

# Perfectly Regular and Perfectly Edge-Regular Intuitionistic Fuzzy Graphs

P. Syamala, K. R. Balasubramanian



**Abstract:** A Perfectly regular intuitionistic fuzzy graph is an intuitionistic fuzzy graph that is both regular and totally regular. In this paper we introduce and classify these types of intuitionistic fuzzy graphs and study several of their properties, including how two classes of intuitionistic fuzzy graphs structurally relate to one another and several of their spectral properties such as isospectral intuitionistic fuzzy graphs and when the energy of intuitionistic fuzzy graph is proportional to the energy of their underlying crisp graphs. These properties are studied in particular due to having at least one constant function  $\mu$  and  $\gamma$ .

**Keywords:** Intuitionistic fuzzy graph, perfectly regular, perfectly edge regular intuitionistic fuzzy graph, Graph energy, Spectral intuitionistic fuzzy graph theory, Intuitionistic fuzzy matrix.

## I. INTRODUCTION

Regular and totally regular fuzzy graphs were first introduced in [5]. The fuzzy edge analog of these concepts, edge regularity and total edge-regularity, were introduced and studied in [18]. These concepts of regularity for both vertices and edges in fuzzy graphs led to many advancements in the structural theory of fuzzy graphs. Several relevant marquee results stemming from this research include [1,3,4,6,8-17,19-23,25-30].

The purpose of this paper is to prepare for a study of those intuitionistic fuzzy graphs that concurrently exhibit both intuitionistic fuzzy vertex and edge-regular properties. These graphs will eventually help link certain intuitionistic fuzzy systems and crisp systems, allowing for greater ease in computing properties of these fuzzy systems for modeling purposes [2] or optimizing these fuzzy networks [7]. We first study perfectly regular intuitionistic fuzzy graph which are both edge regular and totally edge-regular. Spectral properties of these classes of intuitionistic fuzzy graphs in particular will help relate notions of regularity in intuitionistic fuzzy graphs to crisp graphs, thus allowing for a deeper understanding of these special classes of intuitionistic fuzzy graphs.

Perfectly regular intuitionistic fuzzy graphs will be characterized in section 3 along with several initial results on perfectly regular intuitionistic fuzzy graphs. A similar study of perfectly edge-regular intuitionistic fuzzy graphs will be given in section 4.

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From there, we will study the combination of these properties in intuitionistic fuzzy graphs by first studying the relationships between perfectly regular and perfectly edge regular intuitionistic fuzzy graphs in section 5 and then by providing a study of their adjacency matrices in section 6. The intention of this body of work is to serve as the necessary preliminaries for the introduction and study of those intuitionistic fuzzy graphs which are both exhibiting concurrently constant function  $\mu$  and  $\gamma$  including those intuitionistic fuzzy graphs which are both perfectly regular and perfectly edge – regular. For an introduction to fuzzy graph theory and its basic definition, the reader is referred to [8]. For analysis notations and relevant limit theorems, the reader is referred to [24].

## II. PRELIMINARIES

### Definition 2.1

An intuitionistic fuzzy graph is of the form  $G = (V, E)$  where,  
i)  $V = \{v_1, v_2, \dots, v_n\}$  such that  $\mu_1 : V \rightarrow [0,1]$  and  $\gamma : V \rightarrow [0,1]$  denotes the degree of membership and non-membership of the element  $v_i \in V$  respectively and

$$0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1 \forall v_i \in V \quad (i = 1, 2, \dots, n) \rightarrow (1)$$

ii)  $E \subseteq V \times V$  where

$$\mu_2 : V \times V \rightarrow [0,1] \text{ and } \gamma_2 : V \times V \rightarrow [0,1] \text{ are such that}$$

$$\mu_2(v_1, v_2) \leq \min \{ \mu_1(v_1), \mu_1(v_2) \} \rightarrow (2)$$

$$\gamma_2(v_1, v_2) \leq \max \{ \gamma_1(v_1), \gamma_1(v_2) \} \rightarrow (3)$$

and

$$0 \leq \mu_2(v_1, v_2) + \gamma_1(v_1, v_2) \leq 1 \forall (v_1, v_2) \in E (1, 2, \dots, n) \rightarrow (4)$$

### Definition 2.2

The degree of a vertex in an intuitionistic fuzzy graph is  $d(v) = (d_\mu(v), d_\gamma(v))$  where

$$d_\mu(v) = \sum_{u \neq v} \mu_2(u, v) \quad \text{and}$$

$$d_\gamma(v) = \sum_{u \neq v} \gamma_2(u, v)$$

### Definition 2.3

The total degree of a vertex in an intuitionistic fuzzy graph is

$$td(v) = (td_\mu(v), td_\gamma(v)) \text{ Where}$$

$$td_\mu(v) = d_\mu(v) + \mu_1(v)$$

$$\text{and } td_\gamma(v) = d_\gamma(v) + \gamma_1(v).$$

### Definition 2.4

A regular intuitionistic fuzzy graph is an intuitionistic fuzzy graph with if all vertices have same degree.

### Definition 2.5

A totally regular intuitionistic fuzzy graph is an intuitionistic fuzzy graph if all vertices have same total degree.



**Definition 2.6**

A perfectly regular intuitionistic fuzzy graph is an intuitionistic fuzzy graph that is both regular and totally regular.

**Definition 2.7**

The degree of an edge in an intuitionistic fuzzy graph is  $d_\mu(uv) = d_\mu(u) + d_\mu(v) - 2\mu_2(uv)$  and  $d_\gamma(uv) = d_\gamma(u) + d_\gamma(v) - 2\gamma_2(uv)$ .

**Definition 2.8**

The total degree of an edge in an intuitionistic fuzzy graph is

$$td_\mu(uv) = d_\mu(uv) + \mu_2(uv) \text{ and } td_\gamma(uv) = d_\gamma(uv) + \gamma_2(uv).$$

**Definition 2.9**

An edge regular intuitionistic fuzzy graph is an intuitionistic fuzzy graph having

$$d_\mu(uv) = k \text{ and } d_\gamma(uv) = k \forall uv \in E$$

**Definition 3.0**

A perfectly edge-regular intuitionistic fuzzy graph is an intuitionistic fuzzy graph that is both edge-regular and totally edge-regular.

**Definition 3.11**

The order of an intuitionistic fuzzy graph is  $O_\mu(G) = \sum_{v \in V} \mu_1(v)$  and  $O_\gamma(G) = \sum_{v \in V} \gamma_1(v)$ .

**Definition 3.12**

An intuitionistic fuzzy graph is complete if  $\mu_2(uv) = \mu_1(u) \wedge \mu_1(v)$  and  $\gamma_2(uv) = \gamma_1(u) \wedge \gamma_1(v) \forall uv \in E$ .

**III. PERFECTLY REGULAR INTUITIONISTIC FUZZY GRAPH.**

Let  $G = (V, \mu, \gamma)$  be an intuitionistic fuzzy graphs that are not totally regular and of totally regular intuitionistic fuzzy graph that are not regular. In that same work, Theorem 3.2 provided a necessary (but not sufficient) condition on the intuitionistic fuzzy subset  $\gamma$  of  $V$   $\mu_1$  and  $\gamma_1$  of  $V$  for perfectly regular graphs, namely that  $\mu_1 : V \rightarrow [0,1]$  and  $\gamma_1 : V \rightarrow [0,1]$  must be a constant function.

**Theorem 3.1**

Let  $G = (V, \mu, \gamma)$  be a perfectly regular intuitionistic fuzzy graph. Then  $\mu_1 : V \rightarrow [0,1]$  and  $\gamma_1 : V \rightarrow [0,1]$  is a constant function.

**Proof**

Since  $G$  is perfectly regular. We have  $G$  is both  $K_1$  regular and  $K_2$  totally regular. Then we have that,

$$\begin{aligned} td(v) &= [(d_\mu(v) + \mu_1(v)), (d_\gamma(v) + \gamma_1(v))] \\ &= [(d_\mu(u) + \mu_1(u)), (d_\gamma(u) + \gamma_1(u))] \\ &= [td_\mu(u), td_\gamma(u)] \forall u, v \in V \end{aligned}$$

Since,  $[d_\mu(v), d_\gamma(v)] = [d_\mu(u), d_\gamma(u)] = K_1$  and  $[td_\mu(v), td_\gamma(v)] = [td_\mu(u), td_\gamma(u)] = K_2$

We have that,

$$(\mu_1(v), \gamma_1(v)) = (\mu_1(u), \gamma_1(u))$$

Hence if  $G$  is perfectly regular then  $\mu_1$  and  $\gamma_1$  must be a constant function.

**Theorem 3.2**

An intuitionistic fuzzy graph  $G = (V, \mu, \gamma)$  is perfectly regular if and only if it satisfies the following conditions

$$\begin{aligned} \text{i) } \sum_{k \neq i} [\mu_2(v_i v_k), \gamma_2(v_i v_k)] \\ = \sum_{k \neq j} [\mu_2(v_j v_k), \gamma_2(v_j v_k)] \end{aligned}$$

$$\begin{aligned} \text{ii) } (\mu_1(v_i), \gamma_1(v_i)) &= (\mu_1(v_j), \gamma_1(v_j)) \\ \forall i, j \forall \in \{1, 2, \dots, |V|\} \end{aligned}$$

**Proof**

Let  $G$  be perfectly regular.

By definition  $G$  is regular.

Hence it trivially satisfies (i).

Theorem 3.1 implies that condition (ii) is also met.

Conversely,

Let  $G$  be an intuitionistic fuzzy graph satisfies both conditions (i) and (ii)  $(\mu_1, \gamma_1) = c$  is a constant function.

$$[td_\mu(v), td_\gamma(v)] = k + c \forall v \in V.$$

and thus  $G$  is both regular and totally regular.

Hence  $G$  is perfectly regular.

**Corollary 3.3**

Let  $G$  be a perfectly regular intuitionistic fuzzy graph.

Let  $(\mu_1(v), \gamma_1(v)) = c \forall v \in V$ .

Then the order  $G$  is  $O(G) = c |V|$ .

**Theorem 3.4**

Let  $G$  be a perfectly regular intuitionistic fuzzy graph and

Let  $[d_\mu(v), d_\gamma(v)] = k \forall v \in V$ .

Then the size of  $G$  is  $S(G) = \frac{k|V|}{2}$ .

**Proof**

Since  $G$  is perfectly regular, we have that

$$(d_\mu(v), d_\gamma(v)) = k \forall v \in V.$$

Hence,  $\sum_{v \in V} [d_\mu(v), d_\gamma(v)] = k |V|$ .

However, since  $(d_\mu(v), d_\gamma(v)) =$

$$\sum_{u \neq v} (\mu_2(uv), \gamma_2(uv)).$$

We have that  $\sum_{v \in V} [d_\mu(v), d_\gamma(v)] =$

$$\begin{aligned} \sum_{v \in V} \sum_{u \neq v} (\mu_2(uv), \gamma_2(uv)) \\ = 2 \sum_{uv \in E} (\mu_2(uv), \gamma_2(uv)) \\ = 2 S(G). \end{aligned}$$

Thus we conclude that the size of a perfectly regular intuitionistic fuzzy graph is  $\frac{k|V|}{2}$ .

**IV. PERFECTLY EDGE - REGULAR INTUITIONISTIC FUZZY GRAPHS**

**Theorem 4.1**

Let  $G = (V, \mu, \gamma)$  be a perfectly edge-regular fuzzy graph. Then  $\mu_2 : V \times V \rightarrow [0,1]$  and  $\gamma_2 : V \times V \rightarrow [0,1]$  is a constant function.

**Proof**

Since  $G$  is perfectly edge-regular, We have that  $G$  is both  $K_1$  edge regular and  $K_2$  totally edge-regular.

Then we have that

$$\begin{aligned} K_2 = td(uv) &= (d_\mu(uv) + \mu_2(uv), (d_\gamma(uv) + \gamma_2(uv)) \\ &= K_1 + (\mu_2(uv), \gamma_2(uv)) \rightarrow (1) \end{aligned}$$

$$td(xy) = [((d_\mu(xy) + \mu_2(xy)), (d_\gamma(xy) + \gamma_2(xy)))]$$

Since  $uv$  and  $xy$  were arbitrarily chosen edges.

Hence the proof.

**Theorem 4.2**

An intuitionistic fuzzy graph  $G = (V, \mu, \gamma)$  is perfectly edge regular if and only if it satisfies the following conditions.

- i)  $\sum_{z \neq u} (\mu_2(uz), \gamma_2(uz)) + \sum_{z \neq v} (\mu_2(vz), \gamma_2(vz)) - 2 (\mu_2(uv), \gamma_2(uv)) = \sum_{z \neq x} (\mu_2(xz), \gamma_2(xz)) + \sum_{z \neq y} (\mu_2(yz), \gamma_2(yz)) - 2 (\mu_2(xy), \gamma_2(xy))$   
 $\forall uv, xy \in E$
- ii)  $(\mu_2(uv), \gamma_2(uv)) = (\mu_2(xy), \gamma_2(xy))$   
 $\forall uv, xy \in E$ .

**Proof**

Since G is perfectly edge-regular and since (i) is the definition of an edge-regular intuitionistic fuzzy graph.

G obviously satisfies (i). By theorem 4.1, G also satisfies (ii)

Conversely,

Let G be an intuitionistic fuzzy graph satisfies both (i) and (ii) ie) G is k - edge regular and has a constant  $\mu = c$ .

Since (i) is the definition of being edge-regular.

$$\text{iii) } [td_\mu(uv) + td_\gamma(uv)] = k + c$$

$$= (td_\mu(xy), td_\gamma(xy)) \forall uv, xy \in E.$$

**Corollary 4.3**

Let G be a perfectly edge regular intuitionistic fuzzy graph and let  $(\mu_2(uv), \gamma_2(uv)) = c \forall uv \in E$ . Then the size of G is  $S(G) = c |E|$ .

**Theorem 4.4**

Let G be a perfectly edge regular intuitionistic fuzzy graph. Then the order G is bounded between  $\sum_{v \in V} \sum_{x \neq v} (\mu_2(xv), \gamma_2(xv)) \leq O(G) \leq |v|$ .

**Proof**

As the upper bound is obvious, we need only to prove the lower bound.

This follows directly from the definition of  $(\mu_2(uv), \gamma_2(uv)) \leq (\mu_1(u), \gamma_1(u)) \wedge (\mu_1(v), \gamma_1(v))$

Thus we have that  $(\mu_1(v), \gamma_1(v)) \geq \sum_{x \neq v} V (\mu_2(xv), \gamma_2(xv))$  is a lower bound for  $(\mu_1(v), \gamma_1(v)) \forall v \in V$ .

The sum of these individual lower bounds for  $(\mu_1(v), \gamma_1(v))$  is precisely the lower bound stated for O(G) in the theorem.

**Theorem 4.5**

Let G be a perfectly edge – regular complete intuitionistic fuzzy graph and let  $u \in V$  if there exists  $v \in N(u) \ni : (\mu_1(u), \gamma_1(u)) \leq (\mu_1(v), \gamma_1(v))$ . Then there exists an edge  $uv \in E \ni : (\mu_2(uv), \gamma_2(uv)) = (\mu_1(u), \gamma_1(u))$ .

**Proof**

Since G is complete.  
 $((\mu_2(uv), \gamma_2(uv)) = (\mu_1(u), \gamma_1(u)) \wedge (\mu_1(v), \gamma_1(v))$   
choose  $v \ni v \in N(u)$  and  $(\mu_1(v), \gamma_1(v)) \geq (\mu_1(u), \gamma_1(u))$   
Then  $(\mu_2(uv), \gamma_2(uv)) = (\mu_1(u), \gamma_1(u))$ .

**Theorem 4.6**

Let  $P_u$  be an ordering of the  $v \in N(u)$  such that  $x \leq y = (\mu_1(x), \gamma_1(x)) \leq (\mu_1(y), \gamma_1(y))$ . If  $P_u$  does not have a unique greatest element for all  $u \in V$ , then  $\sum_{v \in V} \sum_{x \neq v} (\mu_2(xv), \gamma_2(xv)) = O(G)$ .

**Proof**

Since G is complete and since there is no unique greatest element in  $P_u \forall u \in V$ .

We have that  $\forall v \in V$  there exists  $u \in V \ni (\mu_1(v), \gamma_1(v)) \leq (\mu_1(u), \gamma_1(u))$ .

We have that

$$\sum_{v \in V} \sum_{x \neq v} (\mu_2(xv), \gamma_2(xv)) = O(G)$$

Hence the proof.

**Theorem 4.7**

Let O (G) represent the order of a perfectly edge-regular complete intuitionistic fuzzy graph G with constant  $(\mu_2, \gamma_2) = c$ . Let T be the independent set of all vertices  $v \ni uv \in E \ni (\mu_2(uv), \gamma_2(uv)) = ((\mu_1(v), \gamma_1(v)))$  and let  $|T| = \alpha|V|$ . Then  $(1 - \alpha(1 - c)) O(G) \leq \sum_{v \in V} \sum_{x \neq v} (\mu_2(xv), \gamma_2(xv)) \leq O(G)$ .

**Proof**

As the upper bound was established.

We need only to prove the lower bound.

Let  $V_1$  be the set of vertices  $v \in V \ni uv \in E \ni (\mu_2(uv), \gamma_2(uv)) = (\mu_1(v), \gamma_1(v))$  and let  $V_2 = V \setminus V_1$ .

ie)  $V_2$  is the independent set described in the statement of the theorem.

Hence  $|V| = \alpha|V|$ .

Thus we may write a lower bound for O (G) as  $\sum_{v \in V_1} \sum_{x \neq v} (\mu_2(xv), \gamma_2(xv)) + \sum_{v \in V_2} \sum_{x \neq v} (\mu_2(xv), \gamma_2(xv)) \leq \sum_{v \in V_1} (\mu_1(v), \gamma_1(v)) + \sum_{v \in V_2} (\mu_1(v), \gamma_1(v)) \rightarrow (1)$

By Definition of  $V_1$ .

We have that  $\sum_{v \in V} \sum_{x \neq v} (\mu_2(xv), \gamma_2(xv)) = \sum_{v \in V_1} (\mu_1(v), \gamma_1(v))$

So we may rewrite this lower bound as

$$\sum_{v \in V_1} \sum_{x \neq v} (\mu_2(xv), \gamma_2(xv)) \leq \sum_{v \in V_2} (\mu_1(v), \gamma_1(v))$$
 with equality holding only when  $V_2 = \emptyset$ .

Since  $(\mu_2, \gamma_2) = c$  is a constant function due to the perfect edge-regularity of G.

We may obtain an upper bound on the difference between the two sides of our inequality as

$$\alpha(1 - c)|V| \geq \sum_{v \in V_2} (\mu_1(v), \gamma_1(v)) - \sum_{v \in V_2} \sum_{x \neq v} (\mu_2(xv), \gamma_2(xv)) \rightarrow (2)$$

Thus we may bound  $\sum_{v \in V} \sum_{x \neq v} (\mu_2(xv), \gamma_2(xv))$  in both direction as follows

$$\sum_{v \in V} (\mu_1(v), \gamma_1(v)) - \alpha(1 - c)|V| \leq \sum_{v \in V} \sum_{x \neq v} (\mu_2(xv), \gamma_2(xv)) \leq \sum_{v \in V} (\mu_1(v), \gamma_1(v)) \rightarrow (3)$$

By dividing through by O(G) we obtain

$$1 - \frac{\alpha(1 - c)|V|}{O(G)} \leq \frac{\sum_{v \in V} \sum_{x \neq v} (\mu_2(xv), \gamma_2(xv))}{O(G)} \leq 1 \rightarrow (4)$$

By noticing that  $1 \leq \frac{|V|}{O(G)} \leq \frac{1}{c}$ .

and making a small sacrifice to lower bound, we obtain

$$1 - \alpha(1 - c) \leq \frac{\sum_{v \in V} \sum_{x \neq v} (\mu_2(xv), \gamma_2(xv))}{O(G)} \leq 1 \rightarrow (5)$$

By multiplying through by O (G)

Hence the proof.

**V. RELATING VERTEX AND EDGE -REGULARITY IN INTUITIONISTIC FUZZY GRAPHS**

In this section we study what additional properties are needed for one form of regularity in intuitionistic fuzzy graphs to imply the other form of regularity. A perfectly regular intuitionistic fuzzy graphs that is regular and has a constant function  $\sigma$ .

**Theorem 5.1**

If G is regular and  $\mu = c$  is a constant function then G is perfectly edge – regular.



**Proof**

Let G be a regular intuitionistic fuzzy graph and let  $\mu = c$  be a constant function. Then  $(d_\mu(v), d_\gamma(v)) = k \forall v \in V$ .

Where k is simply a multiple of c. The degree of an arbitrary edge of G is then  $(d_\mu(uv), d_\gamma(uv)) = (d_\mu(u), d_\gamma(u)) + (d_\mu(v), d_\gamma(v)) - 2(\mu_2(uv), \gamma_2(uv)) = 2k - 2c = 2(k - c)$

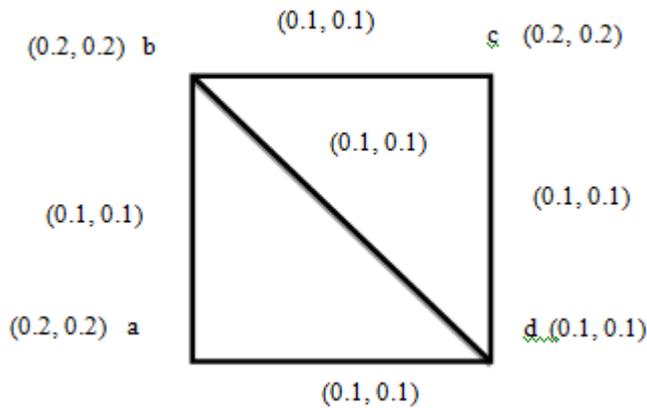
Hence G is edge – regular. Since the total degree of an edge in an intuitionistic fuzzy graph is  $(td_\mu(uv) + td_\gamma(uv)) = (d_\mu(uv), d_\gamma(uv)) + (\mu_2(uv), \gamma_2(uv))$

We have,  $(td_\mu(uv) + td_\gamma(uv)) = 2(k-c) + c = 2k - c$  for all edges in G.

Thus G is totally edge-regular and therefore perfectly edge-regular.

**Example 5.2**

Consider figure is totally regular and not a regular and cannot be totally edge – regular as it is not edge regular.



$(\mu_1(a), \gamma_1(a)) = (\mu_1(c), \gamma_1(c)) = (0.2, 0.2)$   
 $(\mu_1(b), \gamma_1(b)) = (\mu_1(d), \gamma_1(d)) = (0.1, 0.1)$   
 and  $(\mu_2(ab), \gamma_2(ab)) = (\mu_2(bc), \gamma_2(bc)) = (0.1, 0.1)$   
 $(\mu_2(cd), \gamma_2(cd)) = (\mu_2(ad), \gamma_2(ad)) = (\mu_2(bd), \gamma_2(bd)) = (0.1, 0.1)$ .

While the degree of b and d is (0.3, 0.3). The total degree of every vertex is (0.4, 0.4). Hence,

A an constant  $(\mu_2, \gamma_2)$  and is totally regular but is not regular. To see that it is not edge – regular. Consider the edges ab and bd.

We have,  $(d_\mu(ab), d_\gamma(ab)) = 0.3 \neq 0.4 = (d_\mu(bd), d_\gamma(bd))$

Since  $\mu$  is constant. Clearly, this figure cannot be totally edge-regular as it is not edge-regular.

**Theorem 5.3**

If G is perfectly regular and complete, then G is perfectly edge – regular

**Proof**

Since G is perfectly regular.  $\mu_1(u), \gamma_1(u) = (\mu_1(v), \gamma_1(v)) \rightarrow (1)$   
 Since G is complete.  $(\mu_2(uv), \gamma_2(uv)) = (\mu_1(u), \gamma_1(u))$

$$\bigwedge (\mu_1(v), \gamma_1(v)) \forall uv \in E \rightarrow (2)$$

(1) and (2)  $\Rightarrow (\mu_2, \gamma_2)$  is a constant function.

Hence the Proof.

**VI. SOME SPECTRAL PROPERTIES OF EDGE-REGULAR INTUITIONISTIC FUZZY GRAPH**

In this section we study some basic spectral properties of perfectly edge -regular intuitionistic fuzzy graphs. The adjacency matrix of an intuitionistic fuzzy graph A (G) is defined as  $A(G) = (a_{ij}) = (\mu_2(v_i v_j), \gamma_2(v_i v_j))$ .

**Remark 6.1**

Let G be a perfectly edge-regular intuitionistic fuzzy graph with  $(\mu_2, \gamma_2) = c$  and let  $G^*$  be the underlying crisp graph of G. Then  $A(G) = c A(G^*)$

**Theorem 6.2**

Let G be a perfectly edge-regular fuzzy graph and let  $G^*$  be its underlying crisp graph. If  $\lambda$  is an eigen value of G then  $c\lambda$  is an eigen value of  $G^*$ .

**Proof**

From 6.2 we have that the multiplicities of the eigenvalue from will be unchanged and that the eigenvalues of the intuitionistic fuzzy graph will both scaled by c,

Hence  $G_1$  and  $G_2$  are isospectral

**Theorem 6.3**

Let  $G_1$  and  $G_2$  be two perfectly edge-regular intuitionistic fuzzy graphs with respective underlying crisp graphs  $G_1^*$  and  $G_2^*$  respectively. If  $G_1^*$  and  $G_2^*$  are isospectral and  $c_1 = c_2$  then  $G_1$  and  $G_2$  are isospectral.

**Theorem 6.4**

Let G be a perfectly edge-regular intuitionistic fuzzy graph and let  $G^*$  be its underlying crisp graph. If the energy of  $G^*$  is E ( $G^*$ ) then the energy of G is  $E(G) = c E(G^*)$ .

**Proof**

By definition the energy of  $G^*$  given by  $E(G^*) = \sum_{i=1}^n |\lambda_i|$  where the  $\lambda_i$  are the eigen values of  $G^*$ . From theorem 6.2 We have that energy of G is given by  $E(G) = \sum_{i=1}^n |c\lambda_i| = c \sum_{i=1}^n |\lambda_i| = cE(G^*)$ .

**VII. CONCLUSION**

In this paper, we began a systematic study of two classes of intuitionistic fuzzy graph perfectly regular and perfectly edge-regular of intuitionistic fuzzy graphs that link intuitionistic fuzzy graph theory to fuzzy graph in several important aspects, most notably by studying some of the structural and spectral properties of these intuitionistic graphs.

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