Dualities between Laplace Transform and Some Useful Integral Transforms

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Abstract: ***Integral transforms have wide applications in the various disciplines of engineering and science to solve the problems of heat transfer, springs, mixing problems, electrical networks, bending of beams, carbon dating problems, Newton's second law of motion, signal processing, exponential growth and decay problems. In this paper, we will discuss the dualities between Laplace transform and some useful integral transforms namely Kamal transform, Elzaki transform, Aboodh transform, Sumudu transform, Mahgoub (Laplace-Carson) transform, Mohand transform and Sawi transform. To visualize the importance of dualities between Laplace transform and mention integral transforms, we give tabular presentation of the integral transforms (Kamal transform, Elzaki transform, Aboodh transform, Sumudu transform, Mahgoub transform, Mohand transform and Sawi transform) of mostly used basic functions by using mention dualities relations. Results show that the mention integral transforms are strongly related with Laplace transform.***

Keywords: Laplace; Kamal; Elzaki; Aboodh; Sumudu; Mahgoub (Laplace-Carson); Mohand; Sawi transforms.

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I. INTRODUCTION

Many process and phenomenon of science, engineering and real life can be expressed mathematically and solved by using integral transforms. The problems arise in the field of signal processing, statistics, thermal science, medicine, fractional calculus, aerodynamics, civil engineering, control theory, cardiology, quantum mechanics, space science, marine science, biology, gravitation, nuclear magnetic resonance, heat conduction, economics, telecommunications, nuclear reactors, detection of diabetes, chemistry, stress analysis, electricity, physics, potential theory, mathematics, deflection of beams, vibration of plates, defense, Brownian motion and many other fields can be easily handle with the help of integral transforms by converting them into mathematical form. In the advanced time, researchers are interested in solving the advance problems of research, science, space, engineering and real life by introducing new integral transforms. Aggarwal and Chaudhary [1] discussed Mohand and Laplace transforms comparatively by solving system of differential equations using both integral transforms. Recently many scholars [2-7] used different integral transforms namely Kamal transform, Elzaki transform, Aboodh transform, Sumudu transform, Mahgoub (Laplace-Carson) transform and Mohand transform for evaluating improper integrals which contains error function in the integrand. Mahgoub [8] gave Sawi transform which is a new integral transform.

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The aim of this study is to establish duality relations between Laplace transform and some useful integral transforms namely Kamal transform, Elzaki transform, Aboodh transform, Sumudu transform, Mahgoub (Laplace-Carson) transform, Mohand transform and Sawi transform.

II. LAPLACE TRANSFORM

The Laplace transform of the function $Z(\gamma), \gamma \geq 0$ is given by [1]

$$L\{Z(\gamma)\} = \int_0^\infty Z(\gamma)e^{-\epsilon\gamma}d\gamma = B(\epsilon) \quad (1)$$

III. KAMAL TRANSFORM

Kamal transform of the function $Z(\gamma), \gamma \geq 0$ is given by [2]

$$K\{Z(\gamma)\} = \int_0^\infty Z(\gamma)e^{\frac{-\gamma}{\epsilon}}d\gamma = C(\epsilon), \quad (2)$$
$$0 < k_1 \leq \epsilon \leq k_2$$

IV. ELZAKI TRANSFORM

Elzaki transform of the function $Z(\gamma), \gamma \geq 0$ is given by [3]

$$E\{Z(\gamma)\} = \epsilon\int_0^\infty Z(\gamma)e^{-\frac{\gamma}{\epsilon}}d\gamma = D(\epsilon), \quad (3)$$
$$0 < k_1 \leq \epsilon \leq k_2$$

V. ABOODH TRANSFORM

Aboodh transform of the function $Z(\gamma), \gamma \geq 0$ is given by [4]

$$A\{Z(\gamma)\} = \frac{1}{\epsilon}\int_0^\infty Z(\gamma)e^{-\epsilon\gamma}d\gamma = F(\epsilon), \quad (4)$$
$$0 < k_1 \leq \epsilon \leq k_2$$

VI. SUMUDU TRANSFORM

Sumudu transform of the function $Z(\gamma), \gamma \geq 0$ is given by [5]

$$S\{Z(\gamma)\} = \int_0^\infty Z(\epsilon\gamma)e^{-\gamma}d\gamma = G(\epsilon), \quad (5)$$
$$0 < k_1 \leq \epsilon \leq k_2$$

VII. MAHGOUB (LAPLACE–CARSON) TRANSFORM

Mahgoub (Laplace-Carson) transform of the function $Z(\gamma), \gamma \geq 0$ is given by [6]

$$M_*\{Z(\gamma)\} = \epsilon\int_0^\infty Z(\gamma)e^{-\epsilon\gamma}d\gamma = H(\epsilon), \quad (6)$$
$$0 < k_1 \leq \epsilon \leq k_2$$

VIII. MOHAND TRANSFORM

Mohand transform of the function $Z(\gamma), \gamma \geq 0$ is given by [1, 7]

$$M\{Z(\gamma)\} = \epsilon^2\int_0^\infty Z(\gamma)e^{-\epsilon\gamma}d\gamma = I(\epsilon), \quad (7)$$
$$0 < k_1 \leq \epsilon \leq k_2$$

IX. SAWI TRANSFORM

Sawi transform of the function $Z(\gamma), \gamma \geq 0$ is given by [8]

$$S^*\{Z(\gamma)\} = \frac{1}{\epsilon^2}\int_0^\infty Z(\gamma) e^{\frac{-\gamma}{\epsilon}} d\gamma = J(\epsilon), \quad (8)$$

$$0 < k_1 \leq \epsilon \leq k_2$$

X. DUALITIES OF LAPLACE TRANSFORM WITH SOME USEFUL INTEGRAL TRANSFORMS

In this section, we define the dualities between Laplace transform and some useful integral transforms namely Kamal transform, Elzaki transform, Aboodh transform, Sumudu transform, Mahgoub (Laplace-Carson) transform, Mohand transform and Sawi transform.

A. Laplace – Kamal Duality

If Laplace and Kamal transforms of $Z(\gamma)$ are $B(\epsilon)$ and $C(\epsilon)$ respectively then

$$B(\epsilon) = C\left(\frac{1}{\epsilon}\right) \quad (9)$$

$$\text{and } C(\epsilon) = B\left(\frac{1}{\epsilon}\right) \quad (10)$$

Proof: From (1),

$$B(\epsilon) = \int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma = \int_0^\infty Z(\gamma) e^{-\frac{\gamma}{(1/\epsilon)}} d\gamma$$

Now, using (2) in above Equation, we obtain

$$B(\epsilon) = C\left(\frac{1}{\epsilon}\right).$$

To drive (10), we use (2)

$$C(\epsilon) = \int_0^\infty Z(\gamma) e^{\frac{-\gamma}{\epsilon}} d\gamma \quad (11)$$

It is immediately concluded using (1) in (11),

$$C(\epsilon) = B\left(\frac{1}{\epsilon}\right).$$

B. Laplace – Elzaki Duality

If Laplace and Elzaki transforms of $Z(\gamma)$ are $B(\epsilon)$ and $D(\epsilon)$ respectively then

$$B(\epsilon) = \epsilon D\left(\frac{1}{\epsilon}\right) \quad (12)$$

$$\text{and } D(\epsilon) = \epsilon B\left(\frac{1}{\epsilon}\right) \quad (13)$$

Proof: Using (1) follows

$$B(\epsilon) = \int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma$$

$$\Rightarrow B(\epsilon) = \epsilon\left[\frac{1}{\epsilon}\int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma\right] \quad (14)$$

Now, using (3) in above equation, we obtain

$$B(\epsilon) = \epsilon D\left(\frac{1}{\epsilon}\right).$$

To drive (13), we use (3)

$$D(\epsilon) = \epsilon\int_0^\infty Z(\gamma) e^{-\frac{\gamma}{\epsilon}} d\gamma$$

It is immediately concluded using (1) in above equation,

$$D(\epsilon) = \epsilon B\left(\frac{1}{\epsilon}\right).$$

C. Laplace – Aboodh Duality

If Laplace and Aboodh transforms of $Z(\gamma)$ are $B(\epsilon)$ and $F(\epsilon)$ respectively then

$$B(\epsilon) = \epsilon F(\epsilon) \quad (15)$$

$$\text{and } F(\epsilon) = \frac{1}{\epsilon} B(\epsilon) \quad (16)$$

Proof: It is immediately concluded from (1)

$$B(\epsilon) = \int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma$$

$$\Rightarrow B(\epsilon) = \epsilon\left[\frac{1}{\epsilon}\int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma\right]$$

Now, using (4) in above Equation, we have

$$B(\epsilon) = \epsilon F(\epsilon).$$

To drive (16), we use (4)

$$F(\epsilon) = \frac{1}{\epsilon}\int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma$$

It is immediately concluded using (1) in above equation,

$$F(\epsilon) = \frac{1}{\epsilon} B(\epsilon).$$

D. Laplace – Sumudu Duality

If Laplace and Sumudu transforms of $Z(\gamma)$ are $B(\epsilon)$ and $G(\epsilon)$ respectively then

$$B(\epsilon) = \frac{1}{\epsilon} G\left(\frac{1}{\epsilon}\right) \quad (17)$$

$$\text{and } G(\epsilon) = \frac{1}{\epsilon} B\left(\frac{1}{\epsilon}\right) \quad (18)$$

Proof: From (1), we have

$$B(\epsilon) = \int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma$$

Put $\epsilon\gamma = u \Rightarrow d\gamma = \frac{du}{\epsilon}$ in above equation, we have

$$B(\epsilon) = \int_0^\infty Z\left(\frac{u}{\epsilon}\right) e^{-u} \frac{du}{\epsilon}$$

$$\Rightarrow B(\epsilon) = \frac{1}{\epsilon}\int_0^\infty Z\left(\frac{u}{\epsilon}\right) e^{-u} du$$

Now, using (5) in above equation, we have

$$B(\epsilon) = \frac{1}{\epsilon} G\left(\frac{1}{\epsilon}\right).$$

To drive (18), we use (5)

$$G(\epsilon) = \int_0^\infty Z(\epsilon\gamma) e^{-\gamma} d\gamma$$

Put $\epsilon\gamma = u \Rightarrow d\gamma = \frac{du}{\epsilon}$ in above equation, we have

$$G(\epsilon) = \int_0^\infty Z(u) e^{-\frac{u}{\epsilon}} d\gamma \frac{du}{\epsilon}$$

$$\Rightarrow G(\epsilon) = \frac{1}{\epsilon}\int_0^\infty Z(u) e^{-\frac{u}{\epsilon}} du$$

It is immediately concluded using (1) in above equation,

$$G(\epsilon) = \frac{1}{\epsilon} B\left(\frac{1}{\epsilon}\right).$$

E. Laplace – Mahgoub (Laplace – Carson) Duality

If Laplace and Mahgoub transforms of $Z(\gamma)$ are $B(\epsilon)$ and $H(\epsilon)$ respectively then

$$B(\epsilon) = \frac{1}{\epsilon} H(\epsilon) \quad (19)$$

$$\text{and } H(\epsilon) = \epsilon B(\epsilon) \quad (20)$$

Proof: From (1), we have

$$B(\epsilon) = \int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma$$

$$\Rightarrow B(\epsilon) = \frac{1}{\epsilon}\left[\epsilon\int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma\right]$$

Now, using (6) in above equation, we have

$$B(\epsilon) = \frac{1}{\epsilon} H(\epsilon).$$

To drive (20), we use (6)

$$H(\epsilon) = \epsilon\int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma$$

It is immediately concluded using (1) in above equation,

$$H(\epsilon) = \epsilon B(\epsilon).$$

F. Laplace – Mohand Duality

If Laplace and Mohand transforms of $Z(\gamma)$ are $B(\epsilon)$ and $I(\epsilon)$ respectively then

$$B(\epsilon) = \frac{1}{\epsilon^2} I(\epsilon) \quad (21)$$

and $I(\epsilon) = \epsilon^2 B(\epsilon)$ (22)

Proof: From (1), we have

$$B(\epsilon) = \int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma$$

$$\Rightarrow B(\epsilon) = \frac{1}{\epsilon^2}\left[\epsilon^2 \int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma\right]$$

Now, using (7) in above equation, we have

$$B(\epsilon) = \frac{1}{\epsilon^2} I(\epsilon).$$

To drive (22), we use (7)

$$I(\epsilon) = \epsilon^2 \int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma$$

It is immediately concluded using (1) in above equation,

$$I(\epsilon) = \epsilon^2 B(\epsilon).$$

G. Laplace – Sawi Duality

If Laplace and Sawi transforms of $Z(\gamma)$ are $B(\epsilon)$ and $J(\epsilon)$ respectively then

$$B(\epsilon) = \frac{1}{\epsilon^2} J\left(\frac{1}{\epsilon}\right) \quad (23)$$

and $J(\epsilon) = \frac{1}{\epsilon^2} B\left(\frac{1}{\epsilon}\right)$ (24)

Proof: Using (1) follows

$$B(\epsilon) = \int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma$$

$$\Rightarrow B(\epsilon) = \frac{1}{\epsilon^2}\left[\epsilon^2 \int_0^\infty Z(\gamma) e^{-\epsilon\gamma} d\gamma\right]$$

Now, using (8) in above equation, we obtain

$$B(\epsilon) = \frac{1}{\epsilon^2} J\left(\frac{1}{\epsilon}\right).$$

To drive (24), we use (8)

$$J(\epsilon) = \frac{1}{\epsilon^2} \int_0^\infty Z(\gamma) e^{-\frac{\gamma}{\epsilon}} d\gamma$$

Now, using (1) in above equation, we obtain

$$J(\epsilon) = \frac{1}{\epsilon^2} B\left(\frac{1}{\epsilon}\right).$$

XI. APPLICATIONS OF MENTION DUALITY RELATIONS FOR FINDING INTEGRAL TRANSFORMS (KAMAL TRANSFORM, ELZAKI TRANSFORM, ABOODH TRANSFORM, SUMUDU TRANSFORM, MAHGOUB TRANSFORM, MOHAND TRANSFORM AND SAWI TRANSFORM) OF USEFUL BASIC FUNCTIONS

We are giving tabular presentation of the integral transforms of mostly used basic functions by using mention dualities relations to visualize the usefulness of dualities between Laplace transform and mention integral transforms in the application field.

Table-I: Kamal transform of useful basic functions with the help of Laplace – Kamal duality relation

S. N.	$Z(\gamma)$	$L\{Z(\gamma)\} = B(\epsilon)$	$K\{Z(\gamma)\} = C(\epsilon)$
1.	1	$\frac{1}{\epsilon}$	ϵ
2.	γ	$\frac{1}{\epsilon^2}$	ϵ^2
3.	γ^2	$\frac{2!}{\epsilon^3}$	$2!\,\epsilon^3$
4.	γ^n, $n \in N$	$\frac{n!}{\epsilon^{n+1}}$	$n!\,\epsilon^{n+1}$
5.	γ^n, $n > -1$	$\frac{\Gamma(n+1)}{\epsilon^{n+1}}$	$\Gamma(n+1)\epsilon^{n+1}$
6.	$e^{a\gamma}$	$\frac{1}{(\epsilon - a)}$	$\frac{\epsilon}{(1 - a\epsilon)}$
7.	$sina\gamma$	$\frac{a}{(\epsilon^2 + a^2)}$	$\frac{a\epsilon^2}{(1 + a^2\epsilon^2)}$
8.	$cosa\gamma$	$\frac{\epsilon}{(\epsilon^2 + a^2)}$	$\frac{\epsilon}{(1 + a^2\epsilon^2)}$
9.	$sinha\gamma$	$\frac{a}{(\epsilon^2 - a^2)}$	$\frac{a\epsilon^2}{(1 - a^2\epsilon^2)}$
10.	$cosha\gamma$	$\frac{\epsilon}{(\epsilon^2 - a^2)}$	$\frac{\epsilon}{(1 - a^2\epsilon^2)}$

Table-II: Elzaki transform of useful basic functions with the help of Laplace – Elzaki duality relation

S.N.	$Z(\gamma)$	$L\{Z(\gamma)\} = B(\epsilon)$	$E\{Z(\gamma)\} = D(\epsilon)$
1.	1	$\frac{1}{\epsilon}$	ϵ^2
2.	γ	$\frac{1}{\epsilon^2}$	ϵ^3
3.	γ^2	$\frac{2!}{\epsilon^3}$	$2!\,\epsilon^4$
4.	γ^n, $n \in N$	$\frac{n!}{\epsilon^{n+1}}$	$n!\,\epsilon^{n+2}$
5.	γ^n, $n > -1$	$\frac{\Gamma(n+1)}{\epsilon^{n+1}}$	$\Gamma(n+1)\epsilon^{n+2}$
6.	$e^{a\gamma}$	$\frac{1}{(\epsilon - a)}$	$\frac{\epsilon^2}{(1 - a\epsilon)}$
7.	$sina\gamma$	$\frac{a}{(\epsilon^2 + a^2)}$	$\frac{a\epsilon^3}{(1 + a^2\epsilon^2)}$
8.	$cosa\gamma$	$\frac{\epsilon}{(\epsilon^2 + a^2)}$	$\frac{\epsilon^2}{(1 + a^2\epsilon^2)}$
9.	$sinha\gamma$	$\frac{a}{(\epsilon^2 - a^2)}$	$\frac{a\epsilon^3}{(1 - a^2\epsilon^2)}$
10.	$cosha\gamma$	$\frac{\epsilon}{(\epsilon^2 - a^2)}$	$\frac{\epsilon^2}{(1 - a^2\epsilon^2)}$

Table-III: Aboodh transform of useful basic functions with the help of Laplace – Aboodh duality relation

S.N.	$Z(\gamma)$	$L\{Z(\gamma)\} = B(\epsilon)$	$A\{Z(\gamma)\} = F(\epsilon)$
1.	1	$\frac{1}{\epsilon}$	$\frac{1}{\epsilon^2}$
2.	γ	$\frac{1}{\epsilon^2}$	$\frac{1}{\epsilon^3}$
3.	γ^2	$\frac{2!}{\epsilon^3}$	$\frac{2!}{\epsilon^4}$
4.	$\gamma^n, n \in N$	$\frac{n!}{\epsilon^{n+1}}$	$\frac{n!}{\epsilon^{n+2}}$
5.	$\gamma^n, n > -1$	$\frac{\Gamma(n+1)}{\epsilon^{n+1}}$	$\frac{\Gamma(n+1)}{\epsilon^{n+2}}$
6.	$e^{a\gamma}$	$\frac{1}{(\epsilon - a)}$	$\frac{1}{\epsilon(\epsilon - a)}$
7.	$sina\gamma$	$\frac{a}{(\epsilon^2 + a^2)}$	$\frac{a}{\epsilon(\epsilon^2 + a^2)}$
8.	$cosa\gamma$	$\frac{\epsilon}{(\epsilon^2 + a^2)}$	$\frac{1}{(\epsilon^2 + a^2)}$
9.	$sinha\gamma$	$\frac{a}{(\epsilon^2 - a^2)}$	$\frac{a}{\epsilon(\epsilon^2 - a^2)}$
10.	$cosha\gamma$	$\frac{\epsilon}{(\epsilon^2 - a^2)}$	$\frac{1}{(\epsilon^2 - a^2)}$

Table-IV: Sumudu transform of useful basic functions with the help of Laplace – Sumudu duality relation

S.N.	$Z(\gamma)$	$L\{Z(\gamma)\} = B(\epsilon)$	$S\{Z(\gamma)\} = G(\epsilon)$
1.	1	$\frac{1}{\epsilon}$	1
2.	γ	$\frac{1}{\epsilon^2}$	ϵ
3.	γ^2	$\frac{2!}{\epsilon^3}$	$2!\,\epsilon^2$
4.	$\gamma^n, n \in N$	$\frac{n!}{\epsilon^{n+1}}$	$n!\,\epsilon^n$
5.	$\gamma^n, n > -1$	$\frac{\Gamma(n+1)}{\epsilon^{n+1}}$	$\Gamma(n+1)\epsilon^n$
6.	$e^{a\gamma}$	$\frac{1}{(\epsilon - a)}$	$\frac{1}{(1 - a\epsilon)}$
7.	$sina\gamma$	$\frac{a}{(\epsilon^2 + a^2)}$	$\frac{a\epsilon}{(1 + a^2\epsilon^2)}$
8.	$cosa\gamma$	$\frac{\epsilon}{(\epsilon^2 + a^2)}$	$\frac{1}{(1 + a^2\epsilon^2)}$
9.	$sinha\gamma$	$\frac{a}{(\epsilon^2 - a^2)}$	$\frac{a\epsilon}{(1 - a^2\epsilon^2)}$
10.	$cosha\gamma$	$\frac{\epsilon}{(\epsilon^2 - a^2)}$	$\frac{1}{(1 - a^2\epsilon^2)}$

Table-V: Mahgoub (Laplace – Carson) transform of useful basic functions with the help of Laplace – Mahgoub (Laplace – Carson) duality relation

S.N.	$Z(\gamma)$	$L\{Z(\gamma)\} = B(\epsilon)$	$M_*\{Z(\gamma)\} = H(\epsilon)$
1.	1	$\frac{1}{\epsilon}$	1
2.	γ	$\frac{1}{\epsilon^2}$	$\frac{1}{\epsilon}$
3.	γ^2	$\frac{2!}{\epsilon^3}$	$\frac{2!}{\epsilon^2}$
4.	$\gamma^n, n \in N$	$\frac{n!}{\epsilon^{n+1}}$	$\frac{n!}{\epsilon^n}$
5.	$\gamma^n, n > -1$	$\frac{\Gamma(n+1)}{\epsilon^{n+1}}$	$\frac{\Gamma(n+1)}{\epsilon^n}$
6.	$e^{a\gamma}$	$\frac{1}{(\epsilon - a)}$	$\frac{\epsilon}{(\epsilon - a)}$
7.	$sina\gamma$	$\frac{a}{(\epsilon^2 + a^2)}$	$\frac{a\epsilon}{(\epsilon^2 + a^2)}$
8.	$cosa\gamma$	$\frac{\epsilon}{(\epsilon^2 + a^2)}$	$\frac{\epsilon^2}{(\epsilon^2 + a^2)}$
9.	$sinha\gamma$	$\frac{a}{(\epsilon^2 - a^2)}$	$\frac{a\epsilon}{(\epsilon^2 - a^2)}$
10.	$cosha\gamma$	$\frac{\epsilon}{(\epsilon^2 - a^2)}$	$\frac{\epsilon^2}{(\epsilon^2 - a^2)}$

Table-VI: Mohand transform of useful basic functions with the help of Laplace – Mohand duality relation

S.N.	$Z(\gamma)$	$L\{Z(\gamma)\} = B(\epsilon)$	$M\{Z(\gamma)\} = I(\epsilon)$
1.	1	$\frac{1}{\epsilon}$	ϵ
2.	γ	$\frac{1}{\epsilon^2}$	1
3.	γ^2	$\frac{2!}{\epsilon^3}$	$\frac{2!}{\epsilon}$
4.	$\gamma^n, n \in N$	$\frac{n!}{\epsilon^{n+1}}$	$\frac{n!}{\epsilon^{n-1}}$
5.	$\gamma^n, n > -1$	$\frac{\Gamma(n+1)}{\epsilon^{n+1}}$	$\frac{\Gamma(n+1)}{\epsilon^{n-1}}$
6.	$e^{a\gamma}$	$\frac{1}{(\epsilon - a)}$	$\frac{\epsilon^2}{(\epsilon - a)}$
7.	$sina\gamma$	$\frac{a}{(\epsilon^2 + a^2)}$	$\frac{a\epsilon^2}{(\epsilon^2 + a^2)}$
8.	$cosa\gamma$	$\frac{\epsilon}{(\epsilon^2 + a^2)}$	$\frac{\epsilon^3}{(\epsilon^2 + a^2)}$
9.	$sinha\gamma$	$\frac{a}{(\epsilon^2 - a^2)}$	$\frac{a\epsilon^2}{(\epsilon^2 - a^2)}$
10.	$cosha\gamma$	$\frac{\epsilon}{(\epsilon^2 - a^2)}$	$\frac{\epsilon^3}{(\epsilon^2 - a^2)}$

Table-VII: Sawi transform of useful basic functions with the help of Laplace – Sawi duality relation

S.N.	$Z(\gamma)$	$L\{Z(\gamma)\} = B(\epsilon)$	$S^*\{Z(\gamma)\} = J(\epsilon)$
1.	1	$\frac{1}{\epsilon}$	$\frac{1}{\epsilon}$
2.	γ	$\frac{1}{\epsilon^2}$	1
3.	γ^2	$\frac{2!}{\epsilon^3}$	$2!\,\epsilon$
4.	$\gamma^n, n \in N$	$\frac{n!}{\epsilon^{n+1}}$	$n!\,\epsilon^{n-1}$
5.	$\gamma^n, n > -1$	$\frac{\Gamma(n+1)}{\epsilon^{n+1}}$	$\Gamma(n+1)\epsilon^{n-1}$
6.	$e^{a\gamma}$	$\frac{1}{(\epsilon - a)}$	$\frac{1}{\epsilon(1 - a\epsilon)}$
7.	$sina\gamma$	$\frac{a}{(\epsilon^2 + a^2)}$	$\frac{a}{(1 + a^2\epsilon^2)}$
8.	$cosa\gamma$	$\frac{\epsilon}{(\epsilon^2 + a^2)}$	$\frac{1}{\epsilon(1 + a^2\epsilon^2)}$
9.	$sinha\gamma$	$\frac{a}{(\epsilon^2 - a^2)}$	$\frac{a}{(1 - a^2\epsilon^2)}$
10.	$cosha\gamma$	$\frac{\epsilon}{(\epsilon^2 - a^2)}$	$\frac{1}{\epsilon(1 - a^2\epsilon^2)}$

XII. CONCLUSIONS

In the present paper, duality relations between Laplace transform and some useful integral transforms namely Kamal transform, Elzaki transform, Aboodh transform, Sumudu transform, Mahgoub (Laplace-Carson) transform, Mohand transform and Sawi transform are established successfully. Tabular presentation of the integral transforms (Kamal transform, Elzaki transform, Aboodh transform, Sumudu transform, Mahgoub (Laplace-Carson) transform, Mohand transform and Sawi transform) of mostly used basic functions are given with the help of mention dualities relations to visualize the importance of dualities between Laplace transform and mention integral transforms. Results show that the Laplace transform and mention integral transforms in this paper are strongly related to each others. In future using these duality relations, we can easily solved many advanced problems of modern era such as motion of coupled harmonic oscillators, drug distribution in the body, arms race models, Brownian motion and the common health problem such as detection of diabetes.

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