Design of a Robust Controller by LQG/LTR Formalism for Francis hydro Turbine Driving a Synchronous Generator

Yeremou Tamtsia Aurélien, Nneme Nneme Léandre, Samba Aimé Hervé

***Abstract:** This paper presents the design and application of a robust controller by Linear-Quadratic-Gaussian method with Loop-Transfer-Recovery (LQG \LTR) at the same time to carefully attain performance and robustness objectives. To improve Stability, the robust controller has been shown to provide good performance in normal operations conditions. Objectives cannot be suitable unless the controller can perpetuate such quality in the presence of plant uncertainties or any working conditions in the hydroelectric power plants. The approach is based to synthesizing a robust controller minimizing a quadratic criterion (controller LQG) while using the Loop Transfer Recovery (LTR), to restore robustness properties of the Estimator. In this study, we applied this robust control law on the model of a Francis hydro turbine. Computer simulations are carried out to establish and compare the performance and robustness of using the Infinite horizon control (* H_∞*), internal model control (IMC), Proportional Integral Derived (PID) and LQG/LTR controllers.*

***Keywords :** Francis hydro Turbine, speed, LQG/LTR, parametric disturbance hydro- power plant.*

I. INTRODUCTION

The robust control of the dynamic systems plays a significant role in the correct operation of the machines and the industrial processes. Many complex engineering systems are equipped with several actuators that may influence their static and dynamic behavior. The complexity of these systems and the requirements of performance increasingly more strict result in the guaranteeing need for methods of control of the performances raised with a robustness against the risks and unforeseen of the environment. The criteria of stability, robustness and performance are the objectives to be reconciled for any method of effective control. Among the most answered techniques of robust control, [1], [2] outline the solution to a range of optimal control problems. Further algorithms such as loop transfer recovery LTR were developed by [3] to improve the robustness of the more practical LQG when compared to the more robust but less practical LQR.

Since the appearance of the paper by [3] with loop transfer recovery (LTR), many papers have been written on this topic. The most notable ones for continuous time systems are [1], [4], [5] to achieve H_2 and H_∞ norm specifications [1], [6].

In this paper, synthesizing a control law by Linear-Quadratic-Gaussian method with loop-Transfert -Recovery (LQG \LTR) which maintains system response and error signals to within specified tolerances so that they are enable to ensure the robustness and performances for the modes of nominal and disturbed operation. In this context, we applied in simulation this controller to the system (model of hydro turbine of the hydroelectric power plant of Songloulou) [6]-[8].

This paper is organized as follows: We present in section 2 the process model on which we worked; section3 describe the Synthesis of a robust controller by approach LQG \LTR. The results obtained from simulation are given and analyzed in time and frequency domain in section 4, the obtained results are compared with the controllers PID, IMC and H_∞. Finally, section 5 resumes the main conclusions obtained during the development of this work.

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* Correspondence Author

Yeremou Tamtsia Aurelien*, department of Industrial Robotic, Faculty of Industrial Engineering, University of Douala, Douala, Cameroon. Email: ayeremou@yahoo.fr.

Nneme Nneme Léandre, department of electrical engineering, Higher Technical Teacher Training College, University of Douala, Douala, Cameroon. Email: leandren@gmail.com.

Samba Aimé Hervé, department of Industrial Robotic, Faculty of Industrial Engineering, University of Douala, Douala, Cameroon. Email: aimehervesamba@yahoo.fr.

II. PROCEDURE FOR PAPER SUBMISSION MODELING OF THE PLANT

For the realization of our project, we have used the model of the Songloulou hydroelectric power plant. The System modeling is presented in [7], [8]. The process is carried out by association of 4 modules: the Francis hydro turbine, the alternator, the servo-motor-servo valve and power chain, figure 1.

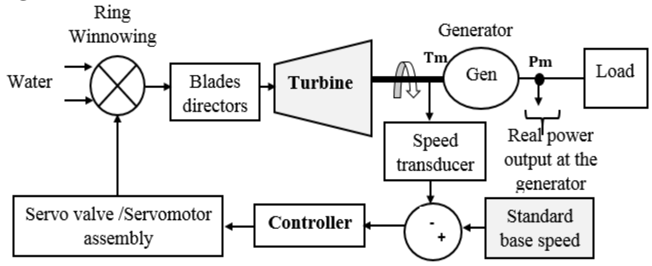

Fig. 1.Schematic representation of the hydro turbine driving a synchronous generator in hydro power plants.

A. Modeling of the Francis turbine model

The reduced Bond graph representation of a Francis hydro turbine shown in Figure 2 [7].

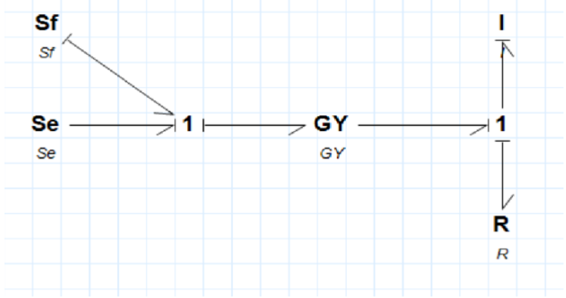

Fig. 2.Bond Graph of the hydro Francis turbine.

Table-1 present the most parameters of the Francis hydro turbine.

Table- I: Parameters of the Francis hydro turbine.

Parameters	Value	Unit
Density of the driving fluid	1000	Kg/m3
Wheel beam at the side of the turbine	2.25	m
Wheel beam at the entry of the turbine	3.2	m
Inertia of turbine	8800000	Kg.m2
Entry flow rate	18.9544	m3/s

The transfer function of the hydro turbine is given by [7]:

$$G_T(s) = \frac{0.663}{1 + 42.55s} \tag{1}$$

Study and modeling of the servo valve, the power chain and the relationship between winnowing (v) and the water flow rate (d) is reported by [7]. Figure 3 present the open loop Simulink model of the hydro turbine's speed control chain of the hydroelectric power plant.

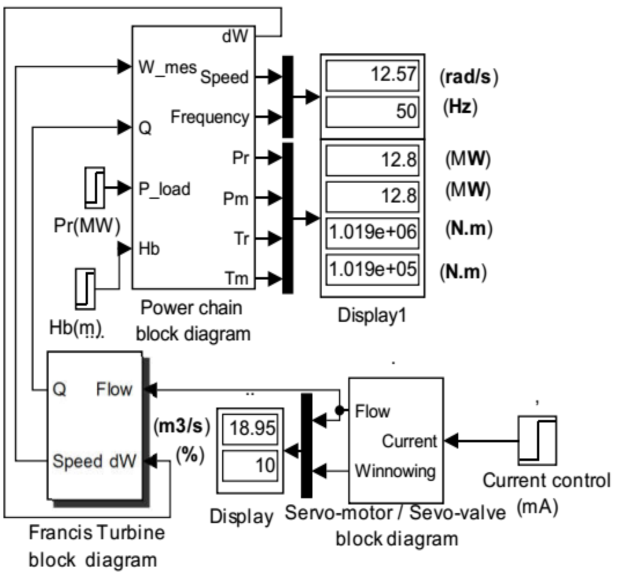

Fig. 3.Simulink model of the open loop hydro power plant.

Table-2 present the most parameters of the hydroelectric plant.

Table- II: Parameters of the hydroelectric plant.

Parameters	Description	Unit
Q	Water flow rate	m^3/s
H_b	Height of waterfall	m
P_r	Power of the load	Watts
T_r	Resistant torque	N.m
P_m	Power produced	Watts
T_m	Motor torque	Nm
ω_{mes}	Speed measured	rad/s
$d\omega$	Disturbance	

III. SYSTEM IDENTIFICATION

From the response curve seen in figure 4, the system is approximately a first order system. Using Broïda parameters, the system response time (T) and time constant is calculated following the formulas given below [9]:

$$T = 2.8t_1 - 1.8t_2 \quad , \quad \tau = 5.5(t_2 - t_1) \tag{2}$$

With K=12.57, T=42.84s and τ=0.538s ,the transfer function of the nominal regime of the Broïda model of the system is given by:

$$G(s) = \frac{12.57}{1 + 42.84s} e^{-0.538s} \tag{3}$$

The first order PADE approximation of the model with time delays is given by:

$$G(s) = \frac{-12.57s + 46.73}{42.84s^2 + 160.3s + 3.717} \tag{4}$$

The simulation results are represented in figure 4.

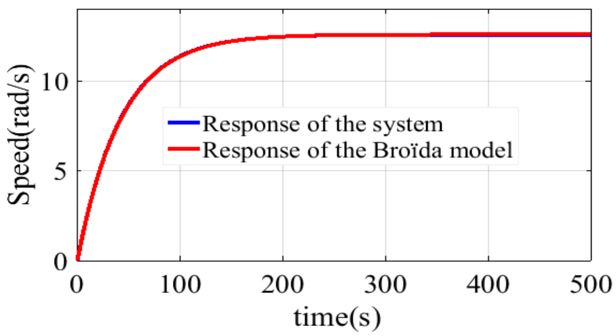

Fig. 4.Index responses of open-loop system and model responses.

IV. METHOD OF SYNTHESIS OF A ROBUST CONTROLLER BY LQG/LTR APPROACH

Consider the stochastic dynamic system of state equations:

$$\begin{cases} \dot{x} = & Ax + Bu + \Gamma w \\ y = & Cx + v \end{cases} \tag{5}$$

Where A, B, C are state space matrices of the plant and Γ is the disturbance, the noise of state w and the measurement noises v are assumed Gaussian noises with zero mean and covariance matrices W and V as follows:

$$E\{ww^T\} = W \geq 0 \ , \qquad E\{vv^T\} = V \geq 0 \tag{6}$$

$$E\{wv^T\} = 0$$

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(7)

The problem is then to devise a feedback control low which minimizes the performance index:

$$J = E\left\{ \lim_{T\to\infty} \frac{1}{T}\int_0^T \left(z^T Q z + u^T R u\right) dt \right\} \quad (8)$$

Where z is a linear combination of the system states, Q and R are weighting matrices.

The mathematical criterion that one seeks to minimize is [10] :

$$\min_{K_f} J_G \quad \text{with} \quad J_G = E\left\{\varepsilon_x{}^T \varepsilon_x\right\} = 0 \quad (9)$$

Where ε_x is the error in estimation of the state of the system in permanent mode.

From the equations (5) and (9), we deduce the equation from evolution of the error in estimation:

$$\dot{\varepsilon}_x = A\varepsilon_x + w - K_f\left(C\varepsilon_x + v\right) \quad (10)$$

$$\dot{\varepsilon}_x = \left(A - K_f C\right)\varepsilon_x + \left[I_n \quad -K_f\right]\begin{bmatrix} w \\ v \end{bmatrix} \quad (11)$$

By applying the theorem of the passage of a white noise in a linear system to the equation (10) and (11) we obtain the equation (12) of continuous LYAPUNOV [3], [11]:

$$\left(A - K_f C\right)P_f + P_f\left(A - K_f C\right)^T + W + K_f V K_f^T = 0$$

(12)

Where Kalman-filter static gain K_f of the optimal observer is as follows:

$$K_f = P_f C^T V^{-1} \quad (13)$$

Where P_f obeys the algebraic Riccati equation.

$$P_f A^T + A P_f - P_f C^T V^{-1} C P_f + W = 0 \quad (14)$$

With $P_f = P_f^T > 0$

The optimal control is given by: $u(t) = -K_c \hat{x}(t)$

Where K_C is an optimal static feedback gain obtained by considering the LQR problem:

$$K_c = R^{-1} B^T P \quad (15)$$

where P is the unique positive definite solution of the Riccati continuous algebraic equation.

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \quad (16)$$

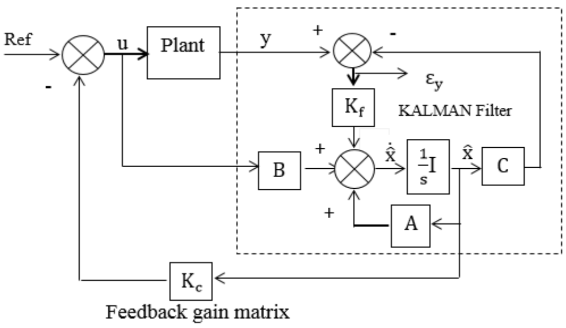


Fig. 5.Functional diagram of Kalman filter.

According to Figure 5, the representation of state of controller LQG is written:

$$\begin{bmatrix} \dot{\hat{x}} \\ u \end{bmatrix} = \begin{bmatrix} A - BK_c - K_c C & K_f \\ -K & 0 \end{bmatrix}\begin{bmatrix} \hat{x} \\ y \end{bmatrix} \quad (17)$$

We deduce from the equation of state (17), the transfer matrix of the controller :

$$K(s) = -K_c\left(sI - A + BK_c + K_f C\right)^{-1} K_f \quad (18)$$

A. Loop Transfer Recovery and Choice of the weighting matrices

Approach LQG/LTR consists with calculated the KALMAN filter by a suitable choice of the matrices of variance noise $E\left\{ww^T\right\} = W = \Gamma\Gamma^T$ and $E\left\{vv^T\right\} = V = \rho I$ by taking of account properties of dual robustness for the choice of the parameters Γ and ρ.

Where $\Gamma \in R^{n\times p}$ is the process noise distribution matrix and ρ is a scaled value; then in calculation, the command by return of state (command LQ) starting from an adjustment of the matrices of weighting Q and R so that the matrix of transfer of the open loop of the unit approaches gradually that obtained by the KALMAN filter.

A first stage of synthesis LQG/LTR relates to the choice of weighting matrices Q and R for the calculation of the command with return of state LQ in the following way [1], [4] :

$$Q = \Gamma\Gamma^T \text{ and } R = \rho I \quad (19)$$

B. Robustness Condition on the Performances in low Frequency (LF) and Robustness Condition on the Performances in High Frequency (HF)

The robustness condition on the performances is given by:

$$\sigma_{min}\left[C\Phi(j\omega)\Gamma\right]/\sqrt{\rho} \le \sigma_{max}\left[W_p(j\omega)\right] \quad (20)$$

Where $\Phi(j\omega) = [sI - A]^{-1}$

The transfer matrix and $\sigma_{max}\left[W_p(s)\right]$ the largest singular value of specification on stability.

For the robustness on stability it is thus necessary, to ensure the following condition [1], [3]:

$$\omega_{c\,max} = \sigma_{max}\left[C\Phi(j\omega)\Gamma\right]/\sqrt{\rho} \quad (21)$$

Where $\omega_{c\,max}$ is the cut-off frequency of the reverse of the singular values of the specification on stability $\left(1/\sigma_{max}\left[W_p(s)\right]\right)$.

Asymptotic Covering

The adjustment is done by a choice of the matrices form [3]:

$$Q = Q_0 + q.C^T C \quad \text{and} \quad R = R_0 \quad (22)$$

From equation (27), we can rewrite the theorem of the dual LTR in the following way [3], [5]:

$$\lim_{q\to\infty} K(s)G(s) \to C\Phi K_f \qquad (23)$$

Figure 6 shows the configuration of a perturbed closed-loop system.

Where u is the commande, y is the process output , ε is the loop's error, G(s) is stamp transfer matrix of the system, K(s) is the transfer matrix of the controller and $\Delta(s)$ is the process uncertainty at the process output.

The maximum singular value of $\Delta(s)$ is given by (24):

$$\sigma_{max}[\Delta(s)] = \sigma_{max}\left(\left[\left([I+\Delta(s)].G(s)\right)\text{-}G(s)\right]G^{-1}(s)\right) \qquad (24)$$

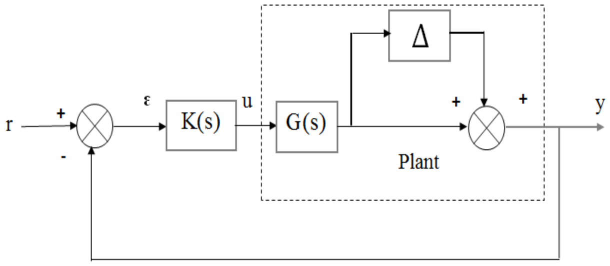


Fig. 6.Feedback configuration of a closed-loop system subjected to multiplicative disturbances.

C. Robust Stability and Robust Performances

The conditions of robustness on stability and the performances corresponding to the parametric disturbance $\Delta(s)$ are :

$$\sigma_{max}[T(s)] < 1/\sigma_{max}[W_t(s)] \qquad (25)$$

$$\sigma_{max}[S(s)] \le 1/\sigma_{max}[W_p(s)] \qquad (26)$$

Where T(s) is the nominal closed loop transfer matrix and S(s) is the sensitivity matrix given by [12], [13]:

$$T(s) = K(s)G(s).[1+K(s)G(s)]^{-1} \qquad (27)$$

$$S(s) = [1+K(s).G(s)]^{-1} \qquad (28)$$

The stability specification matrix $W_t(s)$ is defined by:

$$\sigma_{max}[\Delta_m(s)] \le \sigma_{max}[W_t(s)] \qquad (29)$$

V. DESIGN OF LQG/LTR CONTROLLER FOR FRANCIS HYDRO TURBINE

A. Choice of the specifications on stability and the performances weighting matrices

On the basis of the criteria previously established, the stability specification $W_t(s)$ and the performance specification Wp (s) are represented as follow:

- Stability specification.

$$W_t(s) = 0.9(1+0.03s) \qquad (30)$$

- Performance specification.

$$W_p(s) = \frac{(1+0.1s)}{0.1s} \qquad (31)$$

The frequential gauge of the specifications is represented on figure 7 and figure 8.

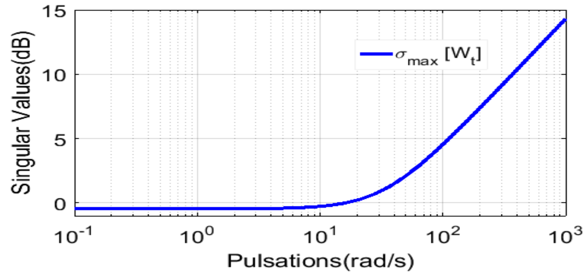


Fig. 7.Maximum singular value of Wt(s).

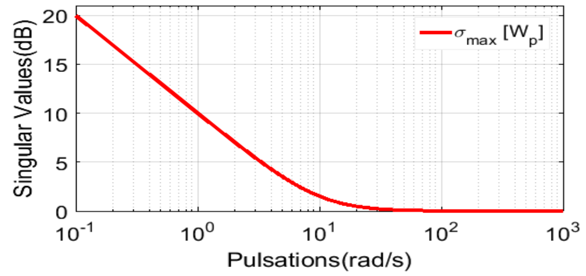


Fig. 8.Maximum singular value of Wp(s).

B. Robustness conditions

The robustness conditions for the speed hydro turbine are represented in figure 9.

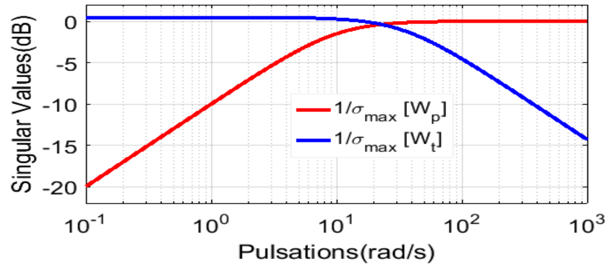


Fig. 9.Robustness conditions.

C. Performances condition in high frequency (HF)

To guarantee this condition it is necessary to ensure the condition of robustness on stability in HF:

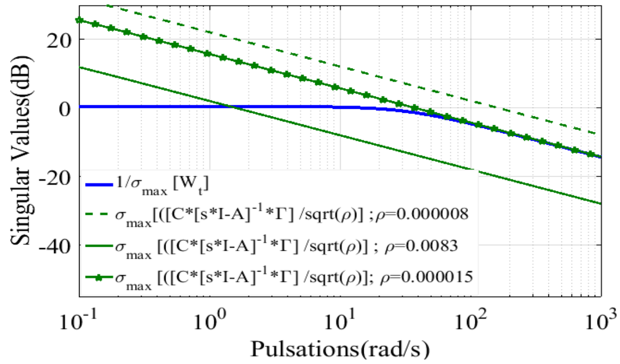


Fig. 10. Robustness condition on the performances in high frequency.

D. Performances condition in low frequency (HF)

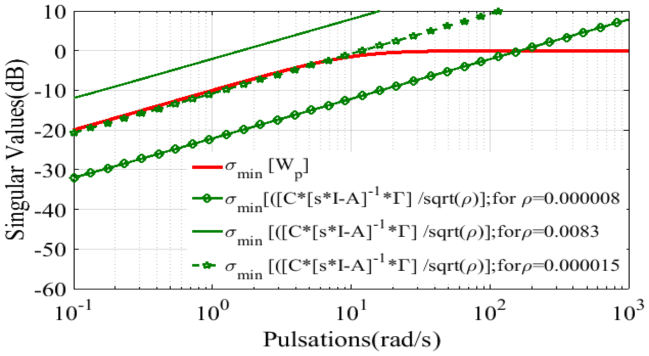


Fig. 11. Robustness condition on the performances in low frequency.

Figures (10) and (11) famous determination of Γ and ρ, by test-error starting from the preceding conditions of robustness is done. One obtains thus Γ=[1;0] and ρ=0.000015.

E. Loop transfer recovery

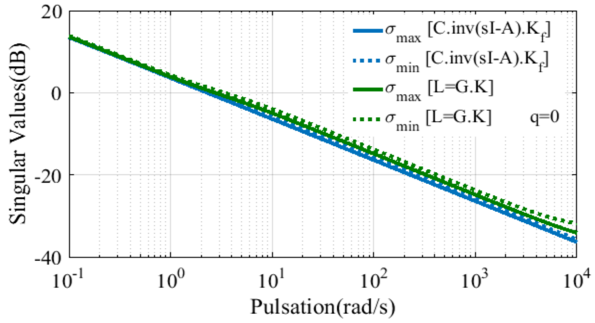


Fig. 12. The recovery of the loop gain for q=0.

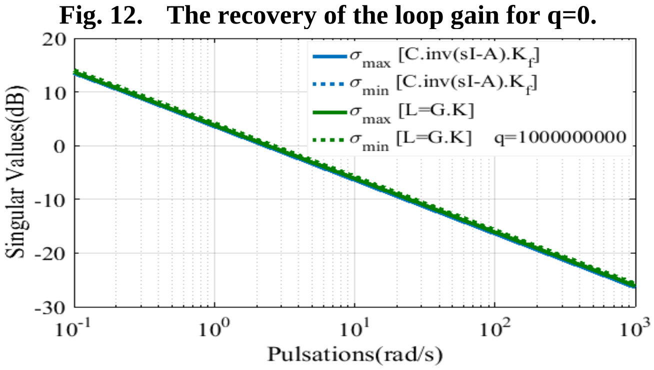


Fig. 13. The recovery of the loop gain for q=109.

According to figures 12 and 13, one observes that the principal gains of $L(s)=K(s).G(s)$ tends towards the principal gains of optimal observer $L_{FK}(s)=C.(sI-A)^{-1}.K_f$ with a total covering.

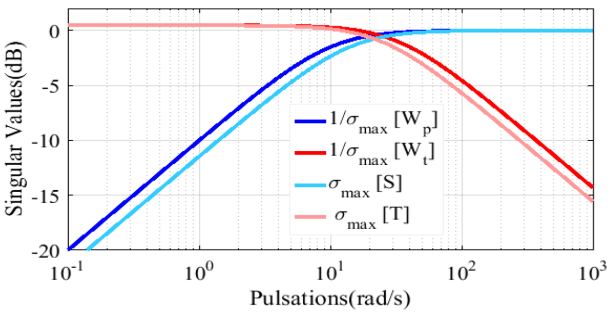


Fig. 14. Principal gains, Sensibility and robustness conditions.

Figure 14 showed that the robustness conditions on stability and the performances are satisfied: the stability is guaranteed if the largest singular value of closed loop transfer matrix function $1/\sigma_{max}[T(s)]$ is lower than the upper bound of the largest singular value of the model uncertainties $1/\sigma_{max}[W_t(s)]$.

The singular values of the sensitivity $\sigma_{max}[S(s)]$ are below the robustness condition on performances $1/\sigma_{max}[Wp(s)]$ what allows the rejection of the disturbances and guarantees the performances desired in the synthesis.

The final controller is then given by

$$K(s)=\frac{5.184s^3+123.1s^2+388.4s+8995}{s^4+62.51s^3+283.6s^2+290.85s+0.02908} \quad (32)$$

Figure 15 shows the simulink model of the closed-loop of the power plant in the presence of the LQG/LTR controller.

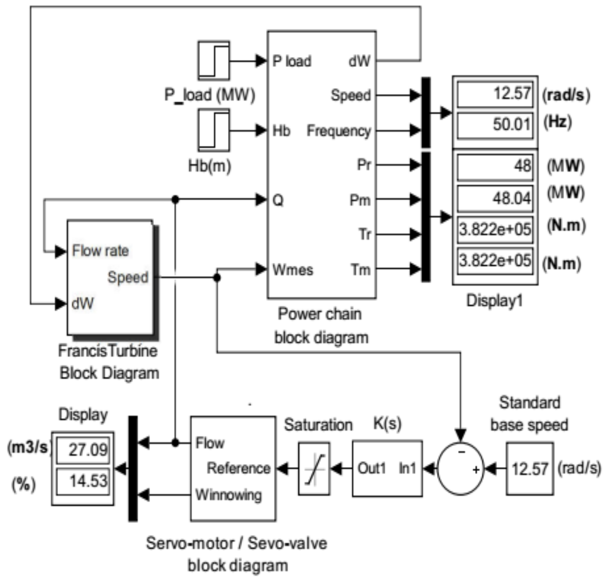


Fig. 15. Simulink model of the closed-loop of the power plant in the presence of the LQG/LTR controller.

VI. RESULT AND DISCUSSION

Figure.16, 17,18 and 19 shows the step responses of nominal of and perturbed regimes of hydro turbine plant.

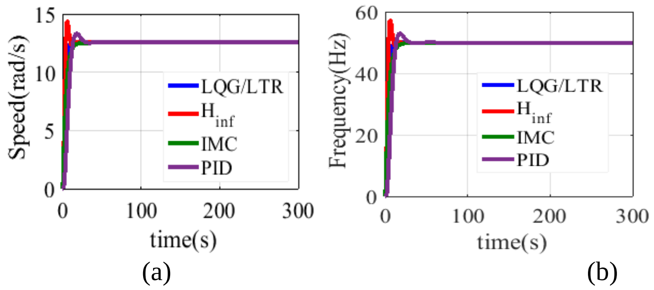
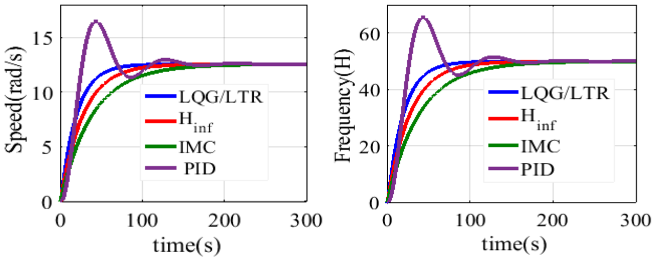
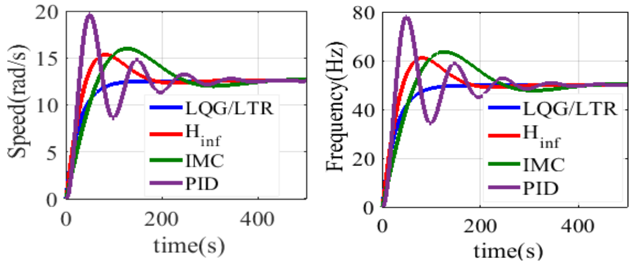


Fig. 16. Closed-loop time responses of the nominal plant G(s) with (K=12.57, T=42.84, and $\tau=0.538$).



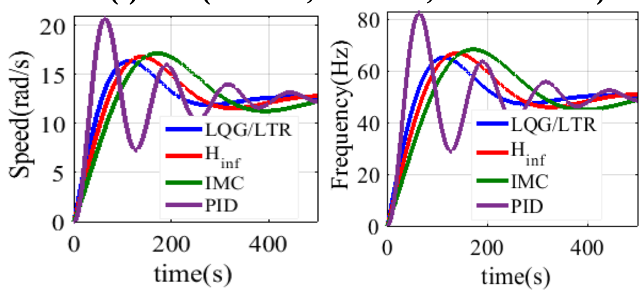
(a) (b)

Fig. 17. Closed-loop time responses of the perturbed plant $G_1(s)$ with (K=1.57, T=42.84, and $\tau = 0.538$).



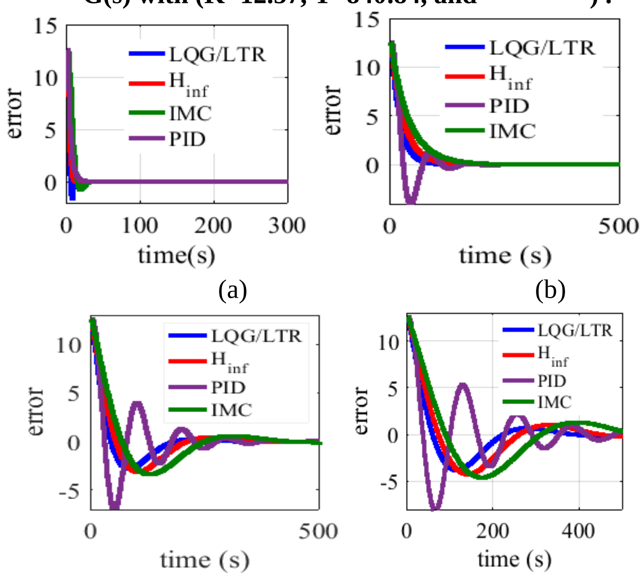
(a) (b)

Fig. 18. Closed-loop responses of the nominal plant G2(s) with (K=10.57, T=420.84, and $\tau = 0.538$).



(a) (b)

Fig. 19. Closed-loop time responses of the nominal plant G(s) with (K=12.57, T=840.84, and $\tau = 0.538$).



(c) (d)

Fig. 20. The disturbance rejection responses of the nominal plant G (a), the perturbed plant G1(b), G2(c), and G3(d).

Figure 18 shows that the LQG/LTR controller has improved the speed of hydro turbine where reduction of overshoot and oscillation are obtained.

The stability of all regimes (figure 16, 17,18 and 19) show a good performance and a fast response time with LQG/LTR controller. It is noted that our obtained results are very encouraging with the PID, CMI and H_∞ controllers reported by [6].

Table- III: Comparison of overshoot, rise time and setting time according to different controllers of closed loop responses of the nominal plant G(s) with (K=12.57 , T=42.84 and τ=0.538).

Controllers	Overshoot (%)	Rise time (s)	4% setting time (s)
PID	36.28	13.47	22
IMC	0	77.2	12.27
H_∞	12.16	3.42	8.9
LQG/LTR	0	9.90	7.29

Table- IV: Comparison of overshoot , rise time and setting time according to different controllers of closed loop responses of the perturbed plant G1(s) with (K=1.57 , T=42.84 and τ=0.538).

Controllers	Overshoot (%)	Rise time (s)	4% setting time (s)
PID	31.11	31.11	69.7
IMC	0	0	132.8
H_∞	0	0	88.54
LQG/LTR	0	0	62.5

Table- V: Comparison of overshoot , rise time and setting time according to different controllers of closed loop responses of the perturbed plant G2(s) with (K=10.57 , T=420.84 and τ=0.538).

Controllers	Overshoot (%)	Rise time (s)	4% setting time (s)
PID	36.28	13.47	22
IMC	0	77.2	12.27
H_∞	12.16	3.42	8.9
LQG/LTR	0	9.90	7.29

Table- VI: Comparison of overshoot , rise time and setting time according to different controllers of closed loop responses of the perturbed plant G3(s) with (K=12.57 , T=840.84 and τ=0.538).

Controllers	Overshoot (%)	Rise time (s)	4% setting time (s)
PID	64.68	28.53	258
IMC	27.13	66.46	227.7
H_∞	23.31	53.17	187.9
LQG/LTR	21.96	42.57	157.5

Figure 20 shows that disturbance rejection due to the LQG/LTR controller is faster than the H_∞, IMC and PID.

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Table 3, Table 4, Table 5 and Table 6 shows that the LQG/LTR controller has eliminated the steady state error faster than the PID, IMC and H_∞ controllers where reduction of overshoot and oscillation are obtained. Settling times of system characteristic component parts by using LQG/LTR controller are shorter, and the stability of the system is better by applying this optimal control method.

VII. CONCLUSION

In this paper a robust controller LQG/LTR to achieve the benefit of feedback in the face of uncertainties has been investigated and successfully applied to a hydro turbine driving a synchronous generator. From the simulation results, it is clear that, the LQG/LTR control exhibits better performance for rotational speed responses than the PID and IMC and H_∞ control.

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AUTHORS PROFILE

YEREMOU TAMTSIA Aurelien received the Master Degree in process engineering and the Ph.D. degree in vision and robotic from agro- food processing engineering school (ENSAI - Cameroon) and Blaise Pascal university Clermont- Ferrand (France) respectively. From 2013, he held an academic position in the Department of Industrial robotics at Faculty of Industrial Engineering, University of Douala (Cameroon), as Lecturer. His current researches lie on Robot Control, robust control, industrial instrumentation and regulations technologies of both deterministic and stochastic dynamic processes.

Pr NNEME NNEME Léandre received the Master Degree in electrical and computer science engineering and the Ph.D. degree in Systems and Automation from Ecole Polytechnique of Montreal (Canada) in 1993 and 1998 respectively. From 1998, he held an academic position in the Advanced Teacher's Training College for Technical Education (ATTCTE) of Douala (Cameroon), as senior Lecturer and Director of the ATTCTE. His current researches lie on Robot Control, industrial instrumentation technologies and remote control of both deterministic and stochastic dynamic processes.

SAMBA Aimé Hervé completed MSc degree in Electronic and Instrumentation from Douala University, Cameroon, in 2016. Currently, he is pursuing for his PhD degree in the Department of Industrial robotics at Faculty of Industrial Engineering, University of Douala, Cameroon. His research interests robust control and networked control systems.