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The Performance on Slope Number of Hypercube, Normal Representation of Butterfly and Benes Networks

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Abstract: Many quality measures have been defined for graph drawings. In order to optimize these measures, slope number is considered to minimize the distinct edge slopes. The edges of the graphs are designed here as straight line segments. A number of distinct slopes required to draw the graph is called slope number. In this paper the slopenumber is discussed for known parallel architectures like hypercube, butterfly and benes networks. In addition to that the characterization of these networks is investigated and the results are observed for the defined problem.

Keywords: Hypercube, Butterfly networks,Benes networks, Diameter.

I.INTRODUCTION

Graph theory is one of the significant part in different fields. There are numerous applications in different controls. A graph can be used to represent relations and processes in physical, biological, social networks and it is used to represent various sorts of interconnection networks. Bearing this in mind, interconnection networks is focussed which comprises of set of processors and information links for exchangingdatas.J.Xu investigated the structures of topology and analysed the network performance [2]. Several interconnection networks have been recommended in writings. Among them hypercube is the most fascinating network, which is deeply surprising as it has many practical applications. Figure.1 shows the hypercube of dimension 3.

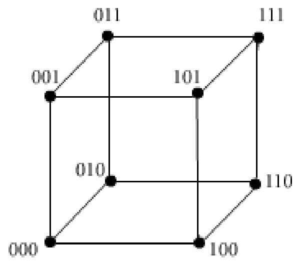

Fig. 1 Hypercube of dimension 3

The variations of hypercube are butterfly networks, benes networks and cube connected cycles. In these, butterfly and benes networks are subordinate system of hypercube. The butterfly network with some modifications becomesbenes network.

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Both butterfly and benes networksareimperative multistage systems which acquires another topologies for data transmissions [3].In the present paper, we consider the normal representation of butterfly and benes network. Wade and Chu introduced the slope number problem in 1994 and they obtained results for complete graphs [1].It is NP-Hard to decide the slope number of any arbitrary graph [4]. In the current paper, we explained how the slope varies for the above mentioned parallel architectures and the results are made elaborately.

II.PRELIMINARIES

Consider finite and connected graph. A graph G with n vertices and m edges consists of vertex set V(G) and an edge set E(G). A sub-graph of G is a graph H such that V(H) ⊆ V(G) and E(H) ⊆ E(G). In a graph G, the largest distance between two vertices of G is called the diameter of G and is denoted by diam(G). A centre of G is a vertex u such that max d(u,v) is as small as possible.

III.OVERVIEW OF THE PAPER

The slope number is framed by setting the vertices on a consistent polygon. In perspective of thatthe defined problem is carried out for parallel architectures namely hypercube, butterfly and benes network. In the present paper, the slopesof benes network and butterfly network is described and characterized.

IV.SLOPE NUMBER

The slope number of a graph G is the minimum number of distinct edge slopes required to draw the graph G. It is denoted by sl[G]. Figure.2is the illustration of slope number.

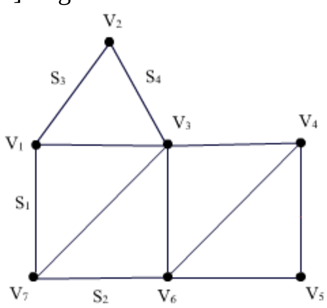

Fig.2sl[G]=4

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The main results are as follows:

Theorem [1]: $sl[K_n] = n$, if $n \geq 3$.

Slope Number of Hypercube Networks

The Hypercube network of n-dimension is denoted by Q_n. It has 2^n vertices and$n2^{n-1}$edges. Theedge of Q_n connects two vertices if and only if the binary representation varyexactly in one bit. It has some of the properties like bipartite graph and is symmetric.The diameter of Q_n is n.The slope number of hypercube is denoted by $sl[Q_n]$.

Theorem[5]:

If Q_n is a graph of hypercube of dimension n, then $sl[Q_n] = n$.

Theorem:

Let Gbe a hypercube network of dimension $n \geq 2$. Then sl[G] is odd or even if and only if it has even number of edges.

Proof:

Case (i): For n is odd

Assume that the slope of the graph G is odd. To prove that|E[G]| is even. Let $S = \{s_1, s_2 \ldots s_n\}$ where $n \in N$, be the set of slopes of G. Since G is n-regular graph with odd degree vertices, the edges incident with n degree vertex shares n slopes such that each n slopes is parallel to 2^{n-1} number of edges. Since the graph of Q_n has $n2^{n-1}$edges, it is always even.

Conversely, let G be a graph with even number of edges. Any two edges incident to the vertex V shares different slopes. Here, each vertex of odd degree is an end point of odd line segment. Choose any vertex $v_i \in V$of G and let $\Delta(G) = n$. The edges incident to the vertex v_i has n slopes. Addition of an edge to the vertex v_i gives n+1 slopes. But this is impossible, since Gis n-regular (i.e the degree of hypercube gets violated). Hence no more edges can be added to the vertex v_i. Hence$sl[Q_{2n-1}] = 2n-1$, which is odd. Refer figure.3

Case (ii): For n is even.

Assume that sl[G] is even. To prove that |E[G]| is even. Let the slopes be $S = \{s_1, s_2 \ldots s_n\}$ where $n \in N$, be the set of slopes of G. Since Gis n-regular graph, the edges incident with n degree vertex shares n slopes such that each n slopes is parallel to 2^{n-1} number of edges. Since the graph of Q_n has $n2^{n-1}$ edges, it is always even.

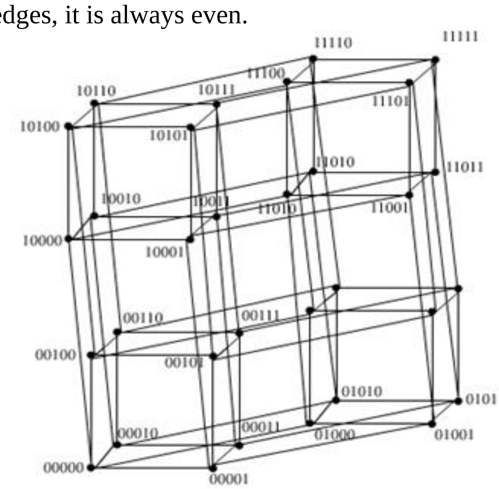

Fig.3Q_5 = 5

Conversely, let G be a graph where |E[G]| is even . Any two edges incident to the vertex V shares different slopes. Thus each vertex of even degree is an end point of even line segment. Choose the vertex v_i. The edges incident to the vertex v_i has n slopes. Addition of an edge to the vertex v_i gives one more slope. But this is impossible, since Gis n-regular (i.e the degree of hypercube gets violated). Hence no more edges can be added to the vertex v_i. Hence,$sl[Q_{2n}] = 2n$, which is even. Refer figure.4

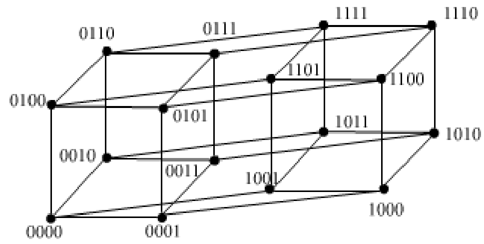

Fig.4Q_4 = 4

Hence the proof.

Observation

For every $n \in N$, $\Delta[Q_n] = sl[Q_n]$

Observation

For every $n \in N$, $diam(Q_n) = sl[Q_n]$.

V.SLOPE NUMBER OF NORMAL REPRESENTATION OF BUTTERFLY NETWORKS

Thebutterfly network of n dimension is denoted by BF(n) which has $V(Q_n) = \{(x ; i) : x \in V(Q_n), 0 \leq i \leq n \}$. Two vertices (x ; i) and (y ; i) are associated by an edge in BF(n) if and only if j = i+1 and either (i) x=y, or (ii) x varies from y in accurately the j^{th}bit, the edge is a straight edge if x=y. If not the edge is a cross edge. Setting i as constant, the vertex (x; i) is i^{th} level vertex.

From the definition, BF(n) has $(n+1)2^n$ vertices since it has n+1 levels and there are $n2^{n+1}$ edges.Figure shows the butterfly network of dimension 2. The slope number of butterfly network is denoted by sl[BF(n)]. Figure.6 shows the butterfly network of dimension 2.

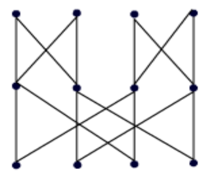

Fig. 5 Butterfly network of BF(2)

Theorem

LetBF(n)be a butterfly network, then the sl[BF(n)] = 2n+1.

Proof:

Let G be a butterfly network. Consider the edges of BF(n) asstraight line edges, x=y. The maximum and minimum degree of BF(n) is 4 and 2 respectively. The slopeline which consists of vertical edges incident with vertices from level 0 to level n are denoted as S_1 and the edges incident

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with the vertex v shares the same slope. Hence the slopes of vertical edges of BF(n) is 1. The remaining crossed edges incident with$\Delta(G)$ varies at each level. The slopes of the crossed edges from level 0 to level n are 2n.Clearly the edges incident with the vertices does not share the same slope. Figure.6 shows he illustration of slope number of BF(3).

i.e. sl[BF(n)]= Slope of the horizontal edges + slopes of crossed edges

$$= S_1 + \sum_{i=2}^{n} S_i$$

$$= S_1 + \{S_2 + S_3 + S_4 + \ldots + S_n\}$$

$$= 1+[2+2+2+\ldots(n \text{ times})]$$

$$= 2n+1$$

Hence the proof.

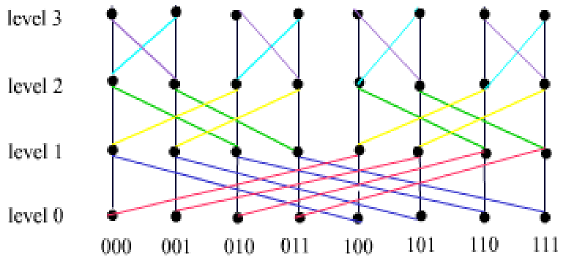

Fig. 6 Slope number of BF(3) = 7

VI.SLOPE NUMBER OF NORMAL REPRESENTATION OF BENES NETWORKS

This network consists of back to back butterflies denoted by BB(n). There number of levels is 2n+1. Each level has 2^n vertices. The initial and final n+1 levels ofBB(n) forms two BF(n) respectively, while the centermost level in BB(n) is shared by these butterfly networks.

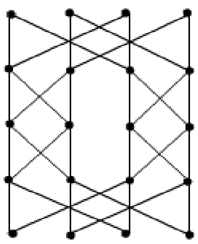

Fig.7Benes network of BB(2)

The n-dimensional benes network consists of $(n+1)2^{n+1}$ vertices and $n2^{n+2}$ edges. Figure.7 shows the benes network of dimension 2.

Theorem

If G is abenes network, then the sl[BB(n)] = 2n+3.

Proof:

Let BB(n) be a benes network.

By Theorem 5.1, sl[BF(n)] = 2n+1. Since, BB(n) has 2n+1 levels, the crossed edges of BF(n) incident with the middle level vertices has 2 slopes.Clearly the edges incident with the vertices does not share the same slope. Figure.8 shows the illustration of slope number of BB(3).

i.e. sl[BB(n)] = sl[BF(n)]]+ slopes of crossed edges of BB(n)

$$= (2n+1)+2$$

$$= 2n+3$$

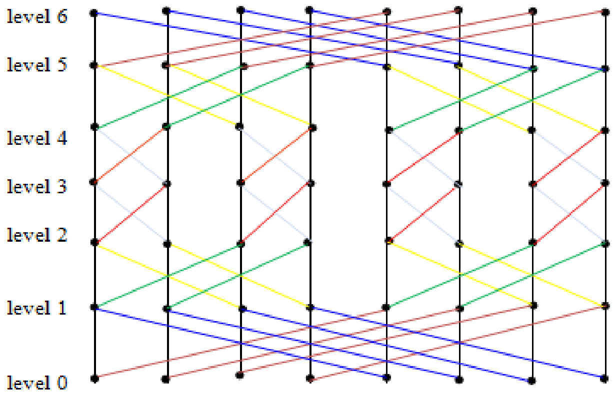

Fig.8Benes network of dimension 2

VII.PARTITION OF BUTTERFLY NETWORKS AND BENES NETWORKS

Butterfly network of n dimension is subdived into two disjoint butterfly of n-1 dimension. When the n^{th} level vertices are removed, two BF(n-1) is obtained.Figure 9 shows the illustration of two disjoint BF(2). Similarly, the removal of the vertices of level 0 and level n of BB(n) results in two disjoint BB(n-1).

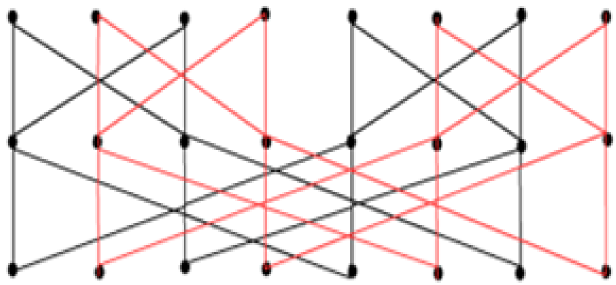

Fig.9Two disjoint BF(2)

Observation

Let BF(n) be a butterfly network. For n >2, the slope number of two disjoint BF(n-1) from BF(n) is 2n-1.

Observation

Let BB(n) be a butterfly network. For n > 2, the slope number of two disjoint BB(n-1) from BB(n) is 2n+1.

VIII.CONCLUSION

Thus we obtained the slope number of interconnection networks such as hypercube and normal representation of butterfly and benes networks. Characterization results for the defined networks are discussed according to the number of edges. Further investigation on diamond representation of butterfly and networks is our future scope.

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