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Denoising of Electrocardiogram Based on Norm Parameter using Wavelet Packet Transform

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Abstract: ***The electrocardiogram is an electrical activity of the human heart and it is mainly used in the diagnosis of different diseases associated with the human heart. In the medical field, it is also known as electrophysiological activity of the human heart. During its recording an unwanted noise known as artifacts is getting contaminated into it, this creates an obstruction in its clinical analysis. As for application of wavelet analysis is concerned, it is then fit to mention that denoising of ECG is effectively done by wavelet tools, so that identification of hidden diseases of the patient becomes possible. In this research article an algorithm is proposed for denoising ECG signal by applying a wavelet transform popularly known as wavelet packet transformation and three wavelet functions: Haar Wavelet, Db-3 Wavelet and Coiflet-3 Wavelet. The performance of this algorithm is studied on the basis of constraints popularly known in the mathematical field as Signal to Noise ratio (SNR), Norm 1 and Norm 2. This algorithm is first applied in deciding the optimum level of decomposition applied to the ECG recording No. S10m.mat, then a comparative study is laid down for a sample of 10 ECG recordings obtained from MIT-BIH database available on www.physio.net.***

Keywords: ***Wavelet Transformation, Electrocardiogram, Thresholding, Norm, Denoising, Wavelet packet transformation***

I. INTRODUCTION

In the world around different kinds of signals exist which are important to analyze and help to get characteristics of them. Few examples are human speech, vibrations, music, data in finance, images, biomedical signals-electrocardiogram (ECG), EKG, seismic signals and others. In this paper, we specifically focus on Electrocardiogram.

The electrocardiogram is developed due to the muscle contractions of the human heart during its function. In reality, it is a graphical representation of two axes, one is a potential difference between two spots on the human body surface over the chest and another is times axis. This signal is obtained by putting electrodes in and around the heart muscle and the potential difference is held between the electrodes. In medical diagnosis, 12- lead Electrocardiogram is obtained in order to assess cardiac status of a patient. This ECG is so difficult in its recognition and analysis due to its size, form and eventual change in it. Also, due to the presence of noise in it makes it difficult to interpret correctly. However, to understand and draw conclusions, there exist many algorithms, tools, methods that help to understand ECG but among them Wavelet Transformation is new and most promising tool for analyzing and interpreting the ECG. The important waveforms a normal ECG consists are P wave, QRS Complex and T-wave. P-wave is the representation of impulse across Atria to AV node with normal duration 80ms. QRS is the representation of the impulse across the ventricles with normal values of R- wave is 1.6mV and that of Q and S is 25 % of R wave[11].The normal duration of the QRS complex is 80ms-120ms. The other features of ECG lies in PR interval with normal duration 120ms-200ms and RR interval with normal duration 0.6s-1.2s[2].The basic issue behind the interpretation of an ECG is unsound recording environment, addition of signals from equipments nearby, poor quality of electrodes and electromagnetic pollution present in the environment like, power-line noise, baseline wander, motion artifacts, etc. These are some of the ground reasons in addition to others behind the unwanted noise that gets fouled in the ECG recording. So the removal of such unwanted noise from ECG is of primary need for experts in the medical field. Since lot of conventional methods have been applied for denoising but they are not too good for the noise having very low amplitude. Reliable techniques should be adopted for the diagnosis of noisy ECG signal. Apart from time-frequency methods available for denoising that are applied for ECG signal processing, Wavelet based method is preferred for denoising due to its better detection and also because of its application in signal compression and feature extraction [14, 16, 10, 17]. In the process of denoising one has to take care about the important features in the signal that may get deleted during removing the unwanted noise. Such features may be relevant and necessary for diagnosis. Since wavelet has ability to split the signal and the noise in it into the wavelet domain. ECG is de-noised by using diverse techniques, for instance Hard [8], Soft [8] Sure Shrink [9] and Hybrid Shrink [3]. They all are popularly known as thresholding techniques. The most important part in ECG is QRS complex and different methods and algorithms have been proposed for its analysis and interpreting ECG [4, 18, 10]. In this paper, we deal with taking out correct ECG signal from the noisy one, for that we investigate wavelet based thresholding design for ECG signal denoising. Because wavelet based techniques allow experts to understand the non- stationary features of the signal and temporal contamination of a noise that was difficult in conventional linear filtering. Let us see how theoretically wavelet transformation is working behind de-noising a signal. In the very first step a signal is to be decomposed at different levels in order to obtain wavelet coefficients, i.e. a signal is broken with the help of wavelet basis.

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Among these obtained wavelet coefficients, the original signal values are concentrated in a small number of large wavelet coefficients and the noise components in the signal (ECG) are concentrated into large number of small wavelet coefficients[6], Then thresholding is applied for removing the noise from raw ECG by deleting the noisy wavelet coefficients. At last a true ECG signal is then calculated from the remaining wavelet coefficients by applying reconstruction process. Let y (t)be a given noise corrupted ECG with x(t) as its original part of ECG and n(t) its noise part, then

$$y(t) = x(t) + n(t) \quad (1)$$

Since different algorithms and methods have been applied for de-noising ECG which includes wavelet transformation and others like in [8, 3, 1] etc.

Let W be the wavelet transformation applied for Denoising y(t) then we get,

$$Wy(t) = Wx(t) + Wn(t) \quad (2)$$

Now in the next section we will actually understand wavelet transformation and its related concepts mathematically and how it works in de-noising ECG. In this paper these concepts will also work as parameters for proposing algorithm.

II. PARAMETERS

2.1 Wavelet

Wavelet is a small wave in which smallness means the wave has finite length and a wave nature means it is oscillatory in nature. Mathematically, in [15] the general wavelet function is defined as,

$$\psi(\tau,s) = \frac{1}{\sqrt{|s|}}\psi\left(\frac{t-\tau}{s}\right) \quad (3)$$

Where, $\psi \in L^2(R)$ and this set is an orthonormal wavelet basis in the function space $L^2(R)$. The translation parameter is used to analyze the signal across the time axis and the scale parameter that gives zoom in, zoom out possibility during the analysis of given signal. Some important wavelets that are used as basis function in wavelet analysis are Daubechies wavelet dbN, Haar wavelet, Symlet Wavelet (SymN), Coiflet Wavelet (CoifN), Biorthogonal Wavelet pairs (Bior. Nr. ND), Mayer Wavelet (Meir), Mexican Hat Wavelet (Mexh) and Morlet wavelet (morl).

2.2 Continuous Wavelet Transform (CWT)

Continuous wavelet transform (CWT) is actually an inner product between the test signal x(t) and the basis function $\psi_{\tau,s}$ i.e. $W_\psi^x(\tau,s) = \langle x, \psi_{\tau,s}^* \rangle$

in otherwords,

$$W_\psi^x(\tau,s) = \int x(t)\psi_{\tau,s}^*(t)\,dt \quad (4)$$

where, $\psi_{\tau,s} = \frac{1}{\sqrt{|s|}}\psi\left(\frac{t-\tau}{s}\right)$

Here for every (τ,s) ,equation (3) gives us values known as wavelet transform coefficients that represents how much the scaled wavelet is similar to the function (signal) at location $t = \frac{s}{\tau}$ [12]. The inner product in the equation (3) shows that the wavelet analysis gives the similarity (frequency component) between wavelets (basis) and the given signal. Therefore the obtained CWT coefficients refer to the nearness of the signal to the selected wavelet function at the current scale [13].If the wavelet coincides with the signal we then get large values otherwise we will get smaller CWT coefficients. In digital signal processing, we need to analyze a signal in discrete form. Let us understand what Discrete wavelet transformation means.

2.3 Discrete Wavelet Transform

Since continuous wavelet transform is a function that analysis a given signal into two parameter representation, `s' and '$\tau$' that also has continuous parametric values of `s' and 'τ'. But it is fit to mention that in signal processing data is represented by a finite number of values. Such a signal f(t) is best analyzed by Discrete wavelet transformation in which scaling 's' and translation parameter τ are discretise by two positive constants a_0 and b_0 such that,

$$f(x) = \sum_{p,q=-\infty}^{\infty} \langle f, \psi_{p,q} \rangle \psi_{p,q}(t) \quad (5)$$

where, $\psi_{p,q}(t) = (a_0)^{\frac{-p}{q}} \psi\left(a^{-q}x - qb_0\right)$

For computation efficiency, we substitute $a_0 = 2$ and $b_0 = 1$ that are generally used so that it gives binary dilation 2^{-p} and dyadic translation q 2^p [7].

2.4 Wavelet Packet Transformation

Wavelet packet transformation is the generalization of wavelet transformation. Its concept was introduced by Coifman, Meyer and Wickerhauser [5]. In this transformation both approximation and detail coefficients are further decomposed into approximation and details at each level, but this is not the case in wavelet transform decomposition. In wavelet transformation, a signal is only decomposed into approximation and details at the very first level, and then on subsequent level only approximation is decomposed further into new approximation and details. In this process of decomposition there are chances of losing required information of the signal that is incorporated into details. Therefore, to overcome this problem, wavelet packet transformation is applied and expected to draw better inferences of the signal for the investigation.

2.5 Thresholding Techniques

Wavelet thresholding actually belongs to wavelet shrinkage, in which small wavelet coefficients obtained in the process of wavelet transformation decomposition are reduced to zero. For this a threshold value is defined that helps in discarding the coefficients carrying noise below the magnitude of the threshold value so that the remaining coefficients will estimate the signal we required [15]. One has to take care about the threshold value in order to minimize the Mean Square Error (MSE). There are mainly two thresholding techniques which act as wavelet shrinkage functions, namely hard threshold and soft threshold.

2.6 Hard Threshold

Let λ be a threshold value that is to be determined by noise variance, then Hard thresholding is defined in [8] by Dohono as below,

$$H_h(j) = \begin{cases} 1 & if\,|y(j)| > \lambda \\ 0 & otherwise \end{cases} \quad (6)$$

`1' indicates we have to keep the wavelet coefficients which are above the threshold value in magnitude and `0' indicates that we have to remove those wavelet coefficients which are below the defined threshold value.

2.7 Soft Thresholding

This technique is also proposed by Donoho [8] and is defined as,

$$H_s(j) = \begin{cases} \text{sign}(y(j)(y(j) - 1)) & \text{if } |y(j)| > \lambda \\ 0 & otherwise \end{cases} \quad (7)$$

In this method the output is further smoothened as the large wavelet coefficients above the threshold value are shrinked. In both the techniques the threshold value is called universal threshold given in [8] and it is calculated as,

$$\lambda = \sqrt{\sigma^2 \log N} \quad (8)$$

Where, N stands for length of the signal and σ^2 is the Variance of the detailed coefficients obtained at the finest level of the wavelet decomposition. The other threshold methods used in wavelet theory are Sure Shrink, Hybrid Threshold, Wavelet Wiener filter mentioned in [14] In this paper we make use of soft threshold method for denoising ECG signal because of its continuity nature.

III. PERFORMANCE PARAMETERS

3.1 Mean Square Error

Mean Square Error is called risk function and it is calculated as an average of the squares of the difference between the estimated observations and actual observations, i.e. Mathematically, MSE is defined as,

$$\text{MSE} = \frac{1}{N}\sum_{1}^{N}(\hat{x}(j) - y(j))^2 \quad (9)$$

Where, $\hat{x}(j)$ is called estimated ECG signal, $y(j)$ is a noisy ECG signal. Lower is the value of MSE, the better is the performance.

3.2 Norm

A norm is a function which assigns strictly positive value to each vector in that space. A norm should satisfy some important properties, namely triangular inequality, absolute homogeneity, and existence of zero vectors. Different types of norm are applied in different physical situation that depends upon the problem in deal. For instance, if one needs to know about the measure of noise on a signal, i.e. to know the measure of the variation from the main signal, then norm plays an important role. Two different types of norms are as follows:

$$\|x\|_1 = |x_1| + |x_2| + |x_3| + \ldots + |x_n| \quad (10)$$

$$\|x\|_2 = \sqrt{|x_1|^2 + |x_2|^2 + |x_3|^2 + \ldots + |x_n|^2} \quad (11)$$

$\|x\|_1$, is called one norm and it would measure the total difference from the main signal. $\|x\|_2$, is called two norm that would measure the added average variation. In this paper, we will apply both the mentioned norms that help us in deciding the better denoising method.

IV. WAVELETS TO BE USED IN THIS PAPER

4.1 Haar Wavelet

This wavelet is explicitly defined in [15] as,

$$\psi(x) = \begin{cases} 1, & 0 \le x < \frac{1}{2} \\ -1, & \frac{1}{2} \le x < 1 \\ 0, & otherwise \end{cases} \quad (12)$$

The corresponding scaling function of this Haar Wavelet is,

$$\phi(x) = \begin{cases} 1, & 0 \le x < \frac{1}{2} \\ 0, & otherwise \end{cases} \quad (13)$$

This wavelet is irreversible, discontinuous with no edge effect and it uses only two scaling and wavelet function coefficients. Because of its discontinuity, the wavelet is useful for the signal with sudden transitions. This wavelet is proposed by Alfred Haar in 1909. Fig.1 [19]

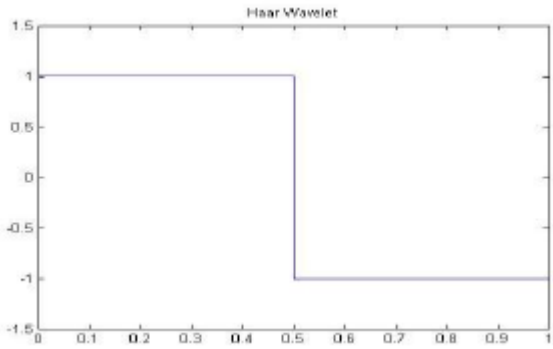

Fig.1. Haar Wavelet

4.2 Daubechies Wavelet

This family is well-liked due to its orthogonality and compact support features. This wavelet is smoother than Haar wavelet because it averages over more pixels. It is frequently used for texture feature analysis because of its properties of orthogonality and compact support abilities. This has four wavelets and scaling coefficients but this wavelet has no explicit form. All Daubechies wavelet is denoted by a symbol dbN, N is natural number Fig. 2 [20].

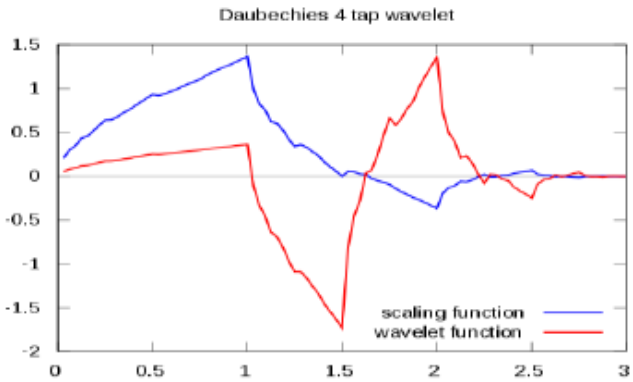

Fig. 2. Daubechies Wavelet

4.3 Coiflet Wavelet

Originally Coiflet wavelet is derived from Daubechies Wavelet. It uses six scaling and wavelet function coefficients [7] they are obtained by imposing vanishing moment conditions on scaling and wavelet functions.

The minimum number of taps in Coiflet wavelet is 6 [7]. A comparative study of these wavelets is discussed, same analysis is then verified and compared along with norm of a signal Fig. [21].

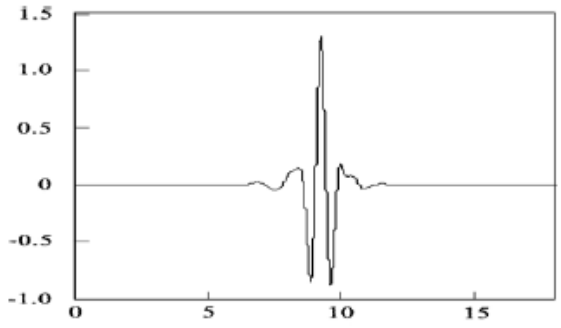

Fig. 3. Coiflet Wavelet

V. ALGORITHM

1. A noisy ECG signal is first decomposed with the help of the wavelet transform at 2nd, 3rd, 4th and 5th level into approximation and detail coefficients.
2. Obtained a threshold value of detailed coefficients at each stage with the help of a formula mentioned in the equation (8).
3. Now apply the wavelet packet transformation for denoising the ECG signal at the threshold value obtained in step 2.
4. Denoised ECG signal so obtained is then reconstructed by applying inverse wavelet packet transformation through MATLAB wavelet tool box.
5. Obtain the SNR values and Norms corresponding to the estimated signal.

$$SNR = 10\log\left(\frac{\sum_{1}^{N}\left(y(j)-x(j)\right)^2}{\sum_{1}^{N}\left(\hat{x}(j)-x(j)\right)^2}\right) \qquad (14)$$

Where, $y(j)$ and $x(j)$ are noisy ECG and original ECG signal and $\hat{x}(j)$ is the synthesized (estimated) ECG signal.

5.1 Analysis

Let us simulate the algorithm for the sample S10m.mat. The values of SNR, the Norm 1 and Norm 2 are then compared to decide the best level as shown in the list of below given tables:

Table 1: Outcomes of de-noised ECG signal S10m.mat by Haar Wavelet

100m.mat	Soft Thresholding (Haar Wavelet)				
	Original Signal	Level-2	Level-3	Level-4	Level-5
SNR		1.201	1.0944	0.8037	0.4093
Norm-1	19584	109584	19584	19584	19584
Norm-2	326.55	326.52	326.55	326.51	326.43

Table 2: Outcomes of de-noised ECG signal S10m.mat by db3 Wavelet

100m.mat	Soft Thresholding (db3 Wavelet)				
	Original Signal	Level-2	Level-3	Level-4	Level-5
SNR		2.1049	2.3528	0.7471	0.3186
Norm-1	19584	19584	19584	19584	19584
Norm-2	326.55	326.52	326.52	326.48	326.43

Table 3: Outcomes of de-noised ECG signal S10m.mat by Coiflet-3 Wavelet.

100m.mat	Soft Thresholding (db3 Wavelet)				
	Original Signal	Level-2	Level-3	Level-4	Level-5
SNR		1.4635	2.2076	0.6119	0.4723
Norm-1	19584	19584	19584	19584	19584
Norm-2	326.55	326.55	326.54	326.48	326.45

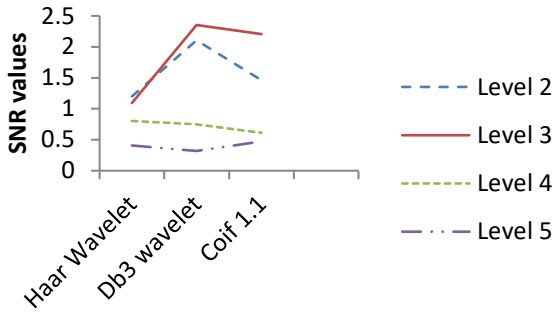

Fig. 4. SNR Values for different wavelets at different levels

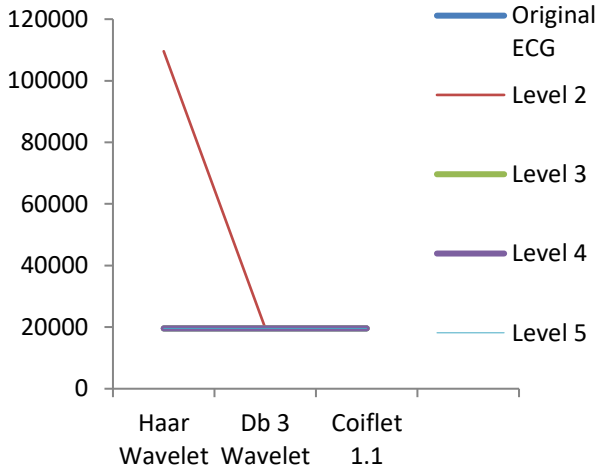

Fig. 5. Norm 1 of wavelets at different levels of decomposition

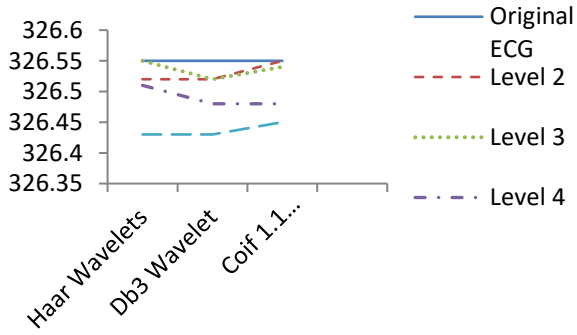

Fig. 6. Norm 2 of three Wavelets at different levels of decomposition

The level of decomposition is shown better at level 3 on the basis of performance parameters SNR, Norm 1 and Norm 2 as shown in Fig.4, Fig.5 and Fig.6.

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In Fig. 4, SNR values are more at level 3 in comparison to other levels. Also in Fig.5 the norm value is equal to the original signal 100m.mat at level 3, 4, 5 but at level 2 it is more. Since SNR value is more at level 3 so we opted level 3 instead of levels 4, 5 even if they are giving same norm value at each level as that of original signal but varies in SNR values simultaneously. In the similar fashion norm 2 reflects that level 3 is better than other levels because norm 2 (Fig. 6) values are very close to the original signal at level 3 than at levels 2, 4, 5.

Let us demonstrate the above Algorithm On taking 10 samples from physio.net for analysis in which threshold value is taken from detail coefficients of each sample at 3rd level for deciding the better wavelet function among the selected one for this paper. Since the samples are obtained in MIT-BIH database available at www.physio.net. The corresponding norms of the original signal before adding the noise is shown in the table 4.

Table 4: Different Norms of 10 samples of original ECG Original Signal

S.No.	100m	101m	102m	103m	104m	105m	106m	107m	108m	109m
Norm 1	19584	19555	19179	19290	19288	19287	19101	19551	19561	19894
Norm 2	326.55	326.14	319.90	322.04	322.00	321.97	319.19	329.93	326.15	332.34

Table 5: Analysis table of 10 ECG Samples with respect to different performance parameters On applying the above proposed algorithm, the analysis table is shown below.

Records	Norm1			Norm2			SNR		
	Haar Wavelet	Db-3 Wavelet	Coif-3 Wavelet	Haar Wavelet	Db-3 Wavelet	Coif-3 Wavelet	Haar Wavelet	Db-3 Wavelet	Coif-3 Wavelet
100m	19584	19584	19584	326.52	326.52	326.54	1.0944	2.3529	2.2076
101m	19555	19555	19555	326.11	326.11	326.11	1.1713	2.8321	1.1713
102m	19179	19179	19180	319.81	319.81	319.84	1.4152	2.2862	1.115
103m	19291	19290	19291	322	322	322	0.6007	1.581	2.1952
104m	19288	19288	19288	321.87	321.89	321.89	1.1194	1.9098	1.4584
105m	19287	19287	19287	321.93	321.97	322	0.9209	1.9882	0.802
106m	19101	19101	19101	319.07	319.12	319.12	0.4297	0.977	1.0593
107m	19551	19551	19551	329.7	329.84	329.9	0.2907	0.4557	1.6653
108m	19561	19561	19561	326.15	326.17	326.2	2.5154	1.5101	0.5847
109m	19894	19894	19894	332.31	332.34	332.38	0.9412	1.8138	0.6479

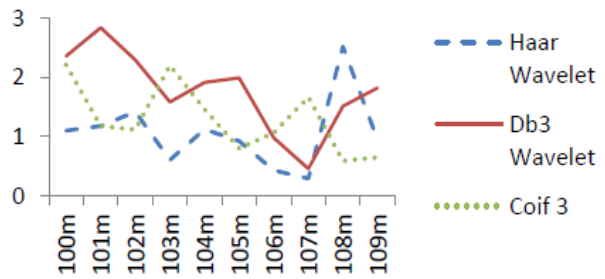


Fig.7. SNR values of three wavelets at decomposition level 3 for 10 samples of ECG

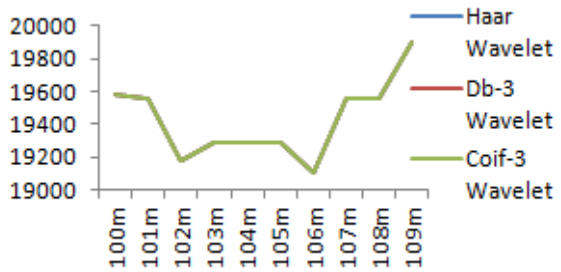


Fig.8. Norm 1 of given three Wavelets

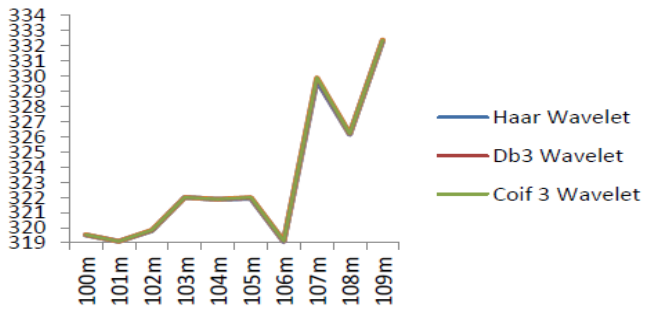


Fig.9. Norm 2 of above mentioned wavelet.

In Fig. 8 the values of norm 1 is same far all the de-noised signals obtained by applying wavelet functions in the algorithm. So we can say norm 1 parameter conveys that all the wavelet functions are equally de-noising the ECG signals, but On the basis of norm 2 that measures the average added variation, it is clear on comparing the middle three columns of

Table 5 with the 2nd row of Table 4 that, the average added variation is less in Coiflet-3 wavelet. Since values of Norm 2 for given 10 Samples are fluctuating randomly for all the three wavelet as shown in Figure 9. So in order to get any clear conclusion, we will take an average of obtained values in the algorithm with the average value of original signal i.e.

$$\text{Difference or Error} = \left| \begin{array}{l} \text{Original ECG} - \text{Synthesised} \\ \text{ECG by Wavelets} \end{array} \right|$$

Table 6: Average norm 2 value of 10 samples.

	Original ECG	Haar Wavelet	Db-3 Wavelet	Coif-3 Wavelet
Average Norm 2	324.621	324.547	324.577	324.598
Difference	0	0.074	0.044	0.023

Here error is less in Coiflet Wavelet followed by Db-3 and then Haar Wavelet. Further, in Fig.7, it seems Db-3 Wavelet shows better results of de-noising the noisy ECG signal because 5 of the samples shows more value of SNR, but in the rest of the samples the bigger SNR values so obtained is fluctuating among other two wavelets. To get rid of this confusion, let us study the performance of these three wavelets in the context of Mean Square Error (MSE).

$$MSE = \left(\frac{1}{N}\right)\sum_{1}^{N}\left(\hat{x}(j) - y(j)\right)^2 \qquad (15)$$

Table 7: MSE values of 10 samples for different Wavelet functions.

Samples	Mean Square Error		
	Haar Wavelet	Db-3 Wavelet	Coif-3 wavelet
100m	0.0308	0.0197	0.0182
101m	0.0214	0.0205	0.0166
102m	0.0199	0.0158	0.0081
103m	0.0253	0.0222	0.0216
104m	0.0231	0.0201	0.0155
105m	0.0229	0.0123	0.0055
106m	0.0278	0.0225	0.0226
107m	0.0364	0.0285	0.0169
108m	0.1771	0.0098	0.0037
109m	0.0238	0.012	0.046

Since the line graph of MSE shows the de-noising performance is better in the order of Coif-3 Wavelet, then Db-3 Wavelet followed by Haar wavelet.

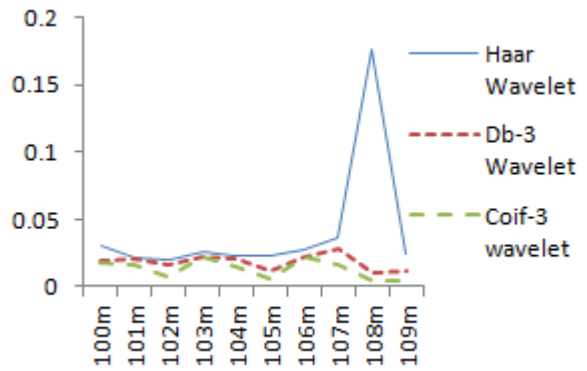


Fig. 10. MSE of three Wavelet functions.

VI. CONCLUSION

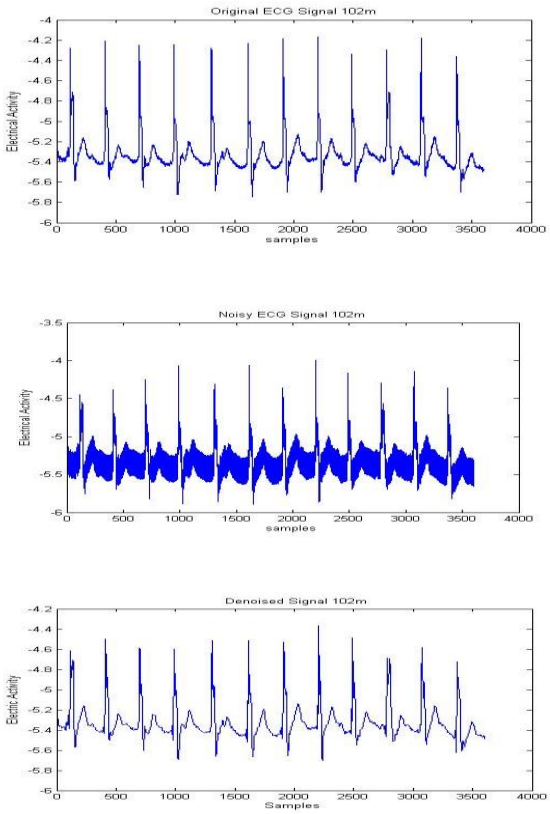
The piece of work in this paper has a couple of concluding points on the basis of outputs of the proposed algorithm simulated in MATLAB under the predefined parameters. The points are as:

The level of decomposition is difficult to decide during the process of de-noising. Because it depends on the nature of signal, noise into it and wavelet functions. Here this is the best way of deciding the level of decomposition at which threshold values can be obtained sensibly. In this piece of work level 3 is opted.

The performance parameters, namely norm 1 and norm 2 conveys that all the wavelet functions are denoising the ECG signal, but Coiflet-3 gives better result.

Since SNR values gives a result with fluctuations at some outputs of 10 samples under simulation, to overcome from this situation, MSE is applied and it shows and justifies that Coiflet-3 wavelet gives optimum results followed by Db3 and Haar Wavelets.

A sample of resulting output under Coiflet-3 wavelet after simulation of the algorithm under defined parameters is shown below.



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