

# A Mathematical Model for Increasing Incidence of Tuberculosis in Poverty Driven Confined Areas and Measures for Control

Ch V Ramana Murthy, K R Kavitha, K Jhansi Rani

**ABSTRACT**--Tuberculosis is observed to be most prevalent airborne infectious diseases killing hundreds and thousands of people every year. Mostly, the disease spreads in poverty driven poor countries and also to some extent in developing nations. The disease transmits in situations where infected persons are in close contact with others in confined spaces and also living in poor and unhygienic conditions. It is well recognized that overcrowding increases the risk of transmission. A detailed analysis is presented in this paper with respect to the various participating parameters. Interestingly, the results presented in this paper illustrate the occurrence and propagation of the disease as a mathematical model. The results are in agreement with the real life situations and the number of new infections is observed to be linear. It is seen that for fixed time ( $t$ ), as the room volume ( $V$ ) increases, the new infections ( $C$ ) decreases gradually. In general it is seen that as the ventilation rate ( $N$ ) increases, the new infections decreases quite rapidly. Further, it is observed that as time increases for a constant ventilation rate, an increase in the new infections is noted. Also, for a constant room volume as time increases, the number of new infections is found to be increasing. An interesting feature is that as the ventilation is progressive, a steep fall in the new infections is noted in the initial stages and subsequently, the drop is not that significant. The influence of time gradually seems to be diminishing as the ventilation rate increases. As the room volume increases, the new infections decrease at a faster rate. However, in each of these observations, it is seen that as ventilation rate increases, the number of new infections are found to be inversely proportional. Such a decrease is more predominant in the initial stages but decreases subsequently

## I. INTRODUCTION

Tuberculosis is observed to be the most prevalent airborne infectious diseases killing hundreds and thousands of people, principally in adults every year [1]. The disease is widespread and is closely associated with poor living conditions of any individual [2]. From the statistics collected world over it is seen that one third of the world population is totally infected by the disease. It is observed that the disease is strongly linked with the overcrowding of the family and individuals living in unhygienic conditions. The mortality rates are directly related to the number of people living in a house in the specific population area [3]. It is seen that, there is a strong relationship between the house and work place overcrowding and TB mortality. The transmission of the disease is generally associated with prolonged close contact with the infected person [4]. The transmission of TB is

found to be increasing at an exponential rate in confined areas [5] with the spreading occurring in situations where the infected persons are in close contact with others. The examples to be cited are generally nursing homes, educational areas, prisons and refugee camps.

The infection is transmitted by inhalation of droplet nuclei, carrying Mycobacterium tuberculosis bacilli having the diameter less than  $5\mu\text{m}$ . Such bacillus settles very slowly and remains suspended in the air for several hours. An infected person coughing severely and frequently will induce thousands of such bacilli into the room space. Therefore, this causes the TB transmission in spaces that are poorly ventilated and overcrowded and also in the poor living conditions viz: slum housing, unhygienic living conditions. Therefore, the poor and inadequate ventilation are also responsible for the propagation of the disease. The overcrowding increases the risk of transmission. However, this could not be effectively quantified. Indeed, no formal definition by the WHO is available in this regard. The extent to which people spend time in overcrowded conditions is driven by the factors such as economics status, financial position, sex, geographical conditions and atmospheric situations. However, it is also observed that the time spent in aircraft, airport lounges, passengers transport vehicles are also some of the other causes for the transmission of the disease.

Despite the availability of the therapy, TB continues to flourish [6]. The spread of the diseases in both developed and developing nations is found to be uneven and highlights the poverty and poor living conditions. Poverty stricken people do not have nutritious food, income stability, access to the protected water supply, sanitation and the health care conditions [7,8]. The devastating effects of TB, mostly in patients with HIV/AIDS, led the WHO to declare the Global emergency for TB [9]. It is also noted that, in countries like Zambia, Zimbabwe, Rwanda (some of the African countries) and south Asian countries suffer with HIV/AIDS simultaneously. A person suffering with HIV/AIDS is more prone to TB at a faster rate when compared to a person not possessing such diseases. It is due to the fact that HIV drastically reduces the immunity of a person causing more exposure and prone for attacking of TB. Once a person attacked with TB and HIV/AIDS becomes poor quickly as they are not able to become economically protective [10,11]. TB control and poverty reduction cannot be achieved by

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seeking improvements of target indicators on population averages. Instead, specific requirements of vulnerable communities/ religions must be addressed with high spirits. In the cases of poverty stricken individuals known to have TB are isolated leading to denial of mixing with people, diagnosis and effective treatment.

Implementation of Directly Observed treatment Strategy (DOTS) in TB control is extremely difficult in overcrowded spaces of poor and geographically remote areas. Number of studies have been conducted to find the link between poverty and TB [12, 13-17]. Beggset *al.* [18] and Simmons [19] applied statistical analysis for the assessment of poverty in relation to the spreading of TB. Most of the studies [20, 21] analyses the link between overcrowding and TB.

Our work is concentrated in establishing a mathematical model between the duration of the contact and the volume of the room and ventilation in which people are living in identifying new infections. This is purely a mathematical model which tries to correlate the results with that of the real life situations.

## II. MATHEMATICAL MODEL

The Mass Action principle quotes that the number of infectious transmission per infected case is a function of number of susceptible individuals into population. As the outbreak progresses, the number of susceptible individuals becomes spares as a result, the outbreak fadeout which is given by

$$C = rIS \quad (1)$$

where 'C' is new infections, 'S' is the dimension of the susceptible people while 'I' is the number of source cases and 'r' is the effective rate of contact. From equation (1) if  $C/I > 1$  the situation is progressing and if  $C/I < 1$  the epidemic is dying out. In the case of  $C/I = 1$ , the outbreak is considered to be in the steady state. In an enclosed space if 'r' is the effective rate of contact and  $\phi$  is the average number of cases generated per infectors, 'p' is the pulmonary ventilation rate and 't' is the duration of exposure to an infected person and 'Q' is the room ventilation rate then

$$r = \frac{p\phi t}{Q} \quad (2)$$

From Eqn (1) and Eqn (2)

$$C = \left( \frac{p\phi t}{Q} \right) IS \quad (3)$$

The same can be simplified to

$$C = \theta pSt \quad (4)$$

where  $\theta$  is consider to be the concentration of infectious doses in the room space under the steady state condition which can further be stated as

$$\theta = \frac{\phi I}{Q} \quad (5)$$

Eqn (5) sometimes represent the infection in the room in terms of total number of quanta, 'n' is given by expression

$$n = V\theta$$

where 'V' is considered to be room volume ( $m^3$ ).

GammaitoniNucci developed a Model that included the change of Quanta level in a room space with respect to the time parameter. This was derived under the fundamental equations for the rate of change of susceptible individuals and quanta with reference to time [22] as

$$\frac{dS}{dt} = \frac{-p}{V} nS \quad (6)$$

$$\frac{dn}{dt} = -Nn + q \quad (7)$$

General solution of Eqn. (7)

$$ne^{Nt} = \int qe^{Nt} dt + c$$

$$ne^{Nt} = q \frac{e^{Nt}}{N} + c$$

with the conditions that when  $t = 0$  and  $n = n_0$

$$n_t = \frac{q}{N} + \left( n_0 - \frac{q}{N} \right) e^{-Nt} \quad (8)$$

Using Eqn. (8) in Eqn. (6)

$$\frac{dS}{dt} = \frac{-p}{V} \left( \frac{q}{N} + \left( n_0 - \frac{q}{N} \right) e^{-Nt} \right) S$$

Integrating on both sides

$$\int \frac{dS}{S} = \int \frac{-p}{V} \left( \frac{q}{N} + \left( n_0 - \frac{q}{N} \right) e^{-Nt} \right) dt$$

$$\log S = \frac{-p}{V} \left( \frac{q}{N} t + \left( n_0 - \frac{q}{N} \right) \frac{e^{-Nt}}{-N} \right) + \log k$$

$$\log S - \log k = \frac{-p}{VN} \left( qt - \left( n_0 - \frac{q}{N} \right) e^{-Nt} \right)$$

$$\log \left( \frac{S}{k} \right) = \frac{-p}{VN} \left( qt - \left( n_0 - \frac{q}{N} \right) e^{-Nt} \right)$$

$$\frac{S}{k} = e^{\left( \frac{-p}{VN} \left( qt - \left( n_0 - \frac{q}{N} \right) e^{-Nt} \right) \right)}$$

$$S = ke^{\left( \frac{-p}{VN} \left( qt - \left( n_0 - \frac{q}{N} \right) e^{-Nt} \right) \right)}$$

Under the assumption that when  $t = 0$ ,  $S = S_0$  and  $n_0 = 0$  which yields that

$$S_t = S_0 e^{\left( \frac{-pq}{V} \left( \frac{Nt + e^{-Nt} - 1}{N^2} \right) \right)} \quad (9)$$

and number of new Infections

$$C = S_0 \left( 1 - e^{\left( \frac{-pq}{V} \left( \frac{Nt + e^{-Nt} - 1}{N^2} \right) \right)} \right) \quad (10)$$

### III. RESULTS AND CONCLUSIONS

1. Fig 1, Fig 2, Fig 3 and Fig 4 illustrates the effect of the time with respect to the room volume on new infections. In all these cases, it is observed that as 't' increases for a fixed room volume, the number of new infections are on

raise. Further, for a fixed t as the room volume increases, new infections decreases gradually. In addition to the above, in general it is seen that as the ventilation rate increases, the new infections decreases quite rapidly. The above phenomenon is in agreement with the real life situation.

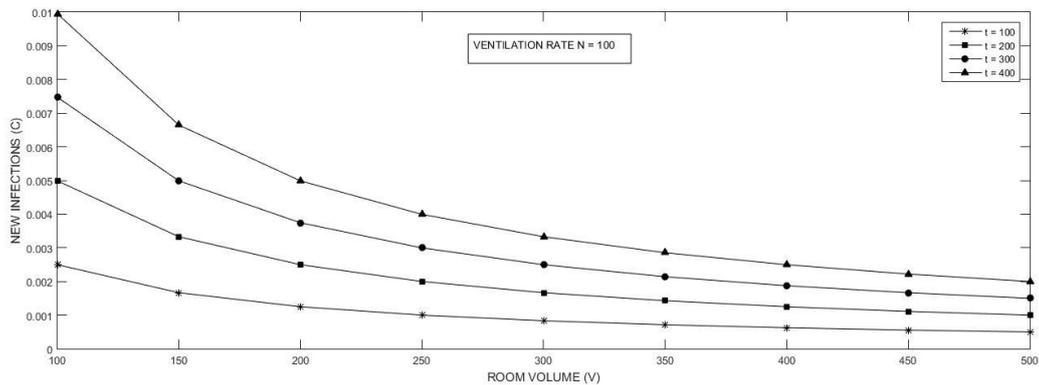


Fig 1: Effect of Time on New infections

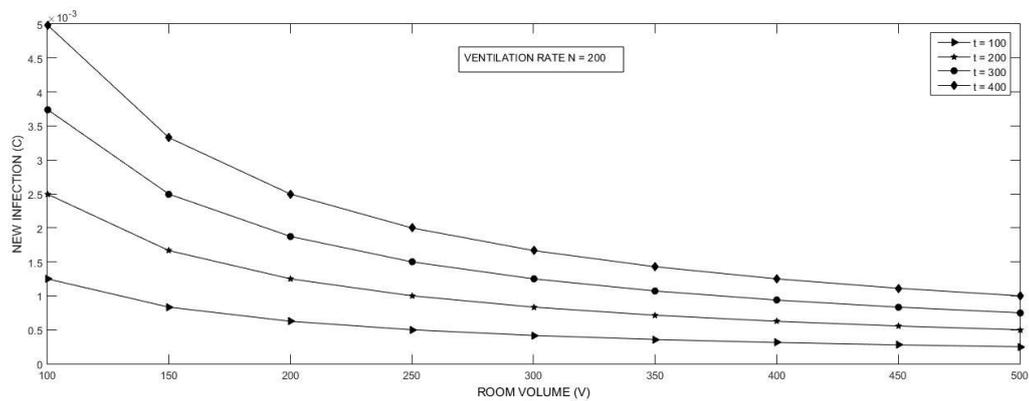


Fig 2: Influence of Time on New infections

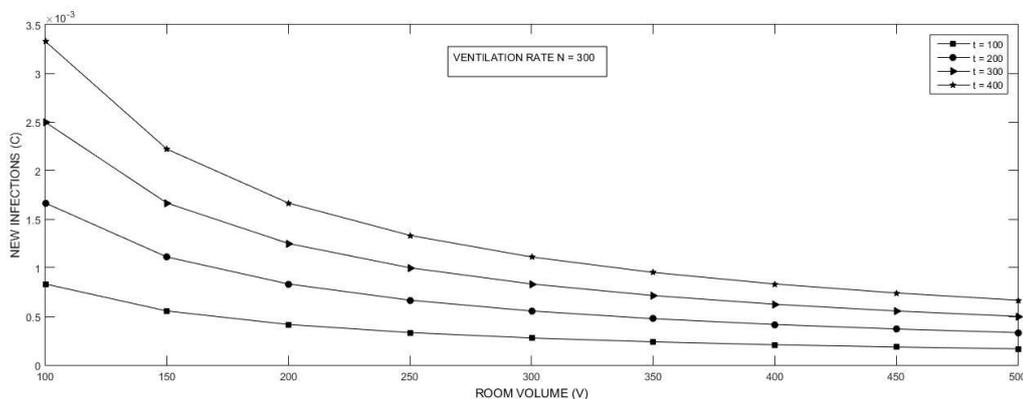
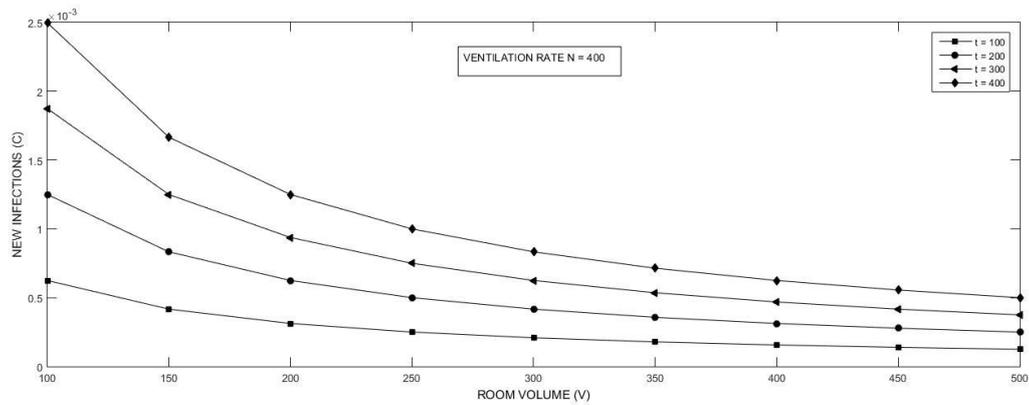


Fig 3: Illustration of Time on New infections for N = 300

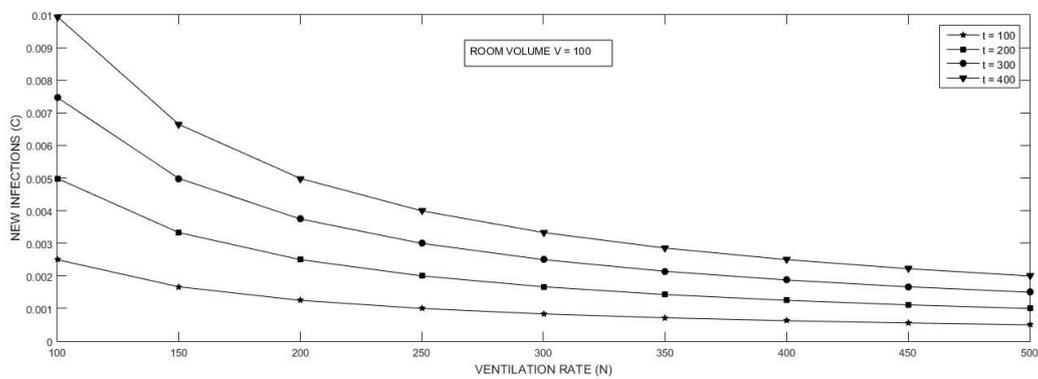
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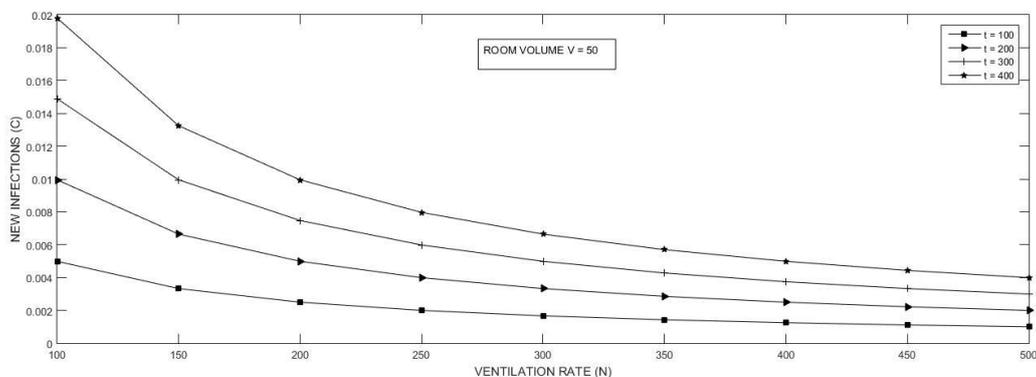
**Fig 4: Room volume Vs New infections (N = 400)**

2. The influence of time with respect to the ventilation rate for a fixed room volume is depicted in Fig 5, Fig 6, Fig 7 and Fig 8. In all this observations, it is seen that as time 't' increases for a constant ventilation rate, an increase in the new infections is noted. Also, for a constant room volume as t increases, the number of new infections is found to be increasing. Seeing all figures, an interesting

feature observed is that there is a steep fall in the new infections in the initial stages and subsequently, the drop is not that significant as the ventilation is progressive. The influence of 't' gradually seems to be diminishing as the ventilation rate increases. The mathematical model developed seems to be in agreement with the real life situation.



**Fig 5: Influence of t on New infections**



**Fig 6: Illustration of t on New infections when V = 50**

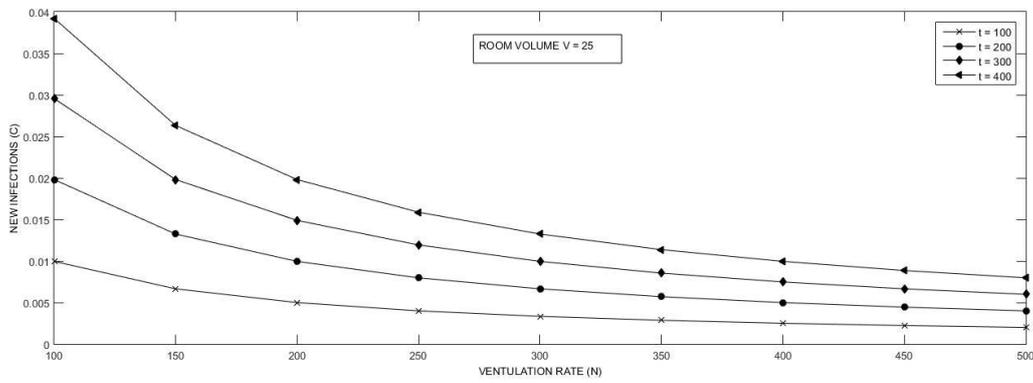


Fig 7: Illustration of t on New Infections for V = 25

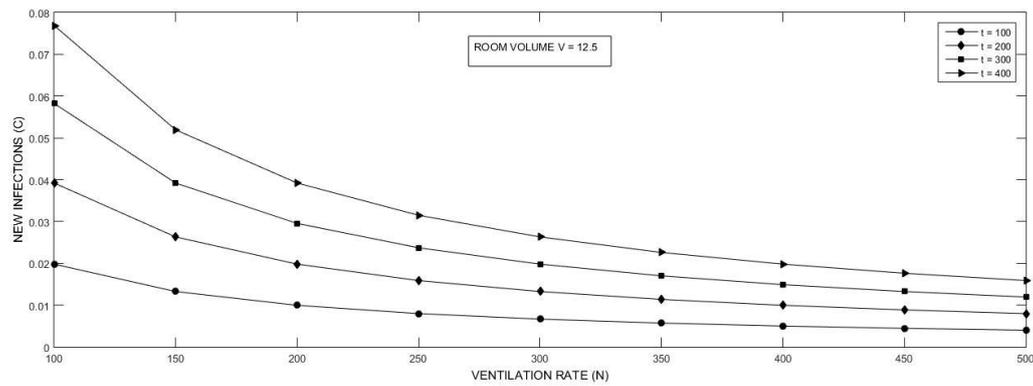


Fig 8: Effect of t Vs New infections (V = 12.5)

3. From Fig 9, Fig 10, Fig 11 and Fig 12 shows the consolidated influence of room volume and time on the new infections. In each of these cases, the number of new infections is observed to be linear. Also it is seen that, in general as N increases the number of new infections

decreases. Further, as the room volume increases the number of new infections decreases rapidly. It is observed that such a decrease is more prominent when the room volume is 200 and 300. But the case is not so when compared to the size of the room where V = 400.

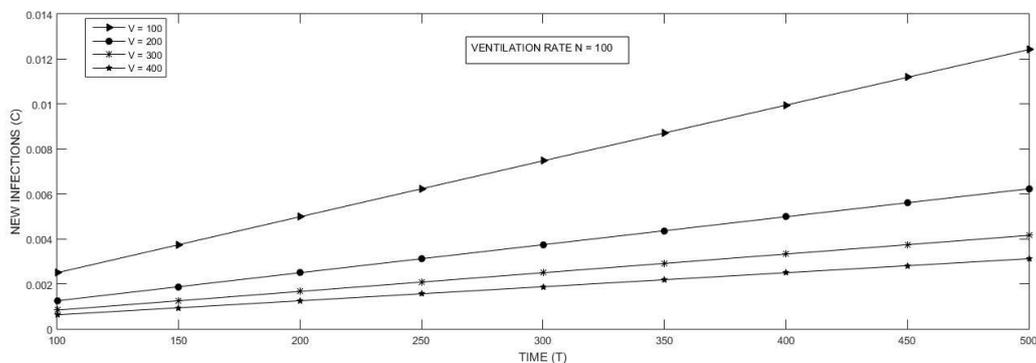


Fig 9: Time Vs New infections for N = 100

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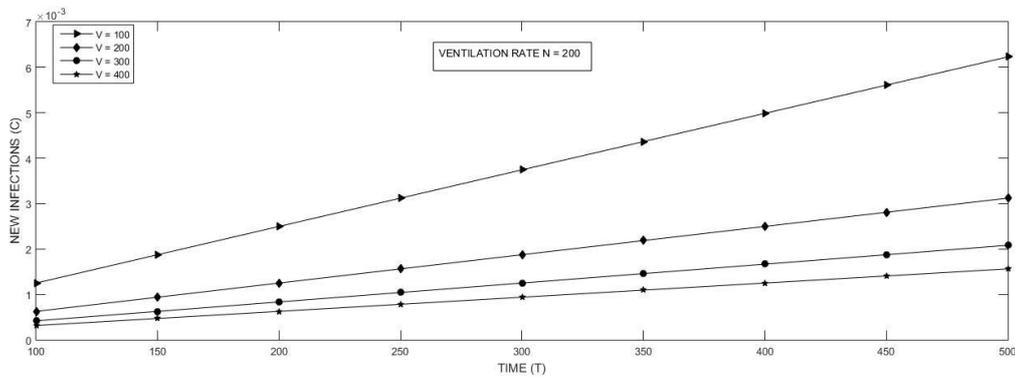


Fig 10: Effect of V on New infections

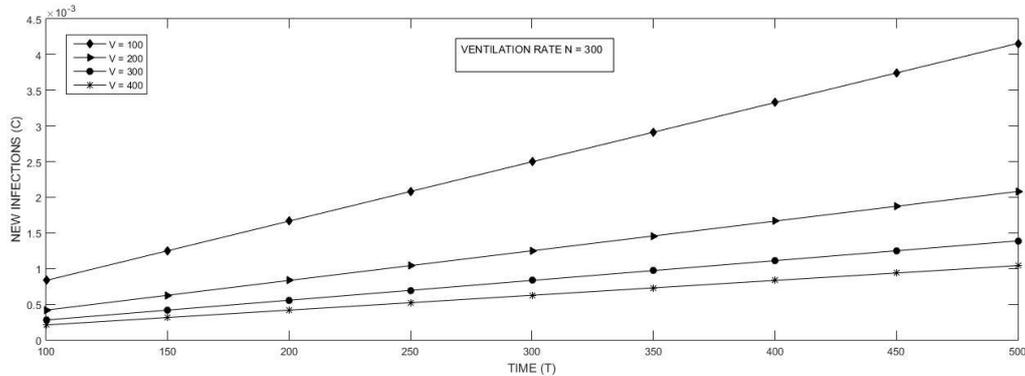


Fig 11: Illustration of Time Vs New infections for N = 300

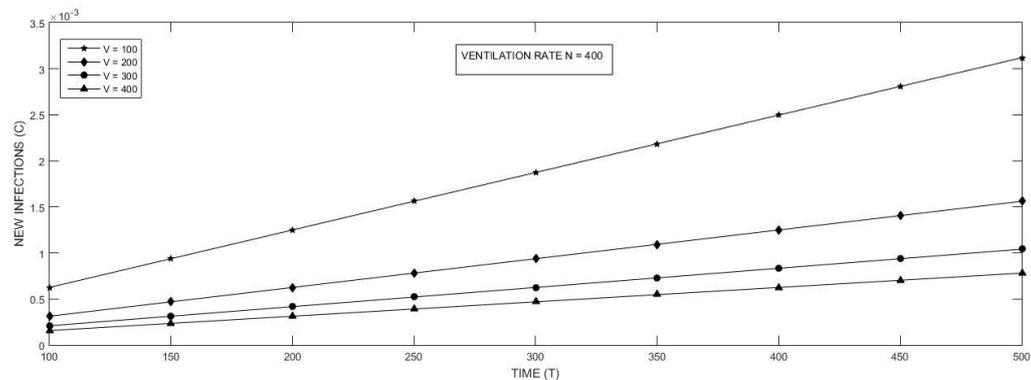


Fig 12: Time Vs New infections for N = 400

4. The consolidated influence of N with respect to time for fixed room volume has been studied in Fig 13, Fig 14, Fig 15 and Fig 16. A combined study shows that, as the room volume increases, the number of new infections C decreases at a faster rate. However, in each of these illustrations it is seen that as N increases, the number of new

infections C are found to be inversely proportional. Such a decrease is more predominant in the initial stages and then decreases subsequently. However, as the time elapses for a fixed volume, the number of new infections is found to be linear and increasing. Such an increase is found to be more significant as N increases.

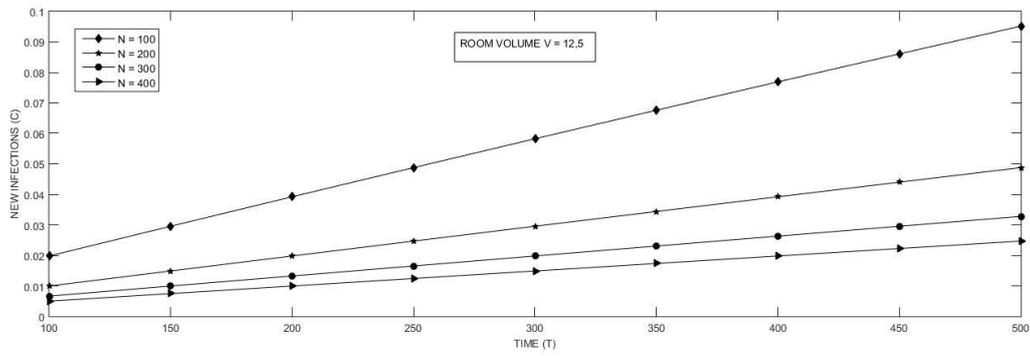


Fig 13: Time Vs New infections for V = 12.5

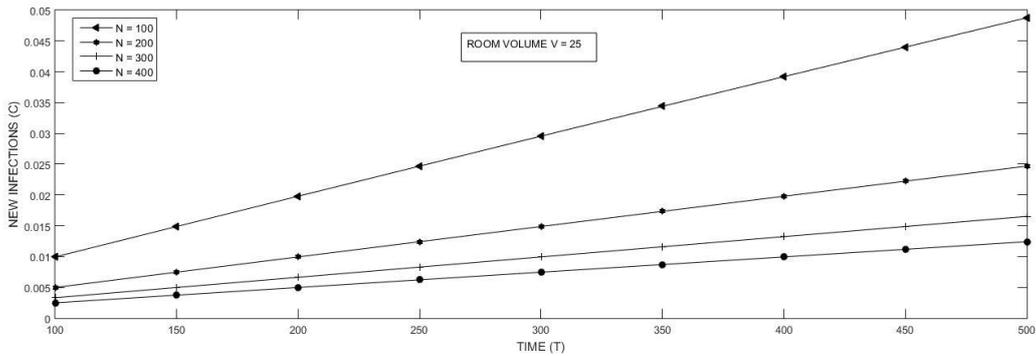


Fig 14: Time Vs New infections for V = 25

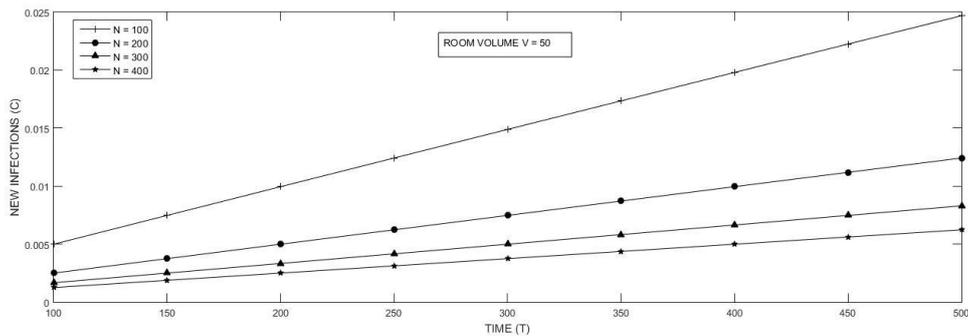


Fig 15: Illustration of N on New Infections

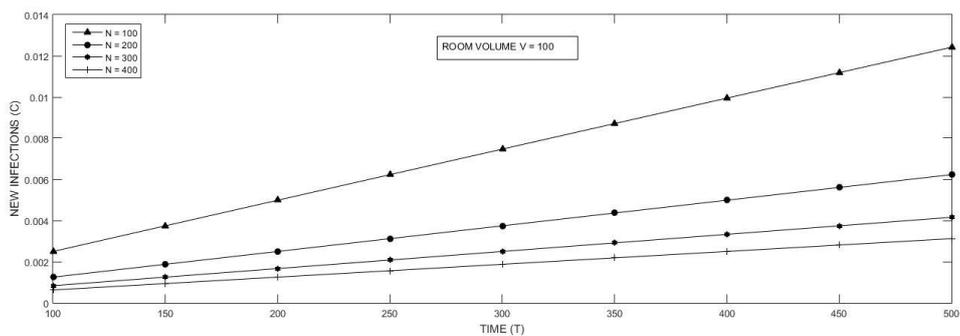


Fig 16: Effect of time Vs New infections for V = 100

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