency. The simulation of the FIR filter circuit is performed using Xilinx system generator with MATLAB2014. The simulation report analysis of FIR filter (sparse and non-sparse) is presented in Table VI.

Table VI. Device utilization of report sparse FIR Filter

Parameter	Sparse	Non-sparse
Number of slice register (607,200)	353	369
Number of slice LUTS(303,600)	100	121
Time delay(nsec)	20.60	27.67
Maximum frequency(MHz)	45.53	36.13

It has been observed from Table 6 that the sparse FIR filter designs take less time delay and less registers in comparison to Non-sparse FIR filter design. Hence, it is concluded that sparse FIR filter is suitable for the high-speed applications of digital signal processing. Further sparse FIR filter is design for filter order 80, 90,100. The filter specifications are given in Table VII.

Table VII. Filter Specifications

Filter order	80
Passband region	0, 0.0436 π
Stopband region	0.0872 π, 0
Passband ripple	Within $\pm$ 0.5 dB of unity
Stopband ripple	-20dB,-25dB, -30dB,-35dB, -40dB

The numbers of non-zero coefficients obtained from the three different algorithms are presented in Table VIII. Table VIII shows that with a SBA of -20dB, number of non-zero coefficients obtained in [18] and [22] are 29 when filter order is 80, 90 and 100. Whereas with the proposed algorithm, 24 non-zero coefficients were achieved for same order set with -20dB SBA. This demonstrates the robustness of the proposed algorithm.

Table VIII. Number of nonzero coefficients of sparse FIR Low Pass filter of order 80

SBA(dB)	SAA[18]	GA[16]	IFFO
-20dB	29	29	24
-25dB	37	37	32
-30dB	47	47	36
-35dB	55	55	40
-40dB	63	65	45

V. CONCLUSION

In this paper, a novel design method based on nature-inspired algorithm proposed for the design of sparse linear phase FIR Low pass filter. The resulting objective function is optimized using FFO and IFFO algorithms. It is proved from the results that the proposed design techniques perform well for filter de- signing. When compared, it has concluded that IFFO algorithm has given best performance among existing optimization methods. The comparative analysis clearly indicates that with 25 % of sparsity, sparse linear phase FIR filter shows the more desirable performance in terms of stopband attenuation, passband ripple and stopband ripple with a very less deviation from non-sparse FIR filter. Later, the simulation analysis on Virtex-7 with Xilinx ISE14.7 also proves that the sparse FIR filters are area and delay efficient in comparison to non-sparse FIR filter. Hence, sparse FIR filter can be used for any practical design problem of digital filter design.

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