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Computation of Vehicle Motion Path upon Entering Turn

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Abstract**: *Curvilinear motion of a vehicle as a part of assembly which takes place mainly on headlands is the most complicated element of its kinematics. Turns are carried out with gradual transition from infinitely large to minimum radius (upon transition from straight motion to motion along simple curve) and from minimum to infinitely large radius (upon transition from motion along simple curve to straight motion). Herewith, curvilinear motion along path of variable radius (upon entering turn and coming out of turn) is a significant portion (quite often more than one half) of all motion of the assembly on the curve. This article presents the procedure of derivation of parametric equations for analytical prediction of theoretical coordinates of motion points of tractor kinematic center upon entering turns. This procedure is based on universal equation for determination of theoretical minimum turning radius of wheeled vehicle. The proposed equations make it possible to determine theoretical path of entering turn of vehicle depending on the following variables: engineering parameters such as base, distance between kingpins, maximum turn angles of internal steered wheels, and operating parameters such as vehicle forward speed, angular turning speed of steered wheels in transverse plane; and to select reasonable properties on this basis.***

Keywords* : *vehicle, transport, vehicle control.

I. INTRODUCTION

In order to achieve high performances of a vehicle with minimum expenses, it is required to provide it with operation mode compatible with its agrotechnical and engineering requirements. In the course of operation, vehicles cover path of significant distance. Herewith, operating runs (productive work) alternate with idle runs and turns. Cyclic alternations of operating runs, idle runs, and turns during land cultivation determine vehicle motion pattern at each land segment. Process technology depending on operational conditions and applied machinery stipulates for provision of motion pattern (kinematics) of vehicle and machinery including on headlands.

Since any practical turning method is based on combination of all-round loopless turn, properties of vehicle curvilinear motion are estimated by its behavior during this type of turn.

In numerous studies, in order to provide simplification, it is assumed that turns are performed at constant (minimum allowable) turning radius. However, it is obvious that the points of assembly move at various speeds and trace various paths. Herewith, the most complicated turn segment is the turn entering.

Experimental studies of the influence of engineering and operating factors of vehicles and assemblies on their basis on properties of curvilinear motion are labor consuming and expensive process, they require for numerous experiments. Predictions are more reasonable: analytical description of curvilinear motion by mathematical simulation [1]-[3].

Since the most complete presentation of vehicle curvilinear motion is provided by path described by its characteristic points (kinematic points, center of gravity, etc.), the authors formulated the problem of its analytical determination upon entering turn.

II. METHODS

A. General description

The studies were based on vehicle universal turning mode when curvilinear motion path was performed by turn of front and rear wheels in different directions with regard to its framework as illustrated in Fig. 1.

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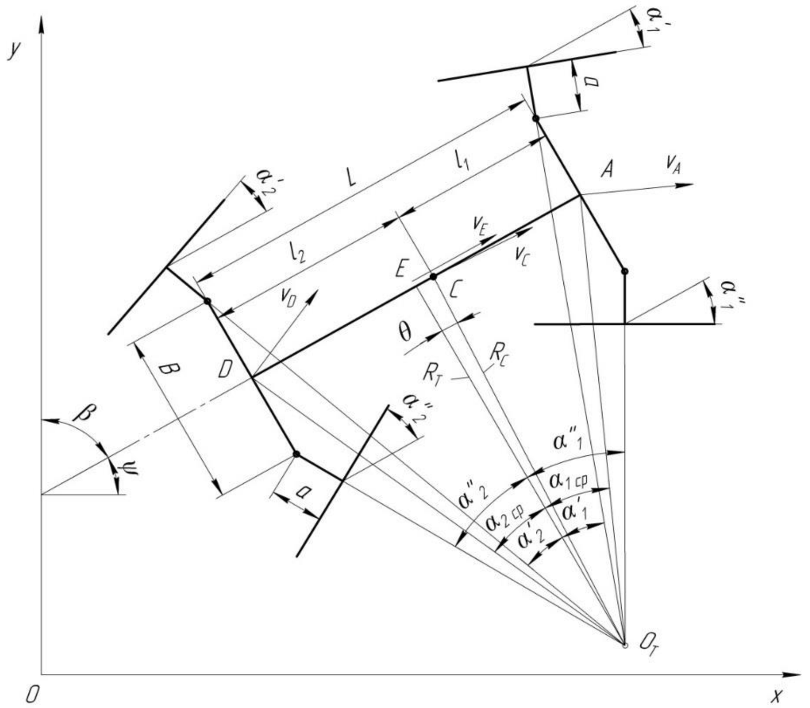

Fig. 1. Computational model of wheeled vehicle with all steered wheels.

B. Algorithm

Motion path of vehicle kinematic center (point *E* in Fig. 1) at variable curve radius, according to the assumption that wheels rotate without slipping and instant turning centers of all vehicle wheels coincide, is determined by solution of the set of two differential equations with two unknown functions of time *x*(*t*) (*m*) and *y*(*t*) (*m*): the current coordinates of vehicle kinematic center [4], [5]:

ρ= |(x(t)2+ y(t)2) 3 2 x (t) y (t)− y (t) x (t)|; v 2= x(t+ y(t}, (1)

where $\rho = \rho(t)$ is the instant vehicle turning radius (curve radius in given point), *m* is the preset function of times *t*, *s*; *v* is the vehicle motion speed, *m/s*.

The instant theoretical vehicle turning radius R_T is the path curve radius and is determined by the first equation of system (1):

RT(t)=|(x(t)2+y(t)2)32x(t)y(t)−y(t)x(t)| . (2)

Since the motion speed is:

v=dSdt,

and vehicle motion distance is:

$$dS = \sqrt{dx(t)^2 + dy(t)^2},$$

then we obtain the second equation of system (1):

$$v^2 = \dot{x}(t)^2 + \dot{y}(t)^2 .$$

Since $\dot{x} = v\cos\psi$ and $\dot{y} = v\sin\psi$ (Fig. 1), then upon uniform motion, let us assume (with accuracy sufficient for practical purposes) that during overall turn, the motion speed of kinematic centers is constant [4]:

$x(t) = v\int_0^t \cos\psi\, d\tau + C_1$; (3)

y(t)=v0tsinψ dτ+C2, (4)

where $\psi = \psi(t)$ is the angle between vehicle longitudinal axis and axis *x*, *rad*.

At initial time, upon transition from straight to curvilinear motion, the vehicle kinematic center is positioned at the origin of coordinates and the turn radius (curve radius) is at the axis *x* equaling to infinity, that is, at:

t = 0, x(*t*) *= x(0) = 0; y*(*t*) *= y(0) = 0; R*(*t*) *= R(0) = ∞;* $\psi(t) = \psi(0) = 90^\circ$.

From the condition *x*(0) = 0 and *y*(0) = 0, on the basis of Eqs. (3) and (4), we obtain that the integration constants are $C_1 = 0$ and $C_2 = 0$. Therefore,

$x(t) = v\int_0^t \cos\psi\, d\tau$; (5)

$y(t) = v\int_0^t \sin\psi\, d\tau$. (6)

Considering (Fig. 1) the ratio of angles ψ and β ($\beta = \frac{\pi}{2} - \psi$) from Eqs. (5) and (6), we have:

$x(t) = v\int_0^t \sin\beta\, d\tau$; (7)

$y(t) = v\int_0^t \cos\beta\, d\tau$, (8)

where $\beta = \beta(t)$ is the course motion angle (between the vehicle longitudinal axis and the axis *y*), *rad*.

Equations (7) and (8) are common solution of the considered system corresponding to the path of vehicle kinematic center.

Taking into account that:

$$dS = R_T\, d\beta$$

we have:

$$\frac{dS}{dt} = R_T \frac{d\beta}{dt} = v$$

and:

$$v\frac{dt}{R} = d\beta,$$

thus:

$$v\int_0^t \frac{d\tau}{R_T} = \beta + C_3.$$

On the basis of the initial conditions: $t = 0, \beta = 0$, we have $C_3 = 0$, that is:

$$\beta = v\int_0^t \frac{d\tau}{R_T}. \qquad (9)$$

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Let us assume that the vehicle driver rotates steering wheel at constant angular speed irrespective of forward speed of the vehicle, then the turn of wheels by angles α''_1 and α''_2 in time *t* is performed approximately (with sufficient accuracy) also at constant angular speeds ω_1 (1/s) and ω_2 (1/s), that is, [6]-[8]:

$$\alpha''_1 = \omega_1 t; \qquad (10)$$

$$\alpha''_2 = \omega_2 t. \qquad (11)$$

On the basis of the obtained universal equation for determination of minimum theoretical turn radius of wheeled vehicles with all steered wheels, differing from the known equations [9], [10] by accounting for the distance between kingpins and turn angles of internal wheels (Fig. 1),

$$R_T = \frac{[\frac{B}{2}(tg\alpha''_1 + tg\alpha''_2) + L]\cos\alpha''_1 \cos\alpha''_2}{\sin(\alpha''_1 + \alpha''_2)}. \qquad (12)$$

where $L = l_1 + l_2$ is the vehicle base, *m; l_1* is the distance from the front axis to the center of gravity *C, m; l_2* is the distance from the rear axis to the center of gravity *C, m; B* is the distance between kingpins, *m;* α''_1 and α''_2 are the turn angles of the front and rear internal steered wheels, respectively (Fig. 1), taking in account Eqs. (10), (11), Eq. (9) will be as follows:

$$\beta = v \int_0^t \frac{\sin(\omega_1\tau + \omega_2\tau)}{[\frac{B'}{2}(tg\omega_1\tau + tg\omega_2\tau) + L]\cdot\cos\omega_1\tau\cos\omega_2\tau} d\tau \ (13).$$

Substituting Eq. (13) into Eqs. (7) and (8), we obtain equations of parametric setting of current coordinates of theoretical curvilinear path of vehicle kinematic center aligned with point *E* (Fig. 1) upon entering turn:

(14)

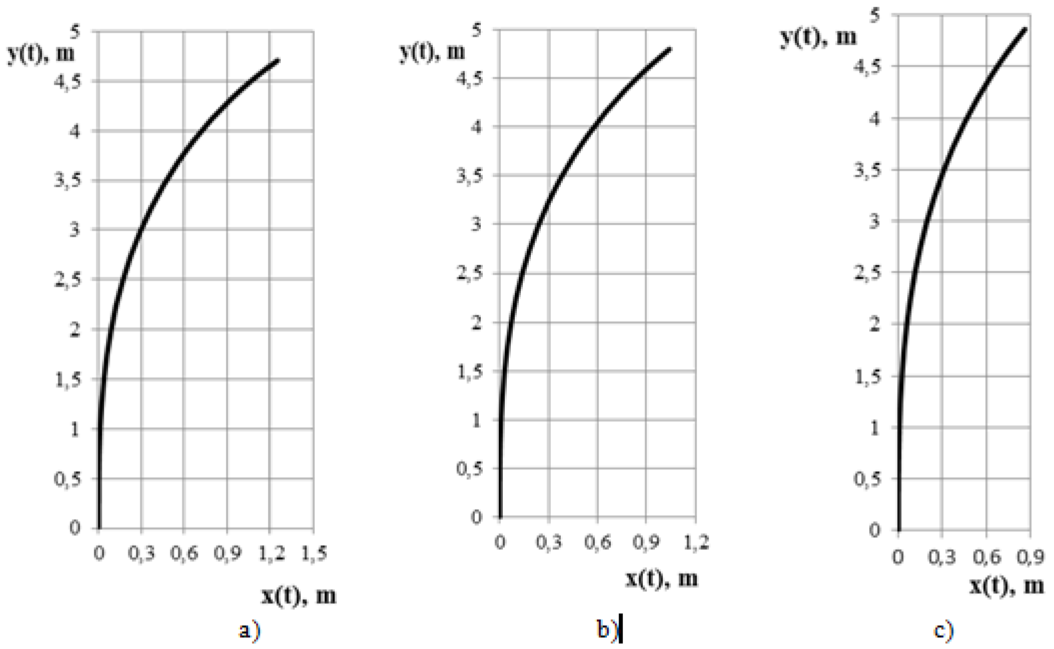
$$y(t) = v \int_0^t \cos[v \int_0^{t_1} \frac{\sin(\omega_1\tau + \omega_2\tau)}{[\frac{B'}{2}(tg\omega_1\tau + tg\omega_2\tau) + L]\cdot\cos\omega_1\tau\cos\omega_2\tau} d\tau] \, dt_1.$$

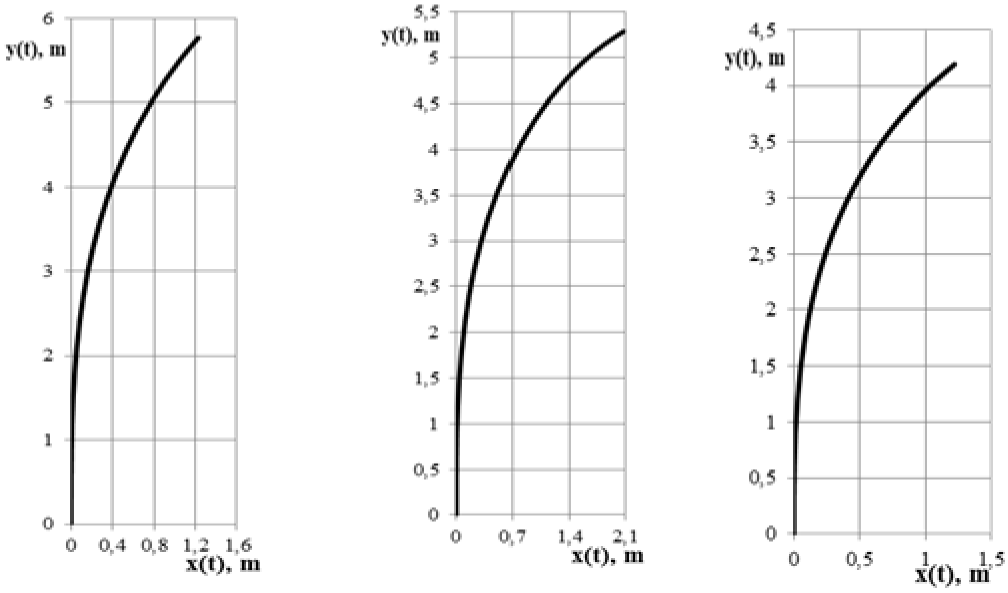
(15)

III. RESULTS AND DISCUSSION

Figure 2 illustrates predicted motion paths of vehicle kinematic center by Eqs. (14) and (15). It has been revealed (Fig. 2 a, b) that with the increase in vehicle base L from 2.0 m to 2.5 m, other initial parameters being equal (B=1.8 m; v=1.25 m/s; ω1=0.157 1/s; ω2=0 1/s; t=4 s; =36°; = 0°), the path abscissa decreases by 16.6% (xmax=1.249 m (Fig. 2 a) and xmax=1.041 m (Fig. 2 b)) and the ordinate increases by 1.9% (ymax=4.708 m (Fig 2 a) and ymax=4.799 m (Fig. 2 b)).

With the increase in the distance between vehicle kingpins B from 1.8 m to 2.2 m at L=2.5 m, other initial parameters being the same, we have (Fig. 2 b d): decrease in abscissa by 17.4% (xmax=1.041 m (Fig. 2 b) and xmax=0.86 m (Fig. 2 c)) and increase in ordinate by 1.4% (ymax=4.799 m (Fig. 2 b) and ymax=4.865 m (Fig. 2 c)).





d) e) f)

Fig. 2. Path of vehicle kinematic center: a – L=2.0 m; B=1.8 m; v=1.25 m/s; ω1=0.157 1/s; ω2=0 1/s; t=4 s; =36°; = 0°; b – L=2.5 m; B=1.8 m; v=1.25 m/s; ω1=0.157 1/s; ω2=0 1/s; t=4 c; =36°; = 0°; c – L=2.5 m; B=2.2 m; v=1.25 m/s; ω1=0.157 1/s; ω2=0 1/s; t=4 s; =36°; = 0°; d – L=2.5 m; B=2.2 m; v=1.5 m/s; ω1=0.157 1/s; ω2=0 1/s; t=4 s; =36°; = 0°; e – L=2.5 m; B=2.2 m; v=1.5 m/s; ω1=0.157 1/s; ω2=0.157 1/s; t=4 s; =36°; =36°; f – L=2.5 m; B=2.2 m; v=1.5 m/s; ω1=0.157 1/s; ω2=0.157 1/s; t=3 s; =36°; =36°.

Herewith, when vehicle forward motion speed v increases from 1,25 m/s (Fig. 2 c) to 1.5 m/s, we have xmax=1.228 m and ymax=5.768 m (Fig. 2 d). The abscissa and the ordinate increase by 30% and 15.7%, respectively.

Upon synchronous turn by front and rear wheels (Fig. 1; ω1=0.157 1/s; ω2=0.157 1/s; =36°; =36°), in this case (Fig. 2 e) xmax=2.086 m and ymax=5.29 m. The abscissa increases by 41.1%, and the ordinate decreases by 8.3%. Herewith, decrease in time of the turn from t=4 s to t=3 s (Fig. 2 e) decreases both the abscissa and the ordinate of the path by 41.5% and 21.9%, respectively (xmax=1.221 m and ymax=4.192 m).

The accuracy of the predictions was verified by computations of arc lengths of entering turn by the equation in [5]:

$$S(t)=\int_0^t \sqrt{(\dot{x}(t)^2+\dot{y}(t)^2)dt}$$

As a consequence, for three variants (Fig. 2 a, b, c) we have S=5 m, which corresponds to v=1.25 m/s and t=4 s, for two variants (Fig. 2 d, e): S=6 m at v=1.5 m/s and t=4 s; and for one variant (Fig. 2 f): S=4.5 m for v=1.5 m/s and t=3 s.

Using Eqs. (14) and (15), on the basis of geometrical relations, it is possible to determine coordinates and differentiate them: speeds, accelerations, and curve radius (by Eq. (2)) of any vehicle point (axle centers (points A and D), center of mass (point C), centers of each wheel (Fig. 1)).

Current path coordinates of vehicle center of mass, for instance, are determined as follows:

$$x_C(t) = x(t)\cos\theta; \qquad (16)$$

$$y_C(t) = y(t)\sin\theta. \qquad (17)$$

With known coordinates of center of mass, Eqs. (16) and (17), we obtain the coordinates of center of each wheel:

- left front:

$$x_1'(t)=x_C(t)+l_1\cdot\sin\beta-B/2\cdot\cos\beta-a\cdot\cos(\beta+\alpha_1');$$

$$y_1'(t)=y_C(t)+l_1\cdot\cos\beta+B/2\cdot\sin\beta+a\cdot\sin(\beta+\alpha_1');$$

- right front:

$$x_1''(t)=x_C(t)+l_1\cdot\sin\beta+B/2\cdot\cos\beta+a\cdot\cos(\beta+\alpha_1'');$$

$$y_1''(t)=y_C(t)+l_1\cdot\cos\beta-B/2\cdot\sin\beta-a\cdot\sin(\beta+\alpha_1'');$$

- left rear:

$$x_2'(t)=x_C(t)-l_2\cdot\sin\beta-B/2\cdot\cos\beta-a\cdot\cos(\beta-\alpha_2');$$

$$y_2'(t)=y_C(t)-l_2\cdot\cos\alpha+B/2\cdot\sin\beta+a\cdot\sin(\beta-\alpha_2');$$

- right rear:

$$x_2''(t)=x_C(t)-l_2\cdot\sin\beta+B/2\cdot\cos\beta+a\cdot\cos(\beta-\alpha_2'');$$

$$y_2''(t)=y_C(t)-l_2\cdot\cos\beta-B/2\cdot\sin\beta-a\cdot\sin(\beta-\alpha_2''),$$

where a is the steering offset, m.

The obtained results can be applied for simulation of curvilinear motion of wheeled vehicles [1]-[3] as well for designing clotoidal paths [11], since the curve expressed by Eqs. (14) and (15) is a clothoid characterized by the fact that the curve radius ρ is inversely proportional to covered distance [5].

IV. CONCLUSION

The obtained equations of theoretical path coordinates of any point of vehicle with all steered wheels during motion along path of variable radius make it possible to select their reasonable engineering and kinematic properties at any design stage and in the course of operation.

In comparison with the previous experimental results, application of the universal equation of minimum turning radius (Eq. (12)) upon derivation of equations of theoretical coordinates of turn entering segment x(t) (Eq. (14)) and y(t) (Eq. 15)) will allow to increase multivariance and computational accuracy on their basis.

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