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Construction of Fuzzy Control Chart with Multinomial Quality using Process Capability

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Abstract: Control charts are the effective and quietest form of statistical process control methods. Many of the times, data are obtained in quantitative form; however there are many quality characteristics that cannot be expressed in numerical measure, such as characteristics for appearance, smoothness and colour, etc. Fuzzy sets theory is an impressive mathematical methodology to evaluate the vagueness related uncertainty that can linguistically express data in these situations. In this paper, we construct a fuzzy control limits under fixed and varying sample size with various quality levels for observing a manufacturing process based on the multinomial distribution using degrees of membership and process capability.

Keywords: Fuzzy set, Linguistic variable, Membership function, Multinomial distribution and Process capability.

I. INTRODUCTION

Control charts are widely used for monitoring and examining a production process. The power of control charts lies in their ability to detect process shifts and to identify abnormal conditions in the process, Amirzadeh et.al (2008). A control chart consists of three horizontal lines called; Upper Control Limit (UCL), Centre Line (CL) and Lower Control Limit (LCL). The centre line in a control chart denotes the average value of the quality characteristic under study, Saravanan and Nagarajan (2012). If a point lies within UCL and LCL, then the process is deemed to be under control. Otherwise, a point plotted outside the control limits can be regarded as evidence representing that the process is out of control and, hence preventive or corrective actions are necessary in order to find and eliminate the assignable cause or causes, which subsequently result in improving quality characteristics, Kolarik (1995).

According to W.A.Shewhart (1924), if w be a sample statistic that measures some quality characteristic of interest the mean of w is μ_w and the standard deviation of w is σ_w, then the control limits are defined as $\mu_w \pm A\sigma_w$, where A is the "distance" of the control limits from the centre line, expressed in standard deviation units. In many situations, control limits could not be so precise. Uncertainty comes from the measurement system including operators and gauges, and environmental conditions. In this context, fuzzy set theory is a useful tool to handle this uncertainty. Numeric control limits can be transformed to fuzzy control limits by using membership functions, Sevil Senturk (2009).

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Pandurangan and Varadharajan (2011) proposed a Fuzzy Multinomial (FM) process with Variable Sample Size (VSS) and the control limits for the FM chart have been obtained using multinomial distribution. They compared with the conventional chart for fraction defective. It is seen that Fuzzy Multinomial chart with Variable Sample Size performs better than the conventional chart. Kawa and Suzan (2014) presented an extension of standard control chart to deal with linguistic categories and the variable sampling size (VSS), it is named as fuzzy multinomial charts (FM-chart), and they compared FM-chart with the conventional p – chart and EWMA Control Chart. It is seen that FM chart with VSS performs better than the conventional charts, this method is more sensitive, accurate and more economic for assisting decision maker to control the operation system as early time, especially when there is a change in sample sizes. Kar et.al. (2018) dealt with the application of fuzzy multinomial control chart (FM-chart) with variable sample size for characterizing the mulberry multi-bivoltine silk cocoons quality and the result is compared with the conventional p-chart. In this research article we introduce the fuzzy control chart under fixed and varying sample size with multinomial quality using process capability with a linguistic variable which is classified in more than two categories by numerical examples.

II. METHODS AND MATERIALS

A linguistic variable differs from a numerical variable in that its values are not numbers but words or phrases in some language, Amirzadeh et.al (2008). In the context of fuzzy set theory, a linguistic variable $\tilde{L}$ is characterized by the set of 'm' mutually exclusive members $\left\{\tilde{l}_1, \tilde{l}_2 ... \tilde{l}_m\right\}$ and each term $\tilde{l}_i$ has a weight $\tilde{L}(l_i)$ that reproduces the degree of membership in the set. It is denoted as

$$\tilde{L} = \left\{\left[\tilde{l}_1, \tilde{L}(l_1)\right], \left[\tilde{l}_2, \tilde{L}(l_2)\right], ... \left[\tilde{l}_m, \tilde{L}(l_m)\right]\right\}.$$

Assume that the production/service process is operating in a stable manner and p_i is the probability that an item is $\tilde{l}_i$, i=1, 2…m and successive items produced are independent and accept that a random sample of n items of the product/service is taken by using sampling techniques, is designed at identifying, regulating unfavourable trends and out of control conditions. In order to do this, the four classifications listed below could cause different degrees of membership.

A random sample of size n_r, r=1, 2,…s and let X_i, i=1, 2,…m, be the number of items of the product/service that are $\tilde{l}_i$, i=1, 2,...m and X_1, X_2,...,X_m has a multinomial distribution with parameters n_1, n_2, …n_s and p_1, p_2,…p_m. It is experienced that each {X_1, X_2,...,X_m} marginally has a binomial distribution with the parameters mean $n_r p_i$ and variance $n_r p_i(1-p_i)$, i=1, 2,...,m and r=1, 2,…s respectively. Further the mean and variance of the linguistic variable with variable sample size will be equal to the mean and variance of the linguistic variable to fixed sample size.

At the present we define the weighted average (Amirzadeh, et. al, 2008) for the degree of membership $\left[\tilde{l}_i, \tilde{L}(l_i)\right]$, i=1, 2,...,m, and as follows:

$$\tilde{\tilde{X}}_{\tilde{L}} = \frac{\sum_{i=1}^{m} X_i \tilde{L}(l_i)}{\sum_{i=1}^{m} X_i} \Rightarrow \tilde{\tilde{X}}_{\tilde{L}} = \frac{\sum_{i=1}^{m} X_i \tilde{L}(l_i)}{n_r},\ i=1,2,...m,\ \ r=1,2,...s$$

In a sample of n_r units, X_i has a binomial distribution with the mean $n_r p_i$, variance $n_r p_i(1-p_i)$ and Cov(Xi, Xj) = $-n_r p_i p_j$, if $i \neq j$. Then variance for the degree of membership $\left[\tilde{l}_i, \tilde{L}(l_i)\right], i=1,\ 2,...,m,$ is given by

$$V\left(\tilde{\tilde{X}}_{\tilde{L}}\right) = V\left(\frac{\sum_{i=1}^{m} X_i \tilde{L}(l_i)}{n_r}\right) = \frac{1}{n_r^2} V\left(\sum_{i=1}^{m} X_i \tilde{L}(l_i)\right)$$

$$= \frac{1}{n_r^2} V\left[X_1 \tilde{L}(l_1) + X_2 \tilde{L}(l_2) + ... + X_m \tilde{L}(l_m)\right]$$

$$V\left(\tilde{\tilde{X}}_{\tilde{L}}\right) = \frac{1}{n_r}\left\{\sum_{i=1}^{m} \tilde{L}^2(l_i)\left[p_i(1-p_i)\right] - 2\sum_{i=1}^{m}\sum_{i<j,j=1}^{m} \tilde{L}(l_i)\tilde{L}(l_j)\left(p_i p_j\right)\right\}, r=1,2,...s$$

The traditional Shewhart (1931) control chart is replaced by fuzzy multinomial control chart, then

$$UCL_S = E\left(\tilde{\tilde{X}}_{\tilde{L}}\right) + 3\sqrt{V\left(\tilde{\tilde{X}}_{\tilde{L}}\right)} \ \ and\ \ CL_S = E\left(\tilde{\tilde{X}}_{\tilde{L}}\right)$$

$$LCL_S = E\left(\tilde{\tilde{X}}_{\tilde{L}}\right) - 3\sqrt{V\left(\tilde{\tilde{X}}_{\tilde{L}}\right)}$$

The derived fuzzy multinomial control chart from the substitution of the derived results of $E\left(\tilde{\tilde{X}}_{\tilde{L}}\right)$ and $V\left(\tilde{\tilde{X}}_{\tilde{L}}\right)$ are as follows:

$$UCL_{FM} = E\left[\frac{\sum_{i=1}^{m} X_i \tilde{L}(l_i)}{n_r}\right] + 3\sqrt{\left\{\frac{1}{n_r}\left[\sum_{i=1}^{m} \tilde{L}^2(l_i)\left[p_i(1-p_i)\right] - 2\sum_{i=1}^{m}\sum_{i<j,j=1}^{m} \tilde{L}(l_i)\tilde{L}(l_j)\left(p_i p_j\right)\right]\right\}}$$

$$CL_{FM} = E\left[\frac{\sum_{i=1}^{m} X_i \tilde{L}(l_i)}{n_r}\right]$$

$$LCL_{FM} = E\left[\frac{\sum_{i=1}^{m} X_i \tilde{L}(l_i)}{n_r}\right] - 3\sqrt{\left\{\frac{1}{n_r}\left[\sum_{i=1}^{m} \tilde{L}^2(l_i)\left[p_i(1-p_i)\right] - 2\sum_{i=1}^{m}\sum_{i<j,j=1}^{m} \tilde{L}(l_i)\tilde{L}(l_j)\left(p_i p_j\right)\right]\right\}}$$

For a specified tolerance level (i.e the difference between upper specification limit and lower specification limit) and process capability of the process, the value of standard deviation $\sigma = \sqrt{V\left(\tilde{\tilde{X}}_{\tilde{L}}\right)}$ (termed as $\tilde{\sigma}_{FM:C_p}$) is calculated from $C_p = \frac{USL - LSL}{6\sigma}$ using a JAVA script (Radhakrishnan and Balamurugan, 2012).

Therefore the resultant of proposed fuzzy multinomial control chart using process capability is given below:

$$UCL_{FM:C_p} = E\left[\frac{\sum_{i=1}^{m} X_i \tilde{L}(l_i)}{n_r}\right] + 3\tilde{\sigma}_{FM:C_p},\ CL_{FM:C_p} = E\left[\frac{\sum_{i=1}^{m} X_i \tilde{L}(l_i)}{n_r}\right] \text{ and}$$

$$LCL_{FM:C_p} = E\left[\frac{\sum_{i=1}^{m} X_i \tilde{L}(l_i)}{n_r}\right] - 3\tilde{\sigma}_{FM:C_p},\ r=1,2,...s$$

III. PERFORMANCE OF THE PROPOSED CONTROL CHART USING PROCESS CAPABILITY FOR FIXED SAMPLE SIZE

The following data includes 20 samples of size 25 each from the turnout of a final assembly line in an automobile company which is located Salem city. The items for assembled in each sample are inspected and classified as each "admirable", "worthy", "normal" and "corruptly" with the degrees of membership 0, 0.25, 0.5 and 1, respectively.

Table 1: The data for final assembly line in an automobile company

S.No	*Admirable*	*Worthy*	*Normal*	*Corruptly*
1	2	9	10	4
2	5	4	5	11
3	13	2	6	4
4	1	3	8	13
5	5	12	1	7
6	8	2	5	10
7	7	4	9	5
8	10	3	5	7
9	2	17	2	4
10	5	1	10	9
11	14	2	1	8
12	3	4	2	16
13	11	4	7	3
14	2	5	14	4
15	1	3	18	3
16	5	9	7	4
17	2	5	14	4
18	4	7	11	3
19	8	10	6	1
20	3	1	15	6

The weighted average for the degree of membership is as follows:

For the first sample,

$$\tilde{\bar{X}}_{\tilde{L}_1} = \frac{\left[\left(\tilde{L}(l_1)\times X_{11}\right)+\left(\tilde{L}(l_2)\times X_{12}\right)+\left(\tilde{L}(l_3)\times X_{13}\right)+\left(\tilde{L}(l_4)\times X_{14}\right)\right]}{n_1}$$

$$= \frac{\left[(1\times 2)+(0.5\times 9)+(0.25\times 10)+(0\times 4)\right]}{25} = 0.360$$

and so on. The variance for the degree of membership is as follows:

For the first sample,

$$V\left(\tilde{\bar{X}}_{\tilde{L}_1}\right) = \frac{1}{n_1}\left\{\begin{array}{l}\left[\begin{array}{l}\tilde{L}^2(l_1)\left[p_1(1-p_1)\right]+\tilde{L}^2(l_2)\left[p_2(1-p_2)\right]\\ +\tilde{L}^2(l_3)\left[p_3(1-p_3)\right]+\tilde{L}^2(l_4)\left[p_4(1-p_4)\right]\end{array}\right]\\ -2\left[\begin{array}{l}\left[\tilde{L}(l_1)\tilde{L}(l_2)(p_1p_2)\right]+\left[\tilde{L}(l_1)\tilde{L}(l_3)(p_1p_3)\right]\\ +\left[\tilde{L}(l_1)\tilde{L}(l_4)(p_1p_4)\right]+\left[\tilde{L}(l_2)\tilde{L}(l_3)(p_2p_3)\right]\\ +\left[\tilde{L}(l_2)\tilde{L}(l_4)(p_2p_4)\right]+\left[\tilde{L}(l_3)\tilde{L}(l_4)(p_3p_4)\right]\end{array}\right]\end{array}\right\}$$

$$= \frac{1}{25}\left\{\begin{array}{l}\left[\begin{array}{l}\left(1^2\left[0.08(1-0.08)\right]\right)+\left(0.5^2\left[0.36(1-0.36)\right]\right)\\ +\left(0.25^2\left[0.40(1-0.40)\right]\right)+\left(0^2\left[0.16(1-0.16)\right]\right)\end{array}\right]\\ -2\left[\begin{array}{l}\left[1\times 0.5(0.08\times 0.36)\right]+\left[1\times 0.25(0.08\times 0.40)\right]\\ +\left[1\times 0(0.08\times 0.16)\right]+\left[0.5\times 0.25(0.36\times 0.40)\right]\\ +\left[0.5\times 0(0.36\times 0.16)\right]+\left[0.25\times 0(0.40\times 0.16)\right]\end{array}\right]\end{array}\right\}$$

$$V\left(\tilde{\bar{X}}_{\tilde{L}_1}\right) = 0.0026$$

and so on.

To supervise the assembly line of the items, 20 samples of fixed size i.e. 25 are chosen. The degrees of membership for the assessment are taken as 1, 0.5, 0.25 and 0 respectively. The data with probabilities, weighted average and variance based on linguistic variables are given in Table-2.

The fuzzy multinomial control limits are as follows:

$$UCL_{FM} = E\left[\frac{\sum_{i=1}^{m} X_i\tilde{L}(l_i)}{n_r}\right] - 3\sqrt{\left\{\frac{1}{n_r}\left[\sum_{i=1}^{m}\tilde{L}^2(l_i)\left[p_i(1-p_i)\right]-2\sum_{i=1}^{m}\sum_{i<j,j=1}^{m}\tilde{L}(l_i)\tilde{L}(l_j)(p_ip_j)\right]\right\}}$$

$$= 0.4070 + 3\sqrt{0.00026} = 0.4553$$

$$CL_{FM} = E\left[\frac{\sum_{i=1}^{m} X_i\tilde{L}(l_i)}{n_r}\right] = 0.4070$$

$$LCL_{FM} = E\left[\frac{\sum_{i=1}^{m} X_i\tilde{L}(l_i)}{n_r}\right] - 3\sqrt{\left\{\frac{1}{n_r}\left[\sum_{i=1}^{m}\tilde{L}^2(l_i)\left[p_i(1-p_i)\right]-2\sum_{i=1}^{m}\sum_{i<j,j=1}^{m}\tilde{L}(l_i)\tilde{L}(l_j)(p_ip_j)\right]\right\}}$$

$$= 0.4070 - 3\sqrt{0.00026} = 0.3587$$

Even the control limit interval hereafter refers to as CLI, is the difference between the control limit value. Thus, control chart for fuzzy multinomial, the control limit interval is 0.0965. From the resulting Figure-1, it is clear that the process is out of control, since the sample numbers 3, 8, 11, 13 and 19 lie outside the upper control limit and the sample numbers 2, 4, 10, 12, 14, 15, 17 and 20 lie outside the lower control limit.

Table 2: Results of probabilities, weighted average, variance based on linguistic variables

S.No	p1	p2	p3	p4	Weighted average $\left(\tilde{\bar{X}}_{\tilde{L}}\right)$	Variance $V\left(\tilde{\bar{X}}_{\tilde{L}}\right)$
1	0.08	0.36	0.40	0.16	0.360	0.0026
2	0.20	0.16	0.20	0.44	0.330	0.0057
3	0.52	0.08	0.24	0.16	0.620	0.0068
4	0.04	0.12	0.32	0.52	0.180	0.0023
5	0.20	0.48	0.04	0.28	0.450	0.0048
6	0.32	0.08	0.20	0.40	0.410	0.0074
7	0.28	0.16	0.36	0.20	0.450	0.0056
8	0.40	0.12	0.20	0.28	0.510	0.0073
9	0.08	0.68	0.08	0.16	0.440	0.0025
10	0.20	0.04	0.40	0.36	0.320	0.0053
11	0.56	0.08	0.04	0.32	0.610	0.0084
12	0.12	0.16	0.08	0.64	0.220	0.0047
13	0.44	0.16	0.28	0.12	0.590	0.0060
14	0.08	0.20	0.56	0.16	0.320	0.0025
15	0.04	0.12	0.72	0.12	0.280	0.0015
16	0.20	0.36	0.28	0.16	0.450	0.0042
17	0.08	0.20	0.56	0.16	0.320	0.0025
18	0.16	0.28	0.44	0.12	0.410	0.0036
19	0.32	0.40	0.24	0.04	0.580	0.0039
20	0.12	0.04	0.60	0.24	0.290	0.0033

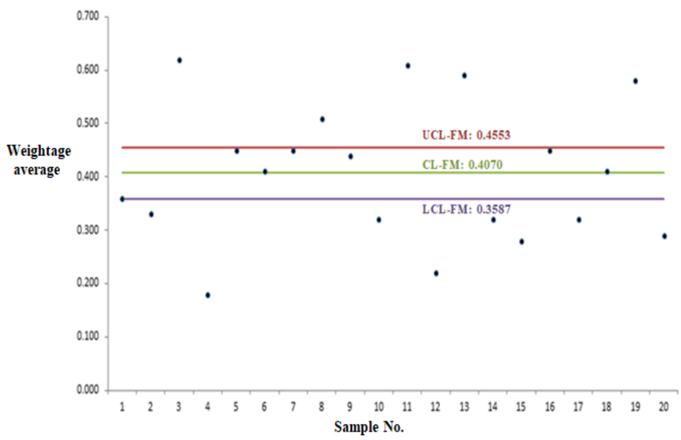

Figure 1: Control limits for fuzzy multinomial

The difference between upper specification and lower specification limits is 0.0535 (USL - LSL = 0.0917-0.0383), which termed as tolerance level (TL) and choose the process capability (Cp) is 2.0, the value of $\tilde{\sigma}_{FM:C_p}$ is 0.0045.

The proposed fuzzy multinomial control limits using process capability for a specified tolerance level are given below.

$$E\left[\frac{\sum_{i=1}^{m} X_i\tilde{L}(l_i)}{n_r}\right] \pm 3\tilde{\sigma}_{FM:C_p}$$

$$UCL_{FM:C_p} = E\left[\frac{\sum_{i=1}^{m} X_i \tilde{L}(l_i)}{n_r}\right] + 3\tilde{\sigma}_{FM:C_p} = 0.4070 + (3\times 0.0045) = 0.4204$$

$$CL_{FM:C_p} = E\left[\frac{\sum_{i=1}^{m} X_i \tilde{L}(l_i)}{n_r}\right] = 0.4070 \ and \ LCL_{FM:C_p} = 0.3936$$

Thus, control chart for fuzzy multinomial using process capability, the control limit interval is 0.0267. From the resulting Figure-2, it is clear that the process is out of control, since the sample numbers 3, 5, 7, 8, 9, 11, 13, 16 and 19 lie outside the upper control limit and the sample numbers 1, 2, 4, 10, 12, 14, 15, 17 and 20 lie outside the lower control limit.

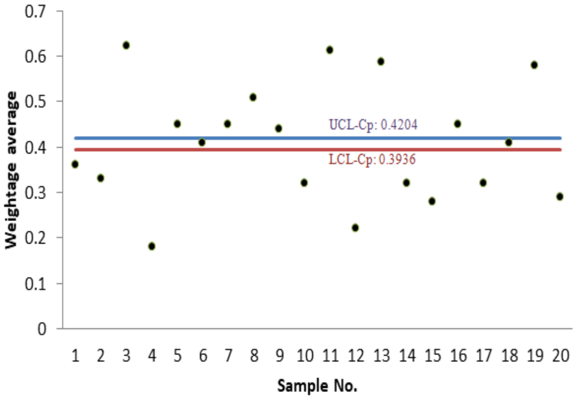

Figure 2: Control limits for fuzzy multinomial using process capability

In general, the expected number of samples taken before the shift is detected is simply the average run length (Montgomery, 2009),

$$ARL = \sum_{r=1}^{\infty} r\beta^{r-1}(1-\beta), \text{ where sample}(r) = 1, 2, \ldots s$$

$$ARL = \frac{1}{1-\beta}$$

Table 3: Efficiency results through average run length (ARL) for control charts

multiple of σ	*Fuzzy multinomial control chart*	*Control chart for Fuzzy multinomial using process capability*
0.0001	366.77	343.59
0.0002	366.56	341.14
0.0003	366.22	337.12
0.0004	365.73	331.64
0.0005	365.11	324.82
0.0006	364.35	316.82
0.0007	363.46	307.81
0.0008	362.44	297.97
0.0009	361.28	287.47
0.0010	360.00	276.49

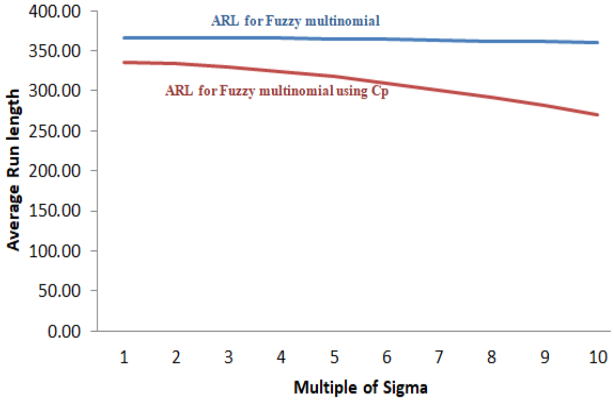

Figure 3: Average run length (ARL) for control charts

In general the expected number of samples needed to detect a shift of 'multiple of σ' under the control chart for fuzzy multinomial using process capability is more agile than the existing fuzzy multinomial control limits.

IV. PERFORMANCE OF THE PROPOSED CONTROL CHART USING PROCESS CAPABILITY FOR VARYING SAMPLE SIZE

A factory is located in Salem District producing spark plug found on the inspection of 15 samples of varying. The spark plugs in each sample are inspected and classified as each "Fail", "Pass", "Good" and "Excellent" with the degrees of membership 1, 0.5, 0.25 and 0, respectively.

Table 4: The data for final assembly line in an automobile company

S.No	*Sample size*	*Fail*	*Pass*	*Good*	*Excellent*
1	20	6	1	6	7
2	21	1	9	5	6
3	33	2	14	9	8
4	25	8	3	12	2
5	29	4	7	9	9
6	30	3	8	7	12
7	19	5	5	5	4
8	27	2	8	9	8
9	28	1	5	20	2
10	26	2	11	8	5
11	19	1	3	10	5
12	32	8	4	10	10
13	34	9	6	14	5
14	31	7	5	12	7
15	29	6	8	11	4

To supervise the assembly line of the items, 15 samples of varying size are chosen.

The degrees of membership for the assessment are taken as 1, 0.5, 0.25 and 0 respectively. The data with probabilities, weighted average and variance based on linguistic variables are given in Table-7.

The weighted average for the degree of membership is as follows:

For the first sample,

$$\tilde{\bar{X}}_{\tilde{L}_1} = \frac{\left[\left(\tilde{L}(l_1)\times X_{11}\right)+\left(\tilde{L}(l_2)\times X_{12}\right)+\left(\tilde{L}(l_3)\times X_{13}\right)+\left(\tilde{L}(l_4)\times X_{14}\right)\right]}{n_1}$$

$$= \frac{\left[(1\times 6)+(0.5\times 1)+(0.25\times 6)+(0\times 7)\right]}{20} = 0.400$$

and so on.

The variance for the degree of membership is as follows:

For the first sample,

$$V\left(\tilde{\bar{X}}_{\tilde{L}_1}\right)=\frac{1}{n_1}\left\{\begin{bmatrix}\tilde{L}^2(l_1)\left[p_1(1-p_1)\right]+\tilde{L}^2(l_2)\left[p_2(1-p_2)\right]\\ +\tilde{L}^2(l_3)\left[p_3(1-p_3)\right]+\tilde{L}^2(l_4)\left[p_4(1-p_4)\right]\end{bmatrix} -2\begin{bmatrix}\left[\tilde{L}(l_1)\tilde{L}(l_2)(p_1p_2)\right]+\left[\tilde{L}(l_1)\tilde{L}(l_3)(p_1p_3)\right]\\ +\left[\tilde{L}(l_1)\tilde{L}(l_4)(p_{11}p_{14})\right]+\left[\tilde{L}(l_2)\tilde{L}(l_3)(p_2p_3)\right]\\ +\left[\tilde{L}(l_2)\tilde{L}(l_4)(p_2p_4)\right]+\left[\tilde{L}(l_3)\tilde{L}(l_4)(p_3p_4)\right]\end{bmatrix}\right\}$$

$$=\frac{1}{20}\left\{\begin{bmatrix}\left(1^2\left[0.30(1-0.30)\right]\right)+\left(0.5^2\left[0.05(1-0.05)\right]\right)\\ +\left(0.25^2\left[0.30(1-0.30)\right]\right)+\left(0^2\left[0.35(1-0.35)\right]\right)\end{bmatrix} -2\begin{bmatrix}\left[1\times 0.5(0.30\times 0.05)\right]+\left[1\times 0.25(0.30\times 0.30)\right]\\ +\left[1\times 0(0.30\times 0.35)\right]+\left[0.5\times 0.25(0.05\times 0.30)\right]\\ +\left[0.5\times 0(0.05\times 0.35)\right]+\left[0.25\times 0(0.30\times 0.35)\right]\end{bmatrix}\right\}$$

$$V\left(\tilde{\bar{X}}_{\tilde{L}_1}\right)=0.009$$

and so on.

The fuzzy multinomial control limits are as given below:

For sample 1:

$$UCL_{FM}=E\left[\frac{\sum_{i=1}^{m}X_i\tilde{L}(l_i)}{n_r}\right]-3\sqrt{\left\{\frac{1}{n_r}\begin{bmatrix}\sum_{i=1}^{m}\tilde{L}^2(l_i)\left[p_i(1-p_i)\right]\\ -2\sum_{i=1}^{m}\sum_{i<j,j=1}^{m}\tilde{L}(l_i)\tilde{L}(l_j)(p_ip_j)\end{bmatrix}\right\}}$$

$$UCL_{FM}=0.372+3\sqrt{0.009}=0.650$$

$$LCL_{FM}=E\left[\frac{\sum_{i=1}^{m}X_i\tilde{L}(l_i)}{n_r}\right]-3\sqrt{\left\{\frac{1}{n_r}\begin{bmatrix}\sum_{i=1}^{m}\tilde{L}^2(l_i)\left[p_i(1-p_i)\right]\\ -2\sum_{i=1}^{m}\sum_{i<j,j=1}^{m}\tilde{L}(l_i)\tilde{L}(l_j)(p_ip_j)\end{bmatrix}\right\}}$$

$$LCL_{FM}=0.372-3\sqrt{0.009}=0.094$$

and so on.

The control limits for fuzzy multinomial using process capability with varying sample size are as follows:

The difference between upper specification and lower specification limits is 0.233 (USL - LSL = 0.414-0.181), which termed as tolerance level (TL) and choose the process capability (Cp) is 2.0, the value of $\tilde{\sigma}_{FM:C_p}$ is 0.019.

The proposed fuzzy multinomial control limits using process capability for a specified tolerance level are given below.

Table 5: Results of probabilities, weighted average, variance based on linguistic variables

S. No	p1	p2	p3	p4	Weighted average $\left(\tilde{\bar{X}}_{\tilde{L}_i}\right)$	Variance $V\left(\tilde{\bar{X}}_{\tilde{L}_i}\right)$	Standard deviation $\tilde{\sigma}_{FM:Cp}\left(\tilde{\bar{X}}_{\tilde{L}_i}\right)$
1	0.30	0.05	0.30	0.35	0.400	0.009	0.414
2	0.05	0.43	0.24	0.29	0.321	0.003	0.258
3	0.06	0.42	0.27	0.24	0.341	0.002	0.260
4	0.32	0.12	0.48	0.08	0.500	0.005	0.361
5	0.14	0.24	0.31	0.31	0.336	0.004	0.323
6	0.10	0.27	0.23	0.40	0.292	0.003	0.310
7	0.26	0.26	0.26	0.21	0.461	0.007	0.365
8	0.07	0.30	0.33	0.30	0.306	0.003	0.275
9	0.04	0.18	0.71	0.07	0.304	0.001	0.181
10	0.08	0.42	0.31	0.19	0.365	0.003	0.262
11	0.05	0.16	0.53	0.26	0.263	0.003	0.236
12	0.25	0.13	0.31	0.31	0.391	0.005	0.385
13	0.26	0.18	0.41	0.15	0.456	0.004	0.356
14	0.23	0.16	0.39	0.23	0.403	0.004	0.357
15	0.21	0.28	0.38	0.14	0.440	0.004	0.326

For sample 1:

$$UCL_{FM:C_p}=E\left[\frac{\sum_{i=1}^{m}X_i\tilde{L}(l_i)}{n_r}\right]+\left[\left(3\big/\sqrt{n_r}\right)\tilde{\sigma}_{FM:C_p}\right]$$

$$UCL_{FM:C_p}=0.372+\left[\left(3\big/\sqrt{20}\right)0.019\right]=0.385$$

$$CL_{FM:C_p}=E\left[\frac{\sum_{i=1}^{m}X_i\tilde{L}(l_i)}{n_r}\right]=0.372$$

$$LCL_{FM:C_p}=E\left[\frac{\sum_{i=1}^{m}X_i\tilde{L}(l_i)}{n_r}\right]-\left[\left(3\big/\sqrt{n_r}\right)\tilde{\sigma}_{FM:C_p}\right]$$

$$LCL_{FM:C_p}=0.372-\left[\left(3\big/\sqrt{20}\right)0.019\right]=0.359$$

and so on.

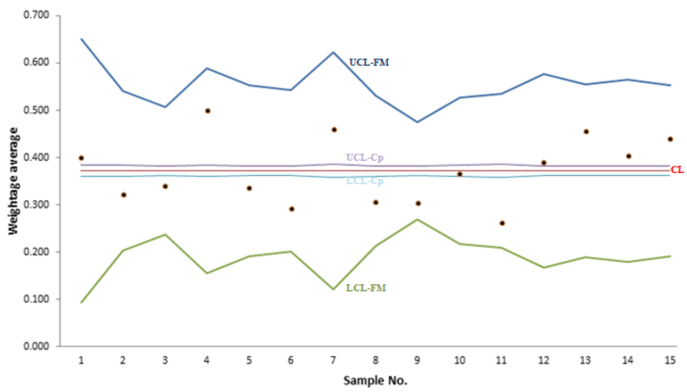

Figure 4: Control limits for fuzzy multinomial and FM using Cp

Table 6: Results of Fuzzy control limits based on linguistic variables

S. No	Sample size (n_r)	Fuzzy multinomial control limits			Control limits for Fuzzy multinomial using process capability		
		UCL	LCL	Class limit Interval (CLI)	UCL	LCL	Class limit Interval (CLI)
1	20	0.650	0.094	0.555	0.385	0.359	0.025
2	21	0.541	0.203	0.337	0.384	0.360	0.025
3	33	0.508	0.236	0.271	0.382	0.362	0.020
4	25	0.588	0.156	0.433	0.383	0.361	0.023
5	29	0.552	0.192	0.360	0.383	0.361	0.021
6	30	0.542	0.202	0.340	0.382	0.362	0.021
7	19	0.623	0.121	0.503	0.385	0.359	0.026
8	27	0.531	0.213	0.318	0.383	0.361	0.022
9	28	0.475	0.269	0.205	0.383	0.361	0.022
10	26	0.526	0.218	0.308	0.383	0.361	0.022
11	19	0.535	0.209	0.325	0.385	0.359	0.026
12	32	0.576	0.168	0.408	0.382	0.362	0.020
13	34	0.555	0.189	0.366	0.382	0.362	0.020
14	31	0.565	0.179	0.385	0.382	0.362	0.020
15	29	0.554	0.190	0.363	0.383	0.361	0.021

It is clear that the process is in-control under fuzzy multinomial control chart, since the entire sample numbers lie inside the control limits. Furthermore, the process is out of control under the control chart for FM using process capability, since the sample numbers 1, 4, 7, 12, 13, 14 and 15 lie above the upper control limit and the sample numbers 2, 3, 5, 6, 8, 9 and 11 lie below the lower control limit.

V. CONCLUSION

The constructed control chart for fuzzy multinomial using process capability in both cases i.e. fixed and varying sample sizes, procedures adopted and discussed in the research article by taking the process capability (C_p) as the base only. In this article offers the possibility of using fuzzy multinomial control chart using process capability, which rules out the weaknesses compared to the existing control charts. Specifically, it presents one of the fuzzy multinomial control charts using process capability and on the real-life data illustrates the simplicity of its usage in practice. The procedures can also be constructed by changing the suitable process capability (C_p) used in this article and the performance with efficiency of the control charts is tested through average run length.

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