

# DOA Estimation on Non-Uniform Linear Array using Extended Kalman Filter



Priyadarshini Raiguru

**Abstract:** This paper presents the performance of the Direction of Arrival estimation for Uniform and Non-Uniform Linear Array using Extended Kalman Filter. A Non-Uniform Linear Array with a specified array arrangement can estimate the signal's DOA and gives better accuracy and resolution as compared to Uniform Linear Array.

**Keywords :** DOA,EKF,NULA,ULA

## I. INTRODUCTION

Estimation of Direction Of Arrival (DOA) is highly pursued the topic of investigations for several decades in various field such as communication [1], target tracking [2], remote sensing [3], biomedical applications [4]. The use of multiple antennas seems to be very helpful in enhancing the performance of transmitting or receiving the system with a communication network. For better accuracy and resolution of DOA estimation following factors such as direction finding Adaptive algorithms, numbers of snapshots, SNR, Array geometric configuration should be taken care of. In general, Array geometry depends on the number of elements, the inter-element spacing between the elements and element position. It plays an important role in the direction of arrival estimation. Uniform Linear Array (ULA) is generally used for DOA estimation. Naturally, the larger ULA, with larger elements gives better accuracy and resolution. However, it is sometimes not suitable for practical application where element numbers are restricted. But Non-Uniform Linear Array (NULA) with specified array arrangement has a larger aperture length with the same elements as compared to ULA. NULA gives better resolutions and accurate angle estimation. In the aspect of practical application also NULA is more preferable as compared to ULA [5,6].

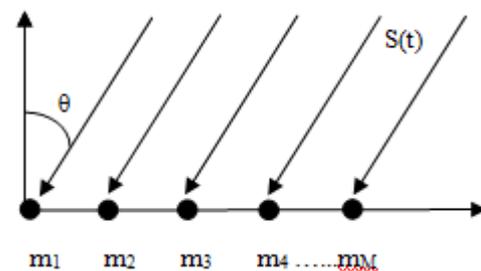
Lots of method for DOA estimation has been proposed in the literature review such as subspace method and model-based approach. The subspace methods, which include MUSIC, ESPRIT and its variations [7,8] gives high resolutions. However, these methods require prior knowledge of the covariance matrix of the signal. The model-based approach such as LMS, RLS and its variations [9] are also used for DOA estimations. These methods cannot be used in a dynamic environment. For practical application, a system is

required to model in such a way that it should have the ability to accurately estimate even in uncertainty scenario. So, the Extended Kalman Filter (EKF) is preferred to use as an estimator. It is an optimal state estimator which updates the state using predictor and corrector method.

The paper compares the performance of DOA estimation on different linearly arranged antenna arrays using EKF Algorithm.

## II. PRELIMINARIES

Let us consider an antenna array composed of M numbers of elements, which is placed in x-y plane and assume that L narrowband signals impinge on the array from the direction with  $\theta_k$  the elevation angle. It is assumed that  $M > L$  and incoming signals are plane waves. The graphical representation of Linear with uniform and non-uniform spacing is shown in figure (1).



**Fig. 1. Schematic representation of Antenna Array**

The element positioned at the origin is considered as reference element and the array has the configuration  $d = (\lambda/2)[m_1, m_2, \dots, m_M]$  and  $m_1$  must be zero, where  $d$  are the multiple integers of  $\lambda/2$ . When  $m$  is uniform i.e.  $\{0, 1, 2, \dots, M\}$ , the array is a Uniform Linear Array (ULA) and if  $m$  is non-uniform i.e.  $\{0, 1, 4, \dots, M\}$ , the array is a Non-Uniform Linear Array (NULA). The spacing of NULA is specific to avoid the ambiguity and this specified spacing NULA is also known as Minimum Redundancy Array [6].

The received signals vector can be written as

$$X = AS + N \tag{1}$$

Where  $X = [X_1, X_2, \dots, X_M]$ ,  $(2)$

$$S = [S_1, S_2, \dots, S_L] \tag{3}$$

$$A = [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_L)] \tag{4}$$

and

$$\mathbf{a}(\theta) = e^{j\left(\frac{2\pi}{\lambda}\right)d \sin \theta} \tag{5}$$

The column of matrix A, S, N represent the array manifold, signal vector, and noise vector respectively and it is assumed that each signal has a different direction of arrival.

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The set of all possible steering vectors is known as the array manifold, which includes the steering vectors  $\mathbf{a}(\theta)$  corresponding to the incoming signals.

### III. DOA ESTIMATION MODEL

Let us assume that the signal sources received by each element are modeled as stationary processes and synthesized by the AR processor [12]. The system model structure for DOA estimation is shown in figure (2).

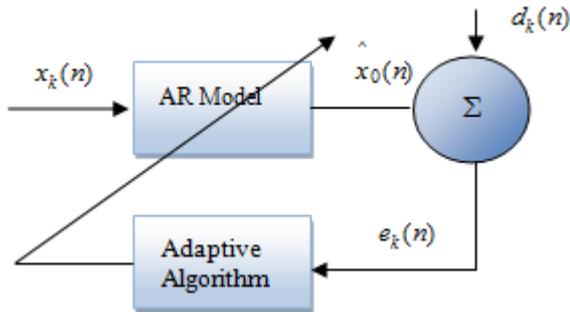


Fig. 2. System Model Structure

The signal sources received by the reference element are considered as desired data  $d_k(n)$  and the signal sources received by other elements of the array, are linearly combined and denoted as  $x_k(n)$ . The output of the AR model is expressed as

$$\hat{x}_0(n) = \sum_{k=1}^M b_k x(n-k) \quad (6)$$

Here the  $b_k$  is the model parameter set. The adaptive algorithms are used for the estimation of these parameters. This paper estimates the parameters by using the EKF algorithm [13] as it is an optimal estimator in the sense of MMSE. This adaptive algorithm is used to minimize the error. Then the power spectrum of DOA estimation is obtained by

$$P(\theta) = \frac{1}{W^H A(\theta) A^H(\theta) W} \quad (7)$$

Where  $W = [1 \quad -b^T]^T$  and  $A(\theta)$  is scan vector which scans over all possible angles and if it matches to any angle, the denominator of (7) becomes very small and it results in a peak corresponding to that the signal's DOA.

### IV. RESULT AND DISCUSSION

The section compares the DOA estimation performance of ULA and NULA, each consists of 7 and 10 elements. The EKF algorithm is used as an adaptive algorithm for DOA estimation. Here we take one narrowband signal to impinge from  $10.05^\circ$  direction on both arrays and observe the accuracy of estimation and power spectrum under different noisy conditions in the range of -10 dB to 10 dB.

Table I shows the comparison of the power spectrum and angle estimation of 7 and 10 elements ULA. The angle estimation is accurate up to 5 dB SNR in the case of 7 elements ULA whereas accuracy is maintained up to -7 dB SNR in the case of 10 elements ULA. The power spectrum of both arrays is also affected as SNR decreases. The power spectrum of 10 elements ULA is more as compared to 7 elements ULA under different specified noisy conditions. It indicates that the larger ULA with larger aperture length

gives better results in the presence of noise. Table II shows the comparison of Non-Uniform Linear Array (NULA) under different SNR conditions.

Table-I: Comparison of Power spectrum estimation of ULA under different SNR conditions.

SNR	ULA			
	7 elements		10 elements	
	theta	Power spectrum in dB	theta	Power spectrum in dB
10 dB	$10.05^\circ$	44.86	$10.05^\circ$	52.59
5 dB	$10.05^\circ$	25.74	$10.05^\circ$	35.44
0 dB	$10.045^\circ$	20.9	$10.05^\circ$	26.05
-3 dB	$10.045^\circ$	18.67	$10.05^\circ$	21.5
-5 dB	$10.045^\circ$	13.42	$10.05^\circ$	19.99
-7 dB	$10.04^\circ$	12.61	$10.05^\circ$	13.62
-10dB	$10.04^\circ$	11.52	$10.045^\circ$	12.47

Table-II: Comparison of Power spectrum estimation of NULA under different SNR conditions.

SNR	NLA			
	7 elements		10 elements	
	theta	Power spectrum in dB	theta	Power spectrum in dB
10 dB	$10.05^\circ$	52.59	$10.05^\circ$	55.38
5 dB	$10.05^\circ$	40.13	$10.05^\circ$	41.05
0 dB	$10.05^\circ$	26.09	$10.05^\circ$	34.48
-3 dB	$10.05^\circ$	24	$10.05^\circ$	29.32
-5 dB	$10.05^\circ$	22.65	$10.05^\circ$	23.95
-7 dB	$10.05^\circ$	17.51	$10.05^\circ$	21.75
-10dB	$10.05^\circ$	15.49	$10.05^\circ$	15.29

In the example we take 7 and 10 element NULA with inter-element spacing set  $\{0 \ 1 \ 2 \ 6 \ 10 \ 14 \ 17\}$  and  $\{0 \ 1 \ 3 \ 6 \ 14 \ 20 \ 27 \ 31 \ 35 \ 36\}$  respectively. It shows that the angle estimation is accurate in all the specified noisy conditions for both the case. The power spectrum of both NULA is affected. But the power spectrum of 10 elements NULA is comparatively large as compared to 7 elements. Again, it is noted that the 7 elements NULA perform better as compared to 10 elements ULA.

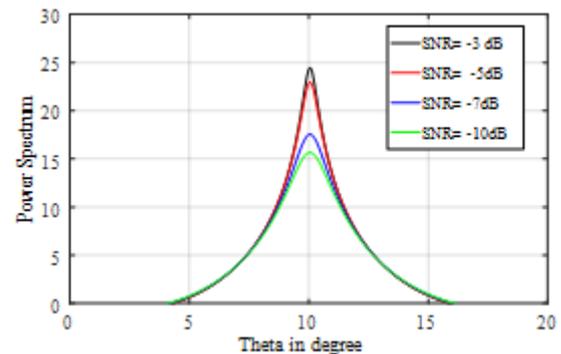
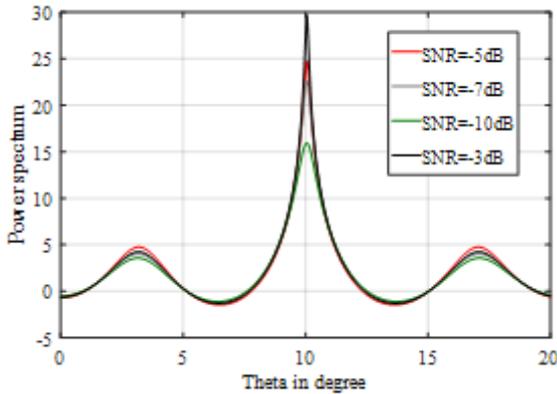


Fig. 3. Comparison of Spectrum obtained for 7 elements NULA under SNR -3 dB, -5 dB, -7 dB, -10 dB

The figure (3) shows the comparison of the power spectrum of 7 elements NULA under severe noisy conditions. The amplitude of power spectrums decreases and peaks are wider with the increase of noise.

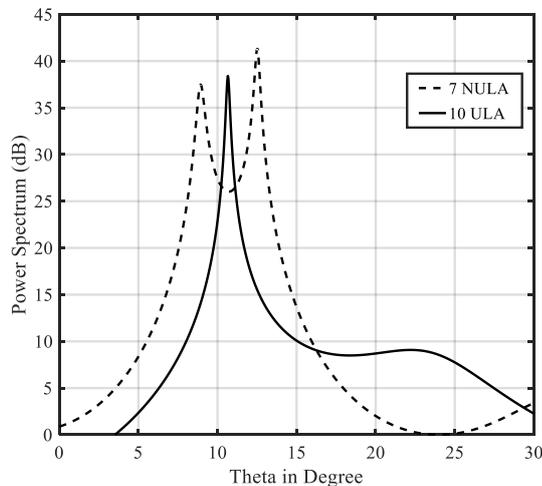
But in severe noise conditions, the DOA estimation is possible. The performance of DOA estimation may degrade if more signal sources are considered.

The figure (4) shows the comparisons of the power spectrum of 10 elements NULA under different severe noise conditions. It shows that the peaks are narrow as compared to peaks of 7 elements NULA which conclude that for sharp narrow peak larger elements are required.



**Fig. 4. Comparison of Spectrum obtained for 10 elements NULA under SNR -3 dB,-5 Db,-7Db,-10 dB**

The resolution between two closed signal sources impinge from  $10^0$  and  $12^0$  on 7 elements NULA and 10 elements ULA are compared with 0 dB SNR in fig. 5. The peaks with only  $2^0$  separations can be detected on 7 elements NULA. But the peaks are merged in the case of 10 ULA. So the resolution is better in 7 elements NULA as compared to 10 elements ULA.



**Fig. 5. Resolution of two closed signals impinge from  $10^0$  and  $12^0$  on 7 elements NULA and 10 elements ULA at SNR=0 dB**

**V. CONCLUSION**

The DOA estimation performance of ULA and NULA each having 7 and 10 elements are compared using the EKF algorithm. The resolution of 7 elements NULA is better as compared to 10 elements ULA. The result shows that the NULA with fewer elements performs well as compared to large element ULA.

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