Analysis and Evaluation of a Coriolis Acceleration Experimental Device

Hafizan Hashim, Hanita Hashim

Abstract*:The Coriolis acceleration is the product of linear and rotational velocities. Acceleration analysis is important because inertial forces are proportional to their rectilinear, angular, and Coriolis accelerations. The magnitudes of Coriolis acceleration vary according to the conditions of motions of an object. This paper presents kinematic analyses of a preliminary design of a Coriolis kit. The Coriolis kit consists of a rotating rod and a slidable collar. A motor is used to rotate the rod and accelerometer is attached to the slider for recording the accelerations. The Coriolis effect is visualized through the mark left by the slider during motion. Common analytical expressions of the Coriolis acceleration are derived and calculated using measured values. Results show that the Coriolis kit is capable to visualize and sketch the travelling path of the object in motion.

Keywords : Coriolis acceleration, force, kinematics and dynamics of mechanism, rotating frame.

I. INTRODUCTION

In classical mechanics there are many ways to derive equations of accelerations. The Coriolis acceleration is the product of the linear motion of an object on the rotating table [1]-[4]. Textbooks in classical mechanics calculate the Coriolis acceleration and a force for common case as follows:

$$a_c = 2\omega v \tag{1}$$

$$F_c = 2mv\omega \tag{2}$$

where a_c is the Coriolis acceleration, ω is the angular velocity of the disc, and v is the linear velocity of the object travelling on the table, m is the mass of the object, and F_c is the Coriolis force. Equation (1) is a classical formula. The components of the angular velocity ω and the linear velocity v are applied for both uniform and accelerated motions of machine elements.

Any course in engineering mechanics should provide a set of fundamental principles and governing equations applicable to a wide variety of problems. The formulation should be clear and consistent, thus empowering the student to meet any technical challenge. In undergraduate courses on rigid body dynamics, the most difficult problems typically involve multi-part systems with 3-D motion, together with sliders or pins moving along rotating parts. In most undergraduate textbooks, this leads to the introduction of rotating frames of reference, and relative acceleration components written in terms of the rotating frames. One of these components is referred to as the Coriolis acceleration. In the textbook by R.C. Hibbeler [2], for example, the Coriolis acceleration is defined as "the difference in the acceleration of [a point] *B* as measured from nonrotating and rotating *x*, *y*, and *z* axes". Other textbooks for undergraduate dynamics contain similar definitions. This method of analysis then requires judicious selection of multiple frames, each with its own angular velocity at a given instant.

This approach is suboptimal in promoting an understanding of the physical nature of the motion, as well as its connection to the mathematical formulation. In addition, considerable confusion can result when comparisons are made between textbooks. Whereas some books use the rotating frames as an intermediate tool to determine the "absolute accelerations" within an inertial frame of reference, others focus on the effects observed from a rotating frame, such as the Earth. Since Newton's laws do not apply in a non-inertial frame of reference, the observed Coriolis effects and Centrifugal effects are accompanied by corresponding, compensating fictitious forces that are only manifestations of the rotating frame. Meanwhile, many patents were and are granted for the inventions that relate to this phenomena [5]-[10].

In this work, we advocate the use of a single inertial frame of reference for analyzing and visualizing the Coriolis effect from the newly develop Coriolis kit. The equations of relative position, velocity, and acceleration are all derived for this single inertial frame. Our approach does not require the selection of rotating frames, and kinematic analyses are performed systematically by applying the same fundamental equations repeatedly. The same method could be applied for mechanism with multiple joints and connecting points, even for three dimensional problems. Results show that the Coriolis kit is capable to visualize and sketch the travelling path of the object in motion and measure some information required to calculate the Coriolis acceleration exhibited by the object.

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II. METHOD

A. Particle Moving on a Slab in Translation

All derivations herein are based on Beer et al. [4]. Let us consider a motion of a particle *P* which describes a path on a slab *S* which is itself in translation. The motion of *P* may be analyzed, either in terms of its coordinates *x* and *y* with respect to a fixed set of axis, or in terms of its coordinates x_1 and y_1 with respect to a set of axes attached to the slab *S* and moving with it as shown in Fig.1. Let us determine the absolute motion of *P* with respect to the fixed axes and its relative motion with respect to the axes moving with *S*. Denoting by x_0 and y_0 the coordinates of *O'* with respect to the fixed axes, we write:

$$x = x_0 + x_1 y = y_0 + y_1 \tag{3}$$

Taking the first derivative of (3):

$$\dot{x} = \dot{x}_0 + \dot{x}_1 \dot{y} = \dot{y}_0 + \dot{y}_1 \tag{4}$$

Now, $\dot{x}$ and $\dot{y}$ represent the component of the absolute velocity $\vec{v}_P$ of *P*, while $\dot{x}_1$ and $\dot{y}_1$ and represent the components of the velocity $\vec{v}_{P/S}$ of *P* with respect to *S*. On the other hand, $\dot{x}_0$ and $\dot{y}_0$ represent the components of the velocity of *O'* or, since the slab *S* is in translation, the components of the velocity of any other point of *S*. We may, for example, consider that $\dot{x}_0$ and $\dot{y}_0$ represent the components of the velocity $\vec{v}_{P'}$ of the point *P' of the slab S which happens to coincide with the particle P at the instant considered*. We thus write:

$$\vec{v}_P = \vec{v}_{P'} + \vec{v}_{P/S} \tag{5}$$

We note that the velocity $\vec{v}_P$ reduces to $\vec{v}_{P/S}$ if the slab is stopped and if the particle is allowed to keep moving on *S*: it reduces to the velocity $\vec{v}_{P'}$ of the coinciding point *P'* if the particle *P* is immobilized on *S* while *S* is allowed to keep moving. Thus, (5) expresses that the velocity $\vec{v}_P$ may be obtained by adding vectorially these two partial velocities. Taking second derivative of (3), we obtain:

$$\ddot{x} = \ddot{x}_0 + \ddot{x}_1 \ddot{y} = \ddot{y}_0 + \ddot{y}_1 \tag{6}$$

Or in vector form:

$$\vec{a}_P = \vec{a}_{P'} + \vec{a}_{P/S} \tag{7}$$

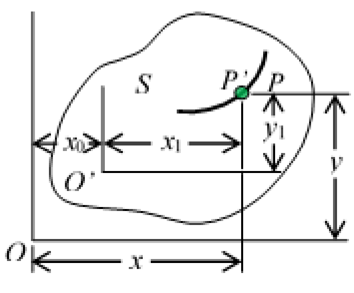

Fig. 1: Slab in translation

Where $\vec{a}_{P/S}$ = acceleration of *P* with respect to *S*
$\vec{a}_{P'}$ = acceleration of point *P'* of *S* coinciding with *P* at the instant considered

Equation (7) may be interpreted in the same way as (5). Equation (5) and (7) actually restate the relative motion of the particle *P* with respect to inertia frame. Since *S* is in translation, the x_1 and y_1 axes, respectively, remain parallel to the fixed *x* and *y* axes, and the velocity $\vec{v}_{P/S}$ is equal to the velocity $\vec{v}_{P/P'}$ relative to point *P'*, similarly, the acceleration $\vec{a}_{P/S}$ is equal to the acceleration $\vec{a}_{P/P'}$ relative to *P'*.

B. Particle Moving on a Rotating Slab – Coriolis Acceleration

Let us consider the motion of a particle *P* which describes a path on a slab *S* which is itself rotating about point *O*. The motion of *P* may be analyzed either in terms of its polar coordinates *r* and θ with respect to fixed axes or in terms of its coordinates *r* and θ_1 with respect to axes attached to the slab *S* and rotating with it as shown in Fig. 2. Again we propose to determine the relation existing between the absolute motion of *P* and its relative motion with respect to *S*. We shall use polar coordinates *r* and θ to express the radial and transverse components of the absolute velocity $\vec{v}_P$ of *P*. Observing that $\theta = \theta_0 + \theta_1$, where θ_0 denotes the angular displacement of the slab at the instant considered, we write:

$$(v_P)_r = \dot{r} (v_P)_\theta = r\dot{\theta} = r\left(\dot{\theta}_0 + \dot{\theta}_1\right) \tag{8}$$

Considering the particular case when *P* is immobilized on *S* and *S* is allowed to rotate, $\vec{v}_P$ reduces to the velocity $\vec{v}_{P'}$ of the point *P'* of the slab *S* which happens to coincide with *P* at the instant considered. Making *r* = constant and θ_1 = constant in (8), we obtain:

$$(v_{P'})_r = 0 \qquad (v_{P'})_\theta = r\dot{\theta}_0 \tag{9}$$

Considering now the particular case when the slab is maintained fixed and *P* is allowed to move, $\vec{v}_P$ reduces to the relative velocity $\vec{v}_{P/S}$. Making θ_0 = constant in (8), we obtain therefore:

$$\left(v_{P/S}\right)_r = \dot{r}\left(v_{P/S}\right)_\theta = r\dot{\theta}_1 \tag{10}$$

Comparing (8) to (10), we find that:

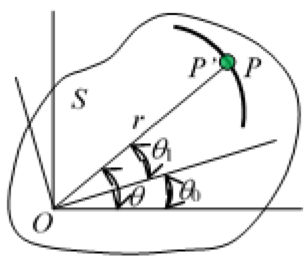

Fig. 2: Particle moving on a rotating slab

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$$(v_P)_r = \dot{r}(v_P)_\theta = r\dot{\theta} = r\left(\dot{\theta}_0 + \dot{\theta}_1\right) \qquad (11)$$

or in vector form:

$$\vec{v}_P = \vec{v}_{P'} + \vec{v}_{P/S} \qquad (12)$$

Equation (12) expresses that $\vec{v}_P$ may be obtained by adding vectorially the velocities corresponding to the two particular cases considered above. The radial and transverse components of the absolute acceleration $\vec{a}_P$ of P can be expressed as:

$$\begin{aligned}(a_P)_r &= \ddot{r} - r\dot{\theta}^2 = \ddot{r} - r\left(\dot{\theta}_0 + \dot{\theta}_1\right)^2 \\ &= \ddot{r} - r\left(\dot{\theta}_0^2 + 2\dot{\theta}_0\dot{\theta}_1 + \dot{\theta}_1^2\right)^2\end{aligned} \qquad (13)$$

$$\begin{aligned}(a_P)_\theta &= r\ddot{\theta} + 2\dot{r}\dot{\theta} \\ &= r\left(\ddot{\theta}_0 + \ddot{\theta}_1\right) + 2\dot{r}\left(\dot{\theta}_0 + \dot{\theta}_1\right)\end{aligned} \qquad (14)$$

Considering the particular case when r = constant and θ_1 = constant (P immobilized on S), we write:

$$(a_{P'})_r = -r\dot{\theta}_0^2 (a_{P'})_\theta = r\ddot{\theta}_0 \qquad (15)$$

Considering now the particular case when θ_0 = constant (slab fixed), we obtain:

$$\begin{aligned}\left(a_{P/S}\right)_r &= \ddot{r} - r\dot{\theta}_1^2 \\ \left(a_{P/S}\right)_\theta &= r\ddot{\theta}_1 + 2\dot{r}\dot{\theta}_1\end{aligned} \qquad (16)$$

Comparing (13) to (16), we find that the absolute acceleration $\vec{a}_P$ cannot be obtained by adding the accelerations $\vec{a}_{P'}$ and $\vec{a}_{P/S}$ corresponding to the two particular cases considered above. We have instead:

$$\begin{aligned}\left(a_{P/S}\right)_r &= \ddot{r} - r\dot{\theta}_1^2 \\ \left(a_{P/S}\right)_\theta &= r\ddot{\theta}_1 + 2\dot{r}\dot{\theta}_1\end{aligned} \qquad (17)$$

where $\vec{a}_c$ is a vector of components:

$$(a_c)_r = -2r\dot{\theta}_0\dot{\theta}_1 (a_c)_\theta = 2\dot{r}\dot{\theta}_0$$

Noting that $\dot{\theta}_0$ represents the angular velocity ω of S, and recalling (10), we have:

$$\begin{aligned}(a_c)_r &= -2\omega\left(v_{P/S}\right)_\theta \\ (a_c)_\theta &= 2\omega\left(a_{P/S}\right)_r\end{aligned} \qquad (18)$$

The vector $\vec{a}_c$ is called the Coriolis acceleration, after the French mathematician de Coriolis (1792-1843). The first of(18) indicates that the vector $(\vec{a}_c)_r$ is obtained by multiplying the vector $\left(\vec{v}_{P/S}\right)_\theta$ by 2ω and rotating it through 90° in the sense of rotation of the slab as shown in Fig. 3; the second formula defines $(\vec{a}_c)_\theta$ from $\left(\vec{v}_{P/S}\right)_r$ in a similar way. *TheCoriolis acceleration* $\vec{a}_c$ *is thus a vector perpendicular to the*

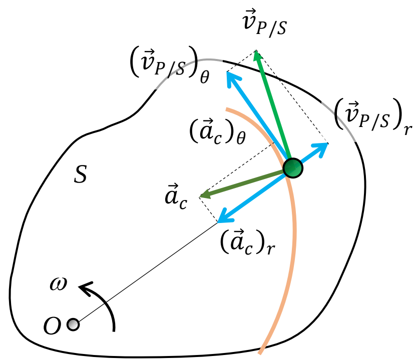

Fig. 3: Coriolis acceleration and its components

relative velocity $\vec{v}_{P/S}$, *and of magnitude equal to* $2\omega v_{P/S}$ *: the sense of* $\vec{a}_c$ *is obtained by rotating the vector* $\vec{v}_{P/S}$ *through 90° in the sense of rotation of* S. Refer to (17), we note that $\vec{a}_{P/S}$ of P relative to the slab S is not equal to the acceleration $\vec{a}_{P/P'}$ of P relative to the point P' of S; this is due to the fact that the first acceleration is defined with resect to *rotating axes*, while the second is defined with respect to *axes of fixed orientation*.

C. Kinematic Formulation of the Coriolis Effect Kit

To further illustrate the physical meaning of (18), we consider a rod-slider assembly, where the rod and slider are rotating with a constant angular velocity $\vec{\omega}$, and the slider is moving along the rod with a constant speed of $\vec{u}$. The first term in (15) is depicted by Fig. 3, while the second term is depicted in Fig. 4. In both figures, the subscript A/O has been omitted in all velocity terms to avoid clutter.

In Fig. 4(a), the difference between the stretch velocity at time $t + \Delta t$ and the stretch velocity at time t gives a resultant vector between the tips of the stretch velocity vectors which, in the limit, is perpendicular to the relative position vector $\vec{r}_{A/O}$. In Fig. 4(b), the difference between vectors $\vec{\omega} \times r'\vec{u}$ and $\vec{\omega} \times r\vec{u}$ gives a net result in the same direction as $\vec{\omega} \times r\vec{u}$,and therefore again perpendicular to the relative position vector $\vec{r}_{A/O}$. Together, (13), Fig. 4(a), and Fig. 4(b), may be used to explain the physical basis of the Coriolis acceleration calculated as a part of kinematic analyses in rigid body dynamics. Thus:

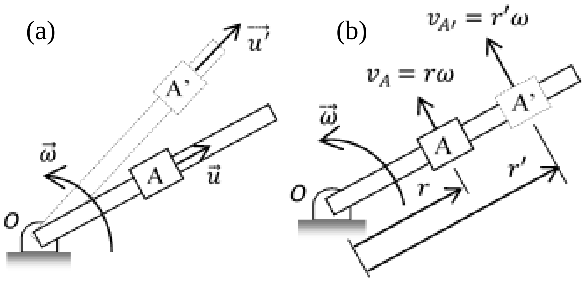

Fig. 4: (a) Change in velocity component due to rotation and (b) change in rotation-based velocity component due to stretch.

$$\left(\omega \times \frac{dr}{dt} + \omega \times \frac{dr'}{dt}\right) = \omega v + \omega v = 2\omega v = a_c$$

Fig. 5 shows the isometric view of the preliminary design of the Coriolis kit. It consists of a double rod with a stopper at each end, a slider mounted with marker and accelerometer, a vertical driving shaft connected to the rotor, and a servo motor. The slider and marker are used to visualize and sketch the traveling path of the slider respectively. The accelerometer measures proper acceleration of the slider in its own instantaneous rest frame. In one embodiment, the two (2) cylindrical rods are secured at center and both ends and they act like a helicopter rotor blade. This type of rotor is more stable and manageable compared to cantilever type. Regardless of the particular embodiment, it will be understood by those knowledgeable and skilled in the art that the device advantageously employs controllable motor and traceable travelling path of slider to enable manipulation of angular speed and effectively utilize analytical technique for solving the problem in acceleration analysis. Other features, objects, and advantages of the present invention will be apparent to those of ordinary skill in the relevant arts.

III. RESULTS AND DISCUSSION

The use of an inertia frame of reference for analysis and physical interpretation of rigid body dynamics is strongly advocated. The same methodology that uses rotating frames are widely used by others [11]. Earth, and the term "Coriolis force" can be found in many engineering sources. In fact, many would argue that the term "Coriolis" should not be used except for effects observed in a rotating frame. The two phenomena represented in Figs. 3 and 4 were presented with all motion observed from a fixed frame in space. The resulting Coriolis acceleration relied on two velocity components: a radial velocity, $\vec{u}'$, and the velocity component related to rotation, namely $\vec{\omega} \times \vec{r}'$. Both the translational and rotational motion were "real", relative to a fixed frame in space, and the resulting accelerations would necessarily be the result of real forces in nature. If the rod OA had been fixed with the slider A still moving along the rod with a stretch velocity, there would be no Coriolis acceleration with respect to the fixed frame. If,however, the fixed rod with translating slider were observed by someone in a rotating frame of reference with angular velocity $\vec{\omega}$(and the observer did not perceive that he or she was rotating), the observer would think that it is the rod which is rotating with angular velocity in the opposite direction; i.e., $-\vec{\omega}$.

A translational analogy is the case where an observer is in a car accelerating past a tree on the side of the road. If the accelerating observer thinks of the car as fixed, then the observer will conclude that it must be the tree which is accelerating backwards. Moreover, from the point of view of this observer, there must be a force *causing* the tree to accelerate. In the same way, a Coriolis acceleration is observed, and a Coriolis force implied, when a particle translating along a straight line is viewed from a rotating frame. We find that this translational analogy effectively emphasizes (to the student) the nature of "Coriolis effects" resulting solely from a rotating point of view. The analogy is fairly direct, in that acceleration can originate kinematically from two sources: changes in velocity due to changes in magnitude (as in the car), and changes in velocity due to rotation. Similarly, a frame can be noninertial due to translational acceleration, and/or rotation. Once the concept of relative motion in terms of frame rotation is understood, the student can better appreciate the Coriolis effects traditionally described by others, in terms of $-2\vec{\Omega} \times \vec{v}_{rel}$, where $\vec{v}_{rel}$ is the velocity viewed from the rotating frame.

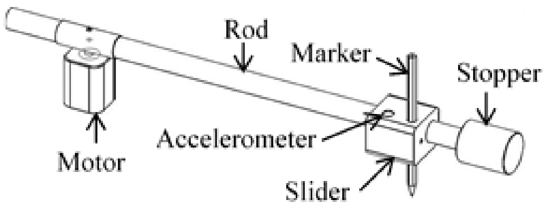

Fig. 5: Isometric view of the preliminary design.

Fig. 6 shows isometric view of the actual prototype and the travelling path of the slider on the rotating rod. In the inertia fame, the slider travels in translational motion from position A to A'. However, the path becomes curve when referring to the rotating rod. To transfer the slider to the intended position, the rotation of the rod must be controllable. Fig. 7 shows the acceleration components of the slider that consist of Coriolis, centrifugal, and net acceleration vectors. The Coriolis acceleration component is always perpendicular to the travelling path of the slider. Table I shows measured values and calculated a_c. The a_c increases as the input step increases. Fig. 8 illustrates the travelling path of the slider with respect to the inertia frame (i.e. *x* and *y* coordinate system) when the rod rotates about the horizontal axis through point *O*.

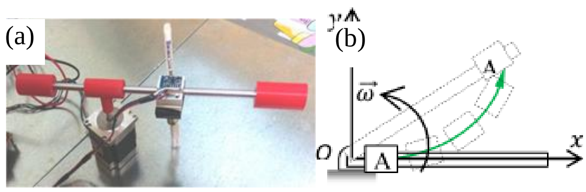

Fig. 6: (a) Actual prototype and (b) travelling path of the slider on the rotating rod.

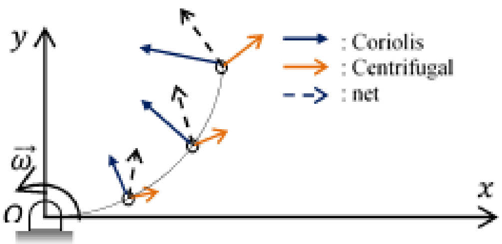

Fig. 7: Acceleration components consist of Coriolis, centrifugal, and net acceleration vectors.

Table I: Measured and calculated value

Steps	$\omega\ (rad/s)$	Velocity (m/s)	$a_c\ (m/s^2)$
300	9.425	0.0147	0.277
500	15.708	0.0211	0.663
700	21.991	0.0271	1.192

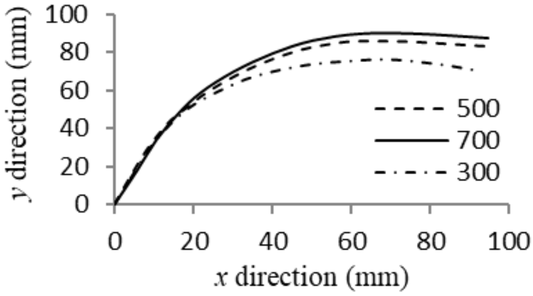

Fig.8: Travelling path of the slider changes with different input step.

IV. CONCLUSION

This paper presents an attempt to measure Coriolis acceleration of an object with respect to a rotating rod using a newly developed Coriolis kit. The instrumentation works accordingly by providing measured values to be used in calculating the Coriolis acceleration. This Coriolis kit is capable to visualize and sketch the travelling path of the object in motion using materials that is typically found in normal classroom. Users could perform direct calculation by applying the theories learnt in class.

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