Certain Properties of Komatu Integral Transform with Negative Coefficients with Reference to Uniform Starlike and Uniform Convex Functions

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ABSTRACT--- The authors obtained a new subclass about strongly starlike and strongly convex functions with respect to Komatu integral transforms and the inclusion properties of these classes such as $\mathcal{S}_pT(\lambda, I_\nu^\rho)$ and $\mathcal{UCVT}(\lambda, I_\nu^\rho)$ were discussed . Furthermore, a new subclass about uniformly starlike functions along uniformly convex functions including negative coefficients defined by the Komatu integral transforms are introduced. The various properties about these classes are obtained here including (for instance) coefficient estimates, extreme points, distortion and covering theorems.

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I. INTRODUCTION

The class of all analytic functions in the unit disk $\mathbb{U} = \{z: |z| < 1\}$, is symbolized as A and each $\varphi \in A$, asserted to be

$$\varphi(z) = z + \sum_{n=2}^{\infty} a_n z^n \qquad (1)$$

Moreover, if each $\varphi \in A$ satisfies the condition of univalent and normalization

i.e.,$\varphi(0) = 0$ and $\varphi'(0) = 1$. Such a class is symbolized as S.

Let T denotes the class that are analytic and univalent in$\mathbb{U}$, characterized in the pattern:

$$\varphi(z) = z - \sum_{n=2}^{\infty} a_n z^n, \quad a_n \geq 0, \quad \forall \quad n \geq 2, \qquad (2)$$

Silverman[18] introduced the above class

Definition 1.1 *When $\varphi \in A$ convince the condition*

$$\left|arg\left(\frac{z\varphi'(z)}{\varphi(z)} - \gamma\right)\right| < \frac{\pi}{2}\beta, \qquad (3)$$

for few γ $(0 < \beta \leq 1)$ and $(0 \leq \gamma < 1)$, then $\varphi(z)$ is termed as strongly starlike with order β and type γ in $\mathbb{U}$ and it is signified as $S^*(\beta, \gamma)$.

Definition 1.2 *When $\varphi \in A$ convince the condition*

$$\left|arg\left(1 + \frac{z\varphi''(z)}{\varphi'(z)} - \gamma\right)\right| < \frac{\pi}{2}\beta, \quad (4)$$

for few $(0 < \beta \leq 1)$ and $\gamma(0 \leq \gamma < 1)$, then $\varphi(z)$ is termed as strongly convex of order β and type γ in $\mathbb{U}$ and it is signified as $C(\beta, \gamma)$.

Definition 1.3 *[5] A function $\varphi \in S$ act as uniformly convex in $\mathbb{U}$ iff*

$$\Re\left\{1 + \frac{z\varphi''(z)}{\varphi'(z)}\right\} \geq \left|\frac{z\varphi''(z)}{\varphi'(z)}\right|, \quad (z \in \mathbb{U})$$

and this class is symbolized as UCV.

Rønning[16] and Ramachandran et al.[13,14] were introduced the following class of starlike functions analogous to UCV as follows:

Definition 1.4 *A function $\varphi \in S$ is termed as uniformly starlike in $\mathbb{U}$ iff*

$$\Re\left\{\frac{z\varphi'(z)}{\varphi(z)}\right\} \geq \left|\frac{z\varphi''(z)}{\varphi'(z)} - 1\right|, \quad (z \in \mathbb{U})$$

and this class is symbolized as $\mathcal{S}_p$.

In [15], Rønning also generalised the class $\mathcal{S}_p$ and is illustrated below:

Definition 1.5 *A function $\varphi \in \mathcal{S}_p(\lambda)$, $0 \leq \lambda \leq 1$, when φ convince the analytic characterization*

$$\Re\left\{\frac{z\varphi'(z)}{\varphi(z)}\right\} - \lambda \geq \left|\frac{z\varphi''(z)}{\varphi'(z)} - 1\right|, \quad (z \in \mathbb{U})$$

and $\varphi \in UCV(\lambda)$ iff $z\varphi' \in \mathcal{S}_p(\lambda)$.

The class $UCV(\lambda)$ is defined in a similar manner(see[3]).

Definition 1.6 *[8] The integral transform of $\varphi \in T$ for $\nu > -1, \rho > 0$ is represented by $\mathcal{I}_\nu^\rho \varphi(z)$, is defined to be*

$$\begin{aligned}\mathcal{I}_\nu^\rho \varphi(z) &= \frac{(1+\nu)^\rho}{z^\nu \Gamma(\rho)} \int_0^z \left(\log\frac{z}{t}\right)^{\rho-1} t^{\nu-1} \varphi(t)dt \\ &= z - \sum_{n=2}^{\infty} \left(\frac{1+\nu}{n+\nu}\right)^\rho a_n z^n, \quad (\nu > -1, \rho > 0),\end{aligned} \qquad (5)$$

is known as Komatu integral transform.

It was introduced and studied by Y. Komatu[8] and extended by the authors

T. N.Shanmugam and C.Ramachandran [17].

For $\rho = 1$, the generalized Bernardi-Libera-Livingston integral operator $\mathcal{I}_\nu^1 \varphi(z) = \mathcal{L}_\nu \varphi(z)$ is given by

$$\begin{aligned}\mathcal{L}_\nu \varphi(z) &= \frac{(1+\nu)}{z^\nu} \int_0^z t^{\nu-1} \varphi(t)dt \\ &= z - \sum_{n=2}^{\infty} \left(\frac{1+\nu}{n+\nu}\right) a_n z^n, \quad (\nu > -1),\end{aligned} \qquad (6)$$

is studied by Bernadi[2] and the operator $\mathcal{L}_\nu \varphi(z)$, for $\nu = 1, \mathcal{L}_1 \varphi(z)$ was investigated by Libera[9].

For $\nu = 1$, Jung, Kim and Srivastava [7] introduced the one parameter family of integral operator $\mathcal{I}_1^{\rho}\varphi(z)$ and also see [12]

$$\mathcal{I}^{\rho}\varphi(z) = \frac{2^{\rho}}{z\Gamma(\rho)}\int_0^z (log\frac{z}{t})^{\rho-1}\varphi(t)dt$$

$$= z - \sum_{n=2}^{\infty}\left(\frac{2}{n+1}\right)^{\rho} a_n z^n, \quad (\rho > 0, f \in A).$$

The operator $\mathcal{I}^{\rho}\varphi(z)$ is considered before by Flett [4].

Using the belief of Bharathi et al.[3] and Jung et al.[7], consider the class $\mathcal{S}_p\mathcal{T}(\lambda, I_\nu^{\rho})$ and $\mathcal{UCVT}(\lambda, I_\nu^{\rho})$ as follows:

Definition 1.7 *Let $\varphi \in \mathcal{S}_p\mathcal{T}(\lambda, I_\nu^{\rho})$, $0 \le \lambda < 1$, arise the class of functions $\varphi \in T$ convince the condition*

$$\left|\frac{z\left(\mathcal{I}_\nu^{\rho}\varphi(z)\right)'}{\mathcal{I}_\nu^{\rho}\varphi(z)} - 1\right| \le \Re\left\{\frac{z\left(\mathcal{I}_\nu^{\rho}\varphi(z)\right)'}{\mathcal{I}_\nu^{\rho}\varphi(z)} - 1\right\} - \lambda, \quad z \in \mathbb{U}. \qquad (7$$

Definition 1.8 *Let $\varphi \in \mathcal{UCVT}(\lambda, I_\nu^{\rho})$, $0 \le \lambda < 1$ arise the class of functions $\varphi \in T$ convince the condition*

$$\left|\frac{z(\mathcal{I}_\nu^{\rho}\varphi(z))''}{(\mathcal{I}_\nu^{\rho}\varphi(z))'}\right| \le \Re\left\{1 + \frac{z(\mathcal{I}_\nu^{\rho}\varphi(z))''}{(\mathcal{I}_\nu^{\rho}\varphi(z))'} - \lambda\right\}, \quad z \in \mathbb{U}. \qquad (8)$$

Lemma 1.9 *If $\mathcal{I}_\nu^{\rho}\varphi(z) \in T$, then*

$$\sum_{n=2}^{\infty}\left(\frac{\nu+1}{\nu+n}\right)^{\rho} na_n \le 1.$$

Proof. Suppose

$$\sum_{n=2}^{\infty}\left(\frac{\nu+1}{\nu+n}\right)^{\rho} na_n > 1,$$

there exits an integer N in such a way

$$\sum_{n=2}^{N}\left(\frac{\nu+1}{\nu+n}\right)^{\rho} na_n > 1 + \varepsilon/2$$

$$\left(\frac{1}{1+\varepsilon/2}\right)^{\frac{1}{N-1}} < z < 1,$$

we obtain

$$(\mathcal{I}_\nu^{\rho}\varphi(z))' = 1 - \sum_{n=2}^{\infty}\left(\frac{\nu+1}{\nu+n}\right)^{\rho} na_n z^{n-1}$$

$$\le 1 - \sum_{n=2}^{N}\left(\frac{\nu+1}{\nu+n}\right)^{\rho} na_n z^{n-1}$$

$$\le 1 - z^{N-1}\sum_{n=2}^{N}\left(\frac{\nu+1}{\nu+n}\right)^{\rho} na_n$$

$$\le 1 - z^{N-1}(1+\varepsilon/2)$$

$$< 0.$$

But $(\mathcal{I}_\nu^{\rho}\varphi(0))' = 1 > 0$.

There exits a real number z_0, $0 < z_0 < 1$, such that

$$(\mathcal{I}_\nu^{\rho}\varphi(z_0)) = 0.$$

Hence $\mathcal{I}_\nu^{\rho}\varphi(z)$ is not univalent.

Remark 1.10 *For $\nu = 1$, Lemma 1.9 were discussed in [10].*

By taking $\rho = 1$ Lemma 1.9, we can conclude the succeeding corollary:

Corollary 1.11 *Suppose $\varphi \in T$ be stated at (2). If $I_\nu \in T$, then*

$$\sum_{n=2}^{\infty}\left(\frac{\nu+1}{\nu+n}\right) na_n \le 1.$$

The prime objective is to investigate some coefficient estimates, distortion bounds, starlikeness and convexity for $\mathcal{S}_pT(\lambda, I_\nu^{\rho})$ and $\mathcal{UCVT}(\lambda, I_\nu^{\rho})$.

II CHARACTERIZATION THEOREM & RESULTS

Aqlan *et al*[1] proposed a finest method to estimate the coefficient estimates for functions in classes $\mathcal{S}_pT(\lambda, I_\nu^{\rho})$ and $\mathcal{UCVT}(\lambda, I_\nu^{\rho})$. The main characterization theorem considering for the above classes are given as follows:

Theorem 2.1 *Suppose $\varphi \in T$. A function $\varphi \in \mathcal{S}_p\mathcal{T}(\lambda, I_\nu^{\rho})$ iff*

$$\sum_{n=2}^{\infty}\left(\frac{\nu+1}{\nu+n}\right)^{\rho}(2n-1-\lambda)a_n \le 1-\lambda, \quad (9)$$

for some $0 \le \lambda < 1$ and $\rho > 0$. The result is sharp for

$$\mathcal{I}_\nu^{\rho}\varphi(z) = z - \frac{1-\lambda}{\left(\frac{\nu+1}{\nu+n}\right)^{\rho}(2n-1-\lambda)}z^n, \quad n \ge 2. \ (10)$$

Proof.

$$\left|\frac{z(\mathcal{I}_\nu^{\rho}\varphi(z))'}{\mathcal{I}_\nu^{\rho}\varphi(z)} - 1\right| - \Re\left\{\frac{z(\mathcal{I}_\nu^{\rho}\varphi(z))'}{\mathcal{I}_\nu^{\rho}\varphi(z)} - 1\right\} \le 2\left|\frac{z(\mathcal{I}_\nu^{\rho}\varphi(z))'}{\mathcal{I}_\nu^{\rho}\varphi(z)} - 1\right|$$

$$\le \left|\frac{z\left(1 - \sum_{n=2}^{\infty}\left(\frac{\nu+1}{\nu+n}\right)^{\rho} na_n z^{n-1}\right)}{z - \sum_{n=2}^{\infty}\left(\frac{\nu+1}{\nu+n}\right)^{\rho} a_n z^n} - 1\right|$$

$$\le \frac{\sum_{n=2}^{\infty}\left(\frac{\nu+1}{\nu+n}\right)^{\rho}(n-1)a_n}{1 - \sum_{n=2}^{\infty}\left(\frac{\nu+1}{\nu+n}\right)^{\rho} a_n}.$$

if (17) holds, we have,

$$\left|\frac{z(\mathcal{I}_\nu^{\rho}\varphi(z))'}{\mathcal{I}_\nu^{\rho}\varphi(z)} - 1\right| - \Re\left\{\frac{z(\mathcal{I}_\nu^{\rho}\varphi(z))'}{\mathcal{I}_\nu^{\rho}\varphi(z)} - 1\right\} \le 1 - \nu$$

which is equivalent to (7).

Conversely, if $\varphi \in \mathcal{S}_p\mathcal{T}(\lambda, I_\nu^{\rho})$,

$$\frac{1 - \sum_{n=2}^{\infty}\left(\frac{\nu+1}{\nu+n}\right)^{\rho} na_n z^{n-1}}{1 - \sum_{n=2}^{\infty}\left(\frac{\nu+1}{\nu+n}\right)^{\rho} a_n z^n} - \lambda$$

$$\ge \frac{\sum_{n=2}^{\infty}\left(\frac{\nu+1}{\nu+n}\right)^{\rho}(n-1))a_n z^{n-1}}{1 - \sum_{n=2}^{\infty}\left(\frac{\nu+1}{\nu+n}\right)^{\rho} a_n z^{n-1}}.$$

Let $z \to 1^-$ along the real line then we have,

$$\frac{1 - \sum_{n=2}^{\infty}\left(\frac{\nu+1}{\nu+n}\right)^{\rho} na_n}{1 - \sum_{n=2}^{\infty}\left(\frac{\nu+1}{\nu+n}\right)^{\rho} a_n} - \frac{\sum_{n=2}^{\infty}\left(\frac{\nu+1}{\nu+n}\right)^{\rho}(n-1)a_n}{1 - \sum_{n=2}^{\infty}\left(\frac{\nu+1}{\nu+n}\right)^{\rho} a_n} \ge \lambda$$

or

$$\sum_{n=2}^{\infty}\left(\frac{\nu+1}{\nu+n}\right)^{\rho} a_n(2n-1-\lambda) \le \lambda - 1$$

Which is the expected result.

Since the proof of the Characterization theorem for the class $UCVT(\lambda, I_\nu^{\rho})$. is alike to the above theorem, it is skipped .

Theorem 2.2 *Suppose the function $\varphi \in T$. A function $\varphi \in UCVT(\lambda, I_\nu^{\rho})$ iff*

$$\sum_{n=2}^{\infty}\left(\frac{\nu+1}{\nu+n}\right)^{\rho}(2n-1-\lambda)na_n \le 1-\lambda,$$

for some $0 \le \lambda < 1$ and $\rho > 0$. The result is sharp for the function

$\mathcal{J}_\nu^\rho \varphi(z) = z - \frac{1-\lambda}{\left(\frac{\nu+1}{\nu+n}\right)^\rho n(2n-1-\lambda)} z^n, \quad n \geq 2.$ (11)

Coefficient estimates for the class $\mathcal{S}_p\mathcal{T}(\lambda, \mathcal{J}_\nu^\rho)$ and $\mathcal{UCVT}(\lambda, \mathcal{J}_\nu^\rho)$ is a consequence of Theorems 3.1 and 3.2 respectively which is given below:

Theorem 2.3 *If $\varphi \in \mathcal{S}_p\mathcal{T}(\lambda, I_\nu^\rho)$, then*

$$a_n \leq \frac{1-\lambda}{\left(\frac{\nu+1}{\nu+n}\right)^\rho (2n-1-\lambda)}, \quad n \geq 2.$$

Equality holds for $\varphi(z)$ is of the form given in (18).

Proof. Since $\varphi \in \mathcal{S}_p\mathcal{T}(\lambda, I_\nu^\rho)$ we have,

$$\left(\frac{\nu+1}{\nu+n}\right)^\rho (2n-1-\lambda)a_n \leq \sum_{n=2}^{\infty} \left(\frac{\nu+1}{\nu+n}\right)^\rho (2n-1-\lambda)a_n \leq 1-\lambda$$

$$a_n \leq \frac{1-\lambda}{\sum_{n=2}^{\infty} \left(\frac{\nu+1}{\nu+n}\right)^\rho (2n-1-\lambda)}$$

$$a_n \leq \frac{1-\lambda}{\left(\frac{\nu+1}{\nu+n}\right)^\rho (2n-1-\lambda)}, \quad n \geq 2.$$

Hence the result

Theorem 2.4 *If $\varphi \in \mathcal{UCVT}(\lambda, \mathcal{J}_\nu^\rho)$, then*

$$a_n \leq \frac{1-\lambda}{\left(\frac{\nu+1}{\nu+n}\right)^\rho n(2n-1-\lambda)}, \quad n \geq 2.$$

Equality holds for the functions of the form given in (19).

Remark 2.5 *For $\nu = 1$, Theorems 3.1, 3.2, 3.3 and 3.4 were discussed in [10].*

By taking $\rho = 1$ in Theorems 3.1, 3.2, 3.3 and 3.4, we can conceive the successive corollaries:

Corollary 2.6 *Let the function $\varphi \in T$. A function $\varphi \in \mathcal{S}_p\mathcal{T}(\lambda, I_\nu)$ iff*

$\sum_{n=2}^{\infty} \left(\frac{\nu+1}{\nu+n}\right)(2n-1-\lambda)a_n \leq 1-\lambda,$ (12)

for some $0 \leq \lambda < 1$. The result is sharp for

$\mathcal{J}_\nu f(z) = z - \frac{1-\lambda}{\left(\frac{\nu+1}{\nu+n}\right)(2n-1-\lambda)} z^n, \quad n \geq 2.$ (13)

Corollary 2.7 *Let the function $\varphi \in T$. A function $\varphi \in \mathcal{UCVT}(\lambda, I_\nu)$ iff*

$\sum_{n=2}^{\infty} \left(\frac{\nu+1}{\nu+n}\right)(2n-1-\lambda)na_n \leq 1-\lambda,$ (14)

for some $0 \leq \lambda < 1$. The result is sharp for

$\mathcal{J}_\nu \varphi(z) = z - \frac{1-\lambda}{\left(\frac{\nu+1}{\nu+n}\right)n(2n-1-\lambda)} z^n, \quad n \geq 2.$ (15)

Corollary 2.8 *If $\varphi \in \mathcal{S}_p\mathcal{T}(\lambda, I_\nu)$, then*

$$a_n \leq \frac{1-\lambda}{\left(\frac{\nu+1}{\nu+n}\right)(2n-1-\lambda)}, \quad n \geq 2.$$

Equality holds true for the functions of the form given in (21).

Corollary 2.9 *Let the function $\varphi \in T$. A function $\varphi \in \mathcal{UCVT}(\lambda, I_\nu)$, then*

$$a_n \leq \frac{1-\lambda}{\left(\frac{\nu+1}{\nu+n}\right)n(2n-1-\lambda)}, \quad n \geq 2.$$

Equality holds true for the functions the form given in (23).

III DISTORTION AND COVERING THEOREMS

Theorem 3.1 *If $\varphi \in \mathcal{S}_p\mathcal{T}(\lambda, I_\nu^\rho)$ and $|z| = k < 1$, the*

$$k - \frac{1-\lambda}{3-\lambda}k^2 \leq |\mathcal{J}_\nu^\rho f(z)| \leq k + \frac{1-\lambda}{3-\lambda}k^2.$$

Equality holds true for

$\mathcal{J}_\nu^\rho \varphi(z) = z - \frac{1-\lambda}{3-\lambda} z^2.$ (16)

Proof. First, it is evident that

$$(3-\lambda)\sum_{n=2}^{\infty} \left(\frac{\nu+1}{\nu+n}\right)^\rho a_n \leq \sum_{n=2}^{\infty} \left(\frac{\nu+1}{\nu+n}\right)^\rho (2n-1-\lambda)a_n.$$

If $\varphi \in \mathcal{S}_p\mathcal{T}(\lambda, I_\nu^\rho)$, using the inequality in Theorem 3.1,

$\sum_{n=2}^{\infty} \left(\frac{\nu+1}{\nu+n}\right)^\rho a_n \leq \frac{1-\lambda}{3-\lambda}.$ (17)

From (2) with $|z| = k < 1$, we get

$$\begin{aligned}|\mathcal{J}_\nu^\rho \varphi(z)| &\leq k + \sum_{n=2}^{\infty} \left(\frac{\nu+1}{\nu+n}\right)^\rho a_n k^n \\ &\leq k + \sum_{n=2}^{\infty} \left(\frac{\nu+1}{\nu+n}\right)^\rho a_n k^2 \\ &\leq k + \leq \frac{1-\lambda}{3-\lambda}k^2\end{aligned}$$

and

$$\begin{aligned}|\mathcal{J}_\nu^\rho \varphi(z)| &\geq k - \sum_{n=2}^{\infty} \left(\frac{\nu+1}{\nu+n}\right)^\rho a_n k^n \\ &\geq k - \sum_{n=2}^{\infty} \left(\frac{\nu+1}{\nu+n}\right)^\rho a_n k^2 \\ &\geq k - \leq \frac{1-\lambda}{3-\lambda}k^2\end{aligned}$$

Theorem 3.2 *If $\varphi \in \mathcal{UCVT}(\lambda, I_\nu^\rho)$ and $|z| = k < 1$, then*

$$\begin{aligned}k - \frac{1-\lambda}{(\nu+1)(3-\lambda)}k^2 &\leq |\mathcal{J}_\nu^\rho \varphi(z)| \\ &\leq k + \frac{1-\lambda}{(\nu+1)(3-\lambda)}k^2.\end{aligned}$$

Equality holds true for

$\mathcal{J}_\nu^\rho \varphi(z) = z - \frac{1-\lambda}{(\nu+1)(3-\lambda)} z^2.$ (18

Remark 3.3 *For $\nu = 1$, Theorems 3.1 and 3.2 were discussed in [10].*

By taking $\rho = 1$ in Theorems 3.1 and 3.2, we can deduce the subsequent corollaries:

Corollary 3.4 *If $\varphi \in \mathcal{S}_p\mathcal{T}(\lambda, I_\nu)$ and $|z| = k < 1$, then*

$$k - \frac{1-\lambda}{3-\lambda}k^2 \leq |\mathcal{J}_\nu(\varphi(z))| \leq k + \frac{1-\lambda}{3-\lambda}k^2.$$

Equality holds true for

$\mathcal{J}_\nu \varphi(z) = z - \frac{1-\lambda}{3-\lambda} z^2.$ (19)

Corollary 3.5 *If $\varphi \in \mathcal{UCVT}(\lambda, I_\nu)$ and $|z| = k < 1$, then*

$$\begin{aligned}k - \frac{1-\lambda}{(\nu+1)(3-\lambda)}k^2 &\leq |\mathcal{J}_\nu \varphi(z)| \\ &\leq k + \frac{1-\lambda}{(\nu+1)(3-\lambda)}k^2.\end{aligned}$$

Equality holds true for

$\mathcal{J}_\nu \varphi(z) = z - \frac{1-\lambda}{(\nu+1)(3-\lambda)} z^2.$ (20).

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