International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249-8958 (Online), Volume-9 Issue-1, October, 2019

Solving Problems with Smooth Sequences that are Subject to Local Restrictions

Egor M. Glukhov

Abstract — This article discusses the options that arise when solving problems with smooth sequences that are subject to local restrictions. It also continues the cycle of work on the study of smooth sequences and supplements the literature in the field of the study of this arithmetic property. The relevance of the current research is that the smooth sequences simulate the motion of the bodies taking into account the resistance of the medium. The methodology is in solving problems and proving theorems by calculating the formulas and building the graphs and providing the comments on them.

It should be noted that when conducting a literature review about such problems and their solution, we noticed a lack of a detailed review and compactness of information. Thus, this work has a scientific novelty and, as a result, practical significance for the learning process. The paper presents a detailed description of the solutions of smooth sequences on the 1st, 2nd, 3rd differences; it also provides evidence with explanations of the theorems, gives illustrations of graphs of sequences under different conditions. The results of the study may be applicated to spaceship building and ballistics. Besides this, the article is supplemented with data in tables shown in the Appendices.

Keywords: final sequence, modules of adjacent numbers, series of differences, sequence height, smooth sequence.

I. INTRODUCTION

Sequences are the most important object of research. People find where they may be applied and can come across them in all branches of mathematics. There are distinguished different classes of sequences: bounded, monotone, convergent, etc. In the work [9], smooth sequences were studied when the second differences are bounded. Among the most popular sequences in mathematics there are the Fibonacci numbers or Fibonacci sequences [28].

Smooth sequences in this sense simulate the motion of bodies. The relevance of this work follows from all the above mentioned.

The main goal of the work is to study smooth sequences in the 1st, 2nd and 3rd differences [1], [2] and to solve the main task of finding the greatest graph height of such a sequence.

The second main goal of this work is another definition of sequence smoothness. It's natural to find the smallest arithmetic mean for such sequences. This understanding of smoothness is apparently new. The first paragraph of the research paper explains the concept of series of differences and defines smooth sequences by the differences of adjacent numbers. In the second paragraph, we address the problem of finding the greatest sequence height of the smooth in the 1st, 2nd, 3rd series of differences. The third paragraph defines smooth sequences by the modules of adjacent numbers and addresses their properties. As it was noted in [29], network analysis is based on graph theory, which is the study of graphs in discrete mathematics. That's why it is important to note that mathematics also affects other sciences, for example, mechanics and sociology. The importance of the Banach spaces in the study of smooth surjections should be mentioned [24], [25], [26], [27].

Revised Manuscript Received on October 30, 2019.
* Correspondence Author
Glukhov Egor*, student, Private educational institution Liceum №1 «Sputnik», Samara, Russia. Email: glukhov_egor@rambler.ru

II. RESEARCH METHODS

Smooth sequences by differences of adjacent numbers

Let's briefly explain the concept of series of difference. Let's have a look at a sequence of numbers [12].

$$a_0, a_1, a_2, a_3, ..., a_n, ... \quad (1)$$

The first series of differences of this sequence is called a sequence of numbers

$$a_0^{(1)}, a_1^{(1)}, a_2^{(1)}, ..., a_n^{(1)}, ... \quad (2)$$

Where

$$a_0^{(1)} = a_1 - a_0, a_1^{(1)} = a_2 - a_1, a_2^{(1)} = a_3 - a_2, ... \\ ..., a_n^{(1)} = a_{n+1} - a_n, ... \quad (3)$$

The second series of differences of this sequence is called a sequence of numbers

$$a_0^{(2)}, a_1^{(2)}, a_2^{(2)}, ..., a_n^{(2)}, ... \quad (4)$$

Where

$$a_0^{(2)} = a_1^{(1)} - a_0^{(1)}, a_1^{(2)} = a_2^{(1)} - a_1^{(1)}, a_2^{(2)} = a_3^{(1)} - a_2^{(1)}, ... \\ ..., a_n^{(2)} = a_{n+1}^{(1)} - a_n^{(1)}, ... \quad (5)$$

Any next p series of differences is defined similarly. The rows of differences are well arranged in the form of a triangle.

In the top zero line of this triangle is the first series of differences and the values in the following lines are written out in accordance with a certain law [8].

$$\begin{array}{ll} & a_0, a_1, a_2 .., a_n, ... \\ \Delta^1 & a_0^{(1)}, a_1^{(1)}, a_2^{(1)}, ..., a_n^{(1)}, a_{n-1}^{(1)}, ... \\ \Delta^2 & a_0^{(2)}, a_1^{(2)}, a_2^{(2)}, ..., a_n^{(2)}, a_{n-1}^{(2)}, a_{n-2}^{(2)}, ... \end{array} \quad (6)$$

Let's generalize this definition to any differences and take any number instead of 1. Thus, we get the following definition.

Definition. Let a > 0, $p \in N$. Sequence of real numbers $\{x_n\}$, n = 1, 2, 3…, is called p smooth differences with a value a (short form, (p, a) — smooth), if the sequence of p differences $\{\Delta^{(p)}x_k\}$ is limited by number a. Every continuous function $\{x_n\}$ can be considered as a superposition of several mappings [11].

$$\left\{\Delta^{(p)}x_k\right\} \le a, k = 1, 2, 3, ... \qquad (7)$$

This definition of smooth sequences modulates the movement of physical bodies. If the sequence represents the x coordinate, then the analogue of 1st differences is $\{\Delta^{(1)}x_k\}$, i.e. the speed of the body at every moment, and the analogue of 2nd differences is $\{\Delta^{(2)}x_k\}$ the change of speeds or acceleration. It is important to evaluate the height of the graph or the maximum term of such a sequence. Such an estimate is a solution to some problems, specific for optimal control theory [9].

Recently, many works have been devoted to weakening the monotonicity condition while preserving the integrability criterion [3], [4], to studying ultimately monotone sets, and also to studying the basic structural properties of ultimately monotone reducibility (lm-reducibility) between a set and a sequence of sets [5].

It is necessary to discuss the characteristic considerations arising while solving problems where certain restrictions are imposed on a sequence since it is forbidden to change it abruptly or turn sharply and it is necessary to evaluate how large its fluctuation may turn out to be in general [9], [10].

The problem of finding the greatest height of a smooth sequence

The numbers of the selected sequence may have different relative positions. In particular, the sequence may turn out to be monotonically increasing, monotonically decreasing, or have alternating extrema [17], [18].

It is sometimes very difficult to prove the existence of a limit for a given sequence. The most common sequences are well studied and are given in reference books. There are important theorems that allow us to conclude that there is a limit to a given sequence (and even calculate it), based on the sequences already studied before [20], [21], [22], [23].

When you start to solve problems, it is useful to review the wording of all the tasks of this cycle [7].

In the work [1], the following problem was solved.

Problem. Let $x_0 = x_1 = x_n = x_{n+1} = 0$, the rest x_k are integrals. If the sequence $\{x_k\}, k = 0, 1, 2, ..., n, n+1 - (2,1)$ is smooth, then what is the greatest value can have the largest member of this sequence. Let's consider $\{x_k\}$ sequence as smooth, what can be done without violating the generality of reasoning [6]. Otherwise, what is the highest sequence graph height of $\{x_k\}$.

Our goal is to solve the general problem where, p and a are arbitrary numbers.

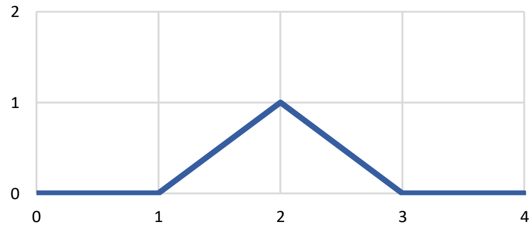
III. RESULTS AND DISCUSSION

1-smooth difference sequences

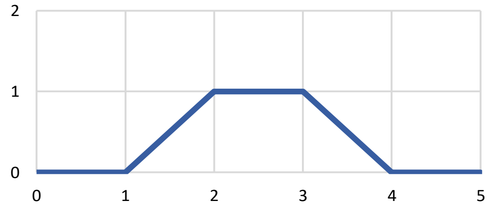
Theorem 1. The greatest height for (1, a) of the smooth sequence is equal to $\left[\frac{n-1}{2}\right]a$.

Proof. The validity of this theorem follows from the fact that, whatever various simple (1, a) may be, you can get non-ending simple [13].

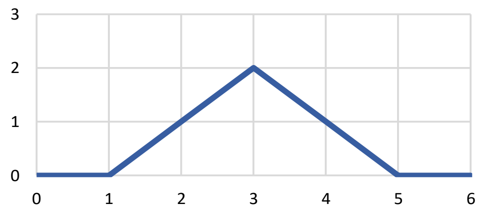
While a = 1 we will build a graph for n (Fig. 1):



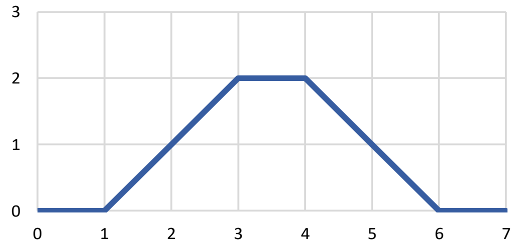
a.



b.



c.



d.

Fig. 1. The graphs for n: a. n=3; b. n=4; c. n=5; d. n=6.

In general:

If n is even, then $\frac{n}{2} - 1$;

If n odd, then $\left[\frac{n-1}{2}\right]$;

Whatever n is we get $\left[\frac{n-1}{2}\right]$. The theorem is proved.

Retrieval Number: A1238109119/2019©BEIESP
DOI: 10.35940/ijeat.A1238.109119
Journal Website: www.ijeat.org

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

2-smooth difference sequences

Theorem 2. The greatest height for (2, a) of the smooth sequence equals to $\left[\left[\frac{n}{2}\right]^2/4\right]a$.

We give a proof from the work [9]. The proof of the theorem requires the following condition [15].

Only symmetric paths can be considered to find the maximum term of a smooth sequence. If $a_m a_{m+1}$ is the top, then the longer of the parts $a_1 ... a_m$ and $a_{m+1} ... a_n$ can be shortened by replacing it with a symmetrical reflection of the other part relative to the middle perpendicular to the segment $a_m a_{m+1}$. Therefore, only the lifting section can be considered (taking no more than $n/2-1$ steps). This shows that for even *n* the maximum lift height will be the same as for the next (odd) *n*.

It is clear that at first it is advantageous to maximize the ascent rate: $\Delta^1 1+2+3+...$; but this can only be done up to half the rise in order to have time to slow down.

Let's evaluate the first row of difference on both sides of the lift:

$$\begin{aligned} a_1 - a_0 &= 0 \\ a_{m+1} - a_m &= 0 \\ a_2 - a_1 &\le 1 \\ a_m - a_{m-1} &\le 1 \\ a_3 - a_2 &\le 2 \\ a_{m-1} - a_{m-2} &\le 2 \end{aligned} \qquad (8)$$

To evaluate the greatest height h_n i.e. values a_m — $a_0 = a_{m+1} - a_0$ you need to add up the estimates of the first differences. Moreover, the corresponding points of the numerical axis form a compact point set [14]. Therefore,

For $n = 4k-2$

$$h_n \le 1+2+...+(k-1)+(k-1)+...+2+1 = k(k-1) \quad (9)$$

For $n = 4k$

$$h_n \le 1+2+...+(k-1)+k+(k-1)+...+2+1 = k^2 \quad (10)$$

In this case, the height h_n for odd n equals the height h_{n-1}

The answer can be written in short form.

$$h_n = \left[\frac{n}{4}\right]\cdot\left[\frac{n+2}{4}\right] = \left[\left[\frac{n}{2}\right]^2/4\right]a \quad (11)$$

The theorem is proved.

Continuous analogue of sequences smooth in 3 differences

1. The formula was created to work with series of differences

$$a_x = a_1 + a_2 + a_3 + ... + a_{x-1} + a_x \quad (0.1)$$

It is proved that the highest point of the highest possible sequence will be located in the middle of the sequence.

2. A program was developed for modeling smooth sequences in 3 series of differences. Different search methods lead to different numbers of patterns found [16].

The basic principle of its work:

Enumeration of all possible 3 rows of differences, consisting of 1, 0, -1.

For each

- Finding the second row of differences using the formula.
- Finding the first row of differences using the formula.
- Calculation of the term in the middle, i.e. the height.
- If the height is greater than the saved, then we save the sequence, its series of difference and its height

The output of the saved row and its height. Some results of the program are given in the form of a table, graph, and series of numbers (Appendix 2)

3. To solve the problem, let's simplify the formula of each term in the sequence up to the integral.

1) $A_0(x) = \int_0^x A1(t)dt$

2) $A_1(x) = \int_0^x A2(t)dt$

3) $A_2(x) = \int_0^x A3(t)dt$

The main condition of the problem remains unnamed: $-1 \le A_3(x) \le 1$

Therefore, the target value

$$A_0(n/2) = \int_0^{n/2} A1(t)dt \quad (13)$$

4. The graphic position of the difference between consecutive primes is studied as the primes tend to infinity [24].

According to the results, the program prompts that the functions will have the following form:

Fig. 2. The graphic position of the difference between consecutive primes is studied as the primes tend to infinity

It is clear that the chart will not rotate anywhere except for the top, which is located exactly in the center. We also note that the vertices of the optimal trajectories in each part of it lie on one parabolic curve [19].

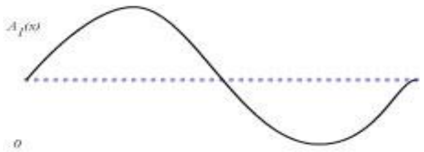


Fig. 3. The vertices of the optimal trajectories in each part of it lie on one parabolic curve

The graph A_1 can be considered as the change rate A_0 or derivative A_0, which is also seen from the formulas. Since A_0 changes direction only in the middle, it means this function is equal to zero also only in the middle.

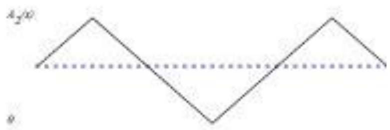


Fig. 4. The graph A_1 for $A1(n/2)=0$

It is clear that the lines of the graph A_2 are tilted at an angle 45° to x axis, which is the maximum lift for the function, the change rate of which does not exceed 1 by a module. It is important to determine under what arguments the graph A_2 changes direction and when it is equal to zero.

5. Shortly speaking, to solve the problem of finding the maximum value

We need to calculate $A_0(n/2)$:

$$A_0(n/2) = \int_0^{n/2} A1(t)dt = \int_0^{n/2}(\int_0^{x} A2(t)dt)dx \quad (14)$$

Where $A_2(t)$ is function according to the schedule; find its exact description below.

We will gradually calculate each area under the graphs to get the answer.

5.1. First you need to clarify the function A_2, with which you can continue the calculation.

$A_1(n/2)=0$

$\int_0^{n/2} A2(t)dt = 0$ by the formula

In other words, the area under the graph $A2(x)$ from 0 to $n/2$ must be equal to zero.

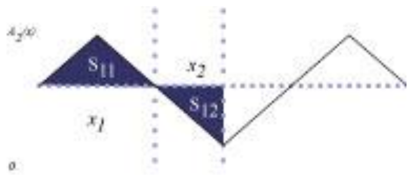


Fig. 5. The graph for A_2(x)

According to the graph

$$S_{11} + S_{12} = 0$$

$$S_{11} = -S_{12}$$

$$|S_{11}| = |S_{12}|$$

In order to find zero value A_1(x) find the area S_{12} and S_{11} according to the area of the triangle and set it equal to:

The area of the first triangle: $S = a*h/2 = x_1*(x_1/2)/2 = x_1^2/4$

Where x_1 is the base.

The area of the second triangle: $S = a*h/2 = x_2*x_2/2 = x_2^2/2$

Where x_2 is the base.

Find the ratio of the bases, considering that the areas are equal:

$$x_2^2/2 = x_1^2/4$$

$$x_2^2*2 = x_1^2$$

$$x_2*\sqrt{2} = x_1$$

The bases relate to both $\sqrt{2}:1$, and in total they give n/2 n/2n/2, which can be seen in the graph. Consequently:

$$x2 = n/2\left(\sqrt{2}+1\right)$$

$$x1 = n*\sqrt{2}/2(\sqrt{2}+1) = \frac{n}{2(2+\sqrt{2}} \quad (15)$$

$\frac{n}{2(2+\sqrt{2}}$ is zero A_1x

And half of this value is the top of the chart.

Let's indicate this on the graph below:

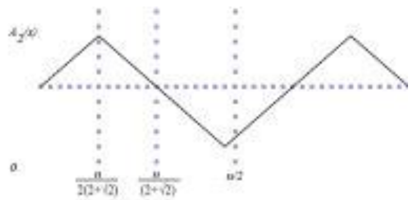


Fig. 6. The half of A_1x is the top of the chart.

Formally:

$A_1(x)$:

While $0 \le x \le \frac{n}{2+\sqrt{2}} \rightarrow A_2(x) = x$

While $\frac{n}{2+\sqrt{2}} \prec x \le \frac{n}{2} \rightarrow$ the graph takes the form of $f(x) = -x$

5.2 Now we can refine the function A_1x by the formula

$A_1(x) = \int_0^x A2(t)dt$

To clearly define this function, we divide it into parts just like in the previous section.

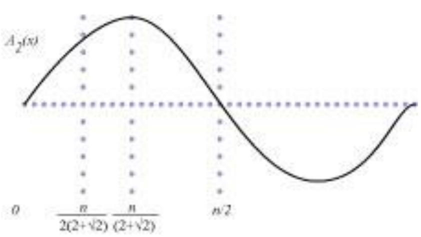


Fig. 7. The graph for function A_1x by the formula

Retrieval Number: A1238109119/2019©BEIESP
DOI: 10.35940/ijeat.A1238.109119
Journal Website: www.ijeat.org

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

$$A_1(x) = \int_0^x A2(t)dt$$

Now we can set the form of the graph in each section using the formula and information about the graph $A_2(x)$:

$A_1(x)$:

While $0 \prec x \prec \frac{n}{2+\sqrt{2}}$ the graph takes the form:

$$\int_0^x tdt = \frac{x^2}{2} \quad (16)$$

While $\frac{n}{2+\sqrt{2}} \prec x \prec \frac{n}{2(2+\sqrt{2})}$ the graph takes the form:

$$\int_0^x \frac{n}{2+\sqrt{2}} - tdt = (n*t)/(2+\sqrt{2}) - t^2/2 \quad (17)$$

While $\frac{n}{2(2+\sqrt{2})} \prec x \prec \frac{n}{2}$ the graph takes the form:

$$\int_0^x -tdt = -\frac{x^2}{2} \quad (18)$$

It is important to remember that only the form of the graph is found here, and not the exact expression of the function. As if we moved a segment of this function to the origin of the axes. Next, we will look for the area under this graph, where such information about the graph will be useful.

5.3 To find the value A_0 in $n/2$, where the maximum point is located, you need to calculate the area under the graph v A_0 from 0 to $n/2$,

You can divide the area under the graph A_1 into several parts, adding up which we get the total area under the schedule A_1 to $n/2$.

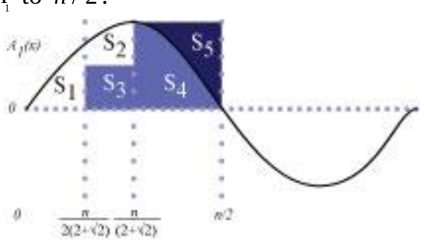


Fig. 8. The total area under the schedule A_1 to $n/2$

Graphs of all functions with highlighted parts are shown in Appendix 1.

We can already find the areas of all the parts; some areas can be found using the formula for the area of the rectangle, others by using the integrals of the functions that define the graph A_1. This affects the interest in the question of the existence of limits of smooth sequences or when a fundamental sequence contains only a countable number of terms [18].

$$s_1 = \int_0^{\frac{n}{2(2+\sqrt{2})}} A1(x) = \int_0^{\frac{n}{2(2+\sqrt{2})}} \frac{x^2}{2} = \frac{n^3}{3(2(2+\sqrt{2}))^3} \quad (19)$$

$$s_2 = \int_{\frac{n}{2(2+\sqrt{2})}}^{\frac{n}{(2+\sqrt{2})}} A1(x) = \int_0^{\frac{n}{2(2+\sqrt{2})}} (n*x)/(2+sqrt(2)) - \quad (20)$$

$$-x\hat{}2/2 = \frac{n^3}{6(2(2+\sqrt{2}))^3}$$

$$s_3 = A1(\frac{n}{2(2+\sqrt{2})})*(\frac{n}{2} - \frac{n}{2(2+\sqrt{2})}) = \frac{n^3}{2(2(2+\sqrt{2}))^3} \quad (21)$$

$$s_4 = A1(\frac{n}{2+\sqrt{2}})*(\frac{n}{2} - \frac{n}{2(2+\sqrt{2})}) = \frac{n^3}{(2(2+\sqrt{2}))^2(2+2\sqrt{2})} \quad (22)$$

$$s_5 = \int_{\frac{n}{2+\sqrt{2}}}^{\frac{n}{2}} A1(x) = \int_0^{\frac{n}{2+2\sqrt{2}}} -\frac{x^2}{2} = -\frac{n^3}{6(2+2\sqrt{2})^3} \quad (23)$$

$$h = A_0(n/2) = S_1 + S_2 + S_3 + S_4 + S_5 =$$

$$n^3(\frac{1}{3(2(2+\sqrt{2}))^3} + \frac{1}{6(2(2+\sqrt{2}))^3} + \frac{1}{2(2(2+\sqrt{2}))^3} +$$

$$+\frac{1}{(2(2+\sqrt{2}))^2(2+2\sqrt{2})} - \frac{1}{6(2+2\sqrt{2})^3}$$

$$= n^3(\frac{1}{3(2(2+\sqrt{2}))^3} + \frac{1}{6(2(2+\sqrt{2}))^3} + \frac{1}{2(2(2+\sqrt{2}))^3} +$$

$$+\frac{\sqrt{2}}{(2(2+\sqrt{2}))^3} - \frac{\sqrt{2}}{3(2(2+2\sqrt{2}))^3} =$$

$$= \frac{n^3}{(2(2+\sqrt{2})^3}*(\frac{1}{3}+\frac{1}{6}+\frac{1}{2}+\sqrt{2}-\frac{\sqrt{2}}{3}) =$$

$$= \frac{n^3}{(2(2+\sqrt{2}))^3}*(1+\frac{2\sqrt{2}}{3}) =$$

$$n^3(\frac{3+2\sqrt{2}}{3*8*(2+\sqrt{2})^3}) = \frac{n^3}{48}(\frac{3+2\sqrt{2}}{10+7\sqrt{2}}) \approx 163{,}88*n^3 \quad (24)$$

That's how we got the formula for height. We got the formula for height. Graphs of this formula in comparison with the output of the program are shown in Appendix 3.

Smooth sequences by modules of adjacent numbers.

Definition. We will call final sequence $\{a_1,....,a_n\}$ smooth by the modules of adjacent numbers if

$$a_1 = 0, |a_2| \le |a_1+1|, |a_3| \le$$
$$\le |a_2+1|, ..., |a_k| \le |a_{k-1}+1|, ..., |a_n| \le |a_{n-1}+1| \quad (25)$$

Problem. Find the smallest value of the arithmetic mean $a_1,...,a_n$ for a fixed *n*, for any sequence $\{a_1,....,a_n\}$ of smooth by the modules of adjacent numbers.

Theorem 3. For any smooth sequence by the modules of adjacent numbers, the following inequality takes place:

$$\frac{a_1+...+a_n}{n} \ge -\frac{1}{2} \quad (26)$$

Proof. The effective application of plausible arguments plays an essential role in decisions [12].

Squaring inequalities

$$|a_2| \le |a_1+1|, |a_3| \le |a_2+1|, ..., |a_n| \le |a_{n-1}+1|.$$

And equality $a_{n+1} = |a_n+1|$, we get

$$a_1^2 = 0$$
$$a_2^2 \le a_1^2 + 2a_1 + 1,$$
$$a_3^2 \le a_2^2 + 2a_2 + 1$$
$$\dots$$
$$a_n^2 \le a_{n-1}^2 + 2a_{n-1} + 1,$$
$$a_{n+1}^2 \le a_n^2 + 2a_n + 1$$

Adding them, we get:

$$a_1^2 + a_2^2 + a_3^2 + ... + a_n^2 + a_{n+1}^2 \le a_1^2 + 2a_1 + 1 + \\ + a_2^2 + 2a_2 + 1 + ... + a_n^2 + 2a_n + 1 \quad (27)$$

Having given similar terms, we get:

$$a_{n+1}^2 \le 2(a_1 + a_2 + ... + a_n) + n,$$
$$2(a_1 + a_2 + ... + a_n) + n \ge 0,$$
$$2\frac{a_1 + ... + a_n}{n} + 1 \ge 0,$$
$$\frac{a_1 + ... + a_n}{n} \ge \frac{1}{2} \quad (28)$$

The theorem is proved.

Theorem 4. Let $\{a_1, ..., a_n\}$ sequence be smooth by the modules of adjacent numbers. If n is even, then the smallest arithmetic mean $a_1, ..., a_n$ equals $-\frac{1}{2}$

Proof. By virtue of Theorem 3:

$$\frac{a_1 + ... + a_n}{n} \ge -\frac{1}{2}$$

Let us prove that for even n the value $-\frac{1}{2}$ is achieved.

Consider the sequence:

$a_1 = 0, a_2 = -1, a_3 = 0, a_4 = -1, ..., a_n = (-1)^{n-1} - 1$. We get:

$$\frac{a_1 + ... + a_n}{n} = \frac{\frac{n}{2}(-1)}{n} = -\frac{1}{2} \quad (29)$$

The theorem is proved.

Theorem 5. Let $\{a_1, ..., a_n\}$ sequence be smooth by the modules of adjacent numbers, and the numbers $a_1, ..., a_n$ are integral. If n is odd, then the smallest arithmetic mean $a_1, ..., a_n$ equals $-\frac{n-1}{2n}$.

Proof. Let $n = 2k - 1, k = 1, 2, ...$

We get $\begin{matrix} a_1 = 0, a_3 = 0, a_5 = 0, ..., a_{2k-1}. \\ a_2 = -1, a_4 = -1, ..., a_{2k-2} = -1. \end{matrix}$.

Then

$$\frac{a_1 + ... + a_n}{n} = \frac{(-1)(k-1)}{n} = -\frac{\frac{n+1}{2} - 1}{2} = -\frac{n-1}{2n} \quad (30)$$

It should be noted that

$$-\frac{n-1}{2n} > -\frac{1}{2}$$

Indeed,

$$-\frac{n-1}{2n} \succ -\frac{1}{2} \Leftrightarrow \frac{n-1}{n} \succ 1 \Leftrightarrow n-1 \prec n \Leftrightarrow -1 \prec 0 \quad (31)$$

Finally, we should note that the value $-\frac{1}{2}$ for integrals $a_1, ..., a_n$ is not achieved for odd n. Indeed, otherwise, we have:

$$\frac{a_1 + ... + a_n}{n} = -\frac{1}{2}$$

Then

$$2(a_1 + a_2 + ... + a_n) = -n \quad (32)$$

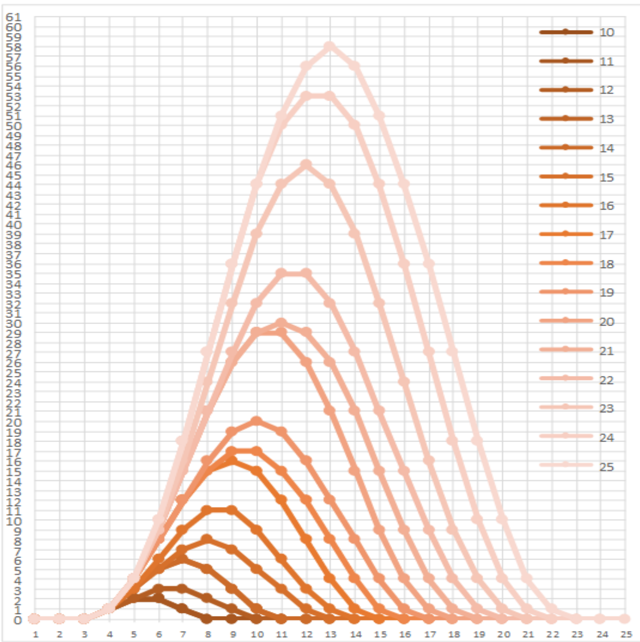
The product on the left is even, and the number on the right is odd, which is impossible.

This method guarantees that the error of restoring the difference of a given order of any sequence from our class does not exceed the calculated error value [22].

IV. CONCLUSION

The main result of the study is that the 5 different theorems were proved. These data can be applied to ballistic sciences and even in spaceship building, since modeling and predicting the motion of physical bodies taking into account the resistance of the medium plays an important role in the development of modern science.

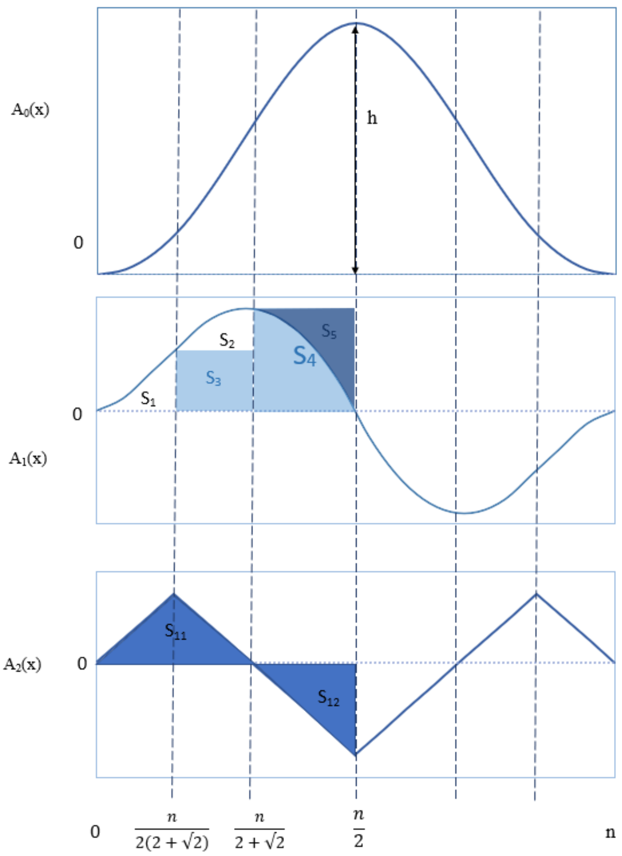
V. APPENDIX



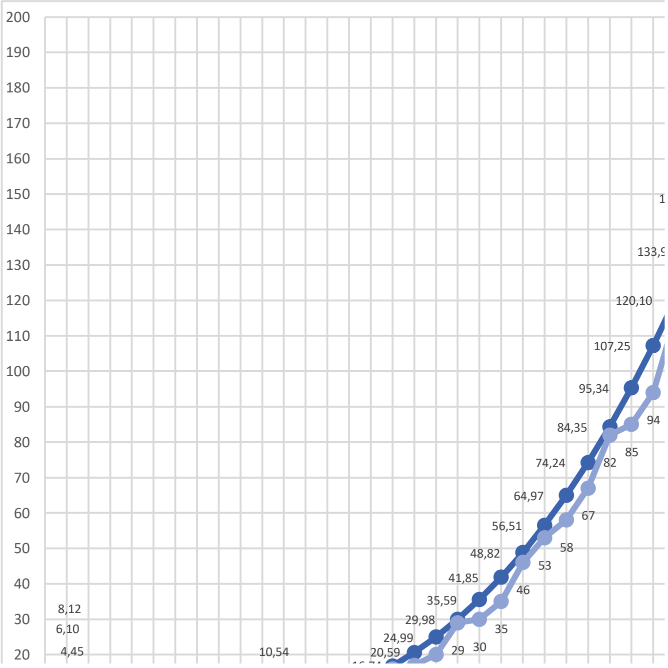
Charts of the third greatest differences of sequences

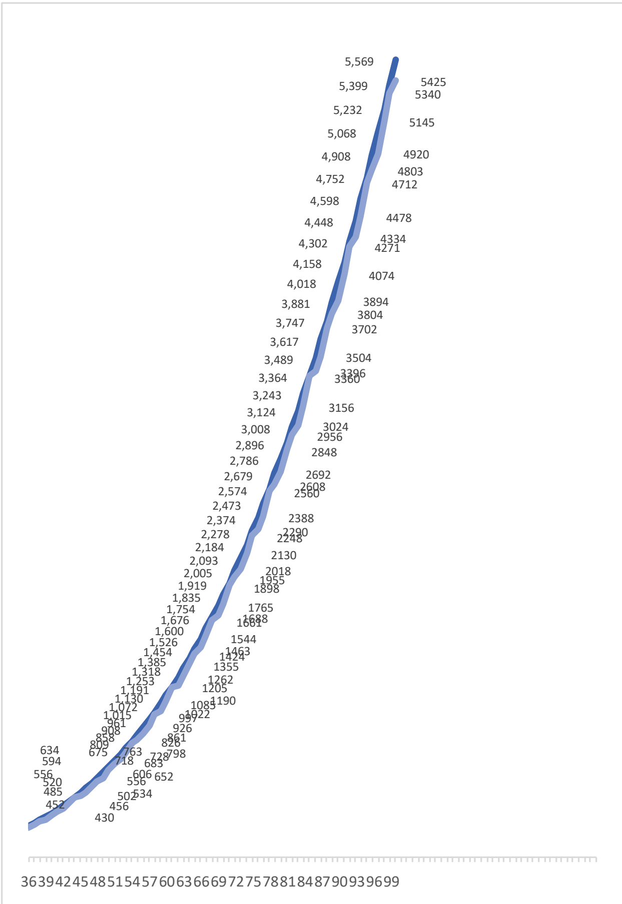
10. 1 -1 -1 0 1 1 -1
11. 1 -1 -1 0 1 1 -1 0
12. 1 -1 0 -1 0 1 0 1 -1
13. 1 0 -1 -1 -1 1 1 1 0 -1
14. 1 0 -1 -1 -1 1 1 1 0 -1 0
15. 1 0 -1 0 -1 -1 1 1 0 1 0 -1
16. 1 0 0 -1 -1 -1 0 1 1 1 0 0 -1
17. 1 1 -1 -1 -1 -1 0 0 1 1 1 1 -1 -1
18. 1 1 -1 -1 -1 0 -1 0 1 0 1 1 1 -1 -1
19. 1 1 -1 -1 0 -1 -1 0 0 1 1 0 1 1 -1 -1
20. 1 1 0 -1 -1 -1 -1 -1 0 1 1 1 1 1 0 -1 -1
21. 1 1 0 -1 -1 -1 -1 0 0 0 0 1 1 1 1 0 -1 -1
22. 1 1 0 -1 -1 0 -1 -1 -1 0 1 1 1 0 1 1 0 -1 -1
23. 1 1 0 0 -1 -1 -1 -1 -1 -1 1 1 1 1 1 1 1 0 0 -1 -1
24. 1 1 1 -1 -1 -1 -1 -1 -1 0 0 0 1 1 1 1 1 1 -1 -1 -1
25. 1 1 1 -1 -1 -1 -1 0 -1 -1 -1 1 1 1 0 1 1 1 1 -1 -1 -1
26. 1 1 1 -1 -1 0 -1 -1 -1 -1 -1 0 1 1 1 1 1 1 0 1 1 -1 -1 -1
27. 1 1 1 0 -1 -1 -1 -1 -1 -1 -1 0 0 1 1 1 1 1 1 1 1 0 -1 -1 -1
28. 1 1 1 0 -1 -1 -1 -1 -1 0 -1 -1 0 1 1 0 1 1 1 1 1 1 0 -1 -1 -1
29. 1 1 1 0 -1 -1 -1 0 -1 -1 -1 -1 0 0 1 1 1 1 0 1 1 1 0 -1 -1 -1
30. 111 0 0 -1 -1 -1 -1 -1 -1 -1 -1 0 1 1 1 1 1 1 1 1 1 1 0 0 -1 -1 -1

Appendix 1. Program output. Charts of the greatest sequences for third difference rows. N is for amount of numbers in the sequence



Appendix 2. Function charts





Appendix 3. Formula for a continuous analogue of smooth sequences in comparison with the program output

Retrieval Number: A1238109119/2019©BEIESP
DOI: 10.35940/ijeat.A1238.109119
Journal Website: www.ijeat.org

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

REFERENCES

1. D. O. Shkljarskij, N. N. Chencov, and I. M. Jaglom. *Izbrannye zadachi i teoremy jelementarnoj matematiki. Chast' 1. Arifmetika i algebra.* Moscow: Fizmatlit, 2001.
2. A. O. Gel'fond. *Ischislenie konechnyh raznostej*. Moscow: Mir, 1972.
3. D. E. Kirk. *Optimal Control Theory: An Introduction*. Mineola: Dover Publications, 2004.
4. S. S. Volosivec, and R. N. Fadeev. Vesovaja integriruemost' summ rjadov po mul'tiplikativnym sistemam, *Izvestiya Saratovskogo universiteta*, 14(2), 2014, pp. 129-136.
5. D. H. Zajnetdinov. Strukturnye svojstva predel'no monotonnoj svodimosti posledovatel'nostej mnozhestv, *Uchenyje zapiski Kazanskogo universiteta*, 160(3), 2018, pp. 517-527.
6. L. E. Portnov. O posledovatel'nostjah monotonnyh funkcij, *Izvestiya TPI*, 131, pp. 108-114.
7. E. B. Dynkin, and V. A. Uspenskij. *Matematicheskie besedy*. Moscow-Leningrad: Gostehizdag, 1952.
8. A. M. Jaglom, and I. M. Jaglom. *Neelementarnye zadachi v elementarnom izlozhenii*. Moscow: Gostehizdat, 1954.
9. N. Vasil'ev, and A. Tolpygo. Plavnye posledovatel'nosti, *Kvant*, 6, 1977, p. 30.
10. V. I. Arnold. O predstavlenii funkcii neskol'kih peremennyh v vide superpositsii funktsij menshego chisla peremennykh, *Matematika, eyo prepodavanije, prilozhenija i istorija*, 2(3), 1958, pp. 41-61.
11. D. Poja. *Matematika i pravdopodobnye rassuzhdenija*. Moscow: Nauka, 1975.
12. A. A. Buhshtab. *Teorija chisel*. Moscow: Prosveshhenie, 1966.
13. I. M. Vinogradov. Osnovy teorii chisel. Moscow: Nauka, 1965.
14. A. V. Birjukov. Fraktal'naja razmernost' posledovatel'nostej, *Prikladnaja matematika, Vestnik Kuzbasskogo gosudarstvennogo tehnicheskogo universiteta*, 2006, p. 29.
15. S. K. Vodop'janov, and A. O. Molchanova. Teorema Stoksa dlja differencial'nyh form proizvol'noj summiruemosti, *Vestnik Kemerovskogo gosudarstvennogo universiteta*, 3(1), 2011, p. 239-243.
16. O. V. Filatov. Raschjot chislennostej poiskovyh shablonov v paradokse Penni, *Problemy sovremennoj nauki i obrazovanija*, 11(41), 2015, pp. 40-50.
17. S. V. Dronov. O svojstve fundamental'nosti segmentov klassa natural'nyh chisel, *Izvestija Altajskogo gosudarstvennogo universiteta*, 2009, pp. 46-49.
18. A. V. Birjukov. K analizu arifmeticheskogo *haosa, Vestnik Kuzbasskogo gosudarstvennogo tehnicheskogo universiteta*, 2003, pp. 3-4.
19. S. A. Inozemcev, and K. V. Chasov. Issledovanie posledovatel'nostej v interaktivnom obuchajushhem dokumente, *Mezhdunarodnyy studencheskiy nauchnyy vestnik*, 4(6), pp. 837-841.
20. S. S. Chudova. Optimal'noe vosstanovlenie raznostej posledovatel'nostej po netochnoj informacii, *Vestnik Rossijskogo universiteta druzhby narodov. Serija: Matematika, informatika, fizika*, 3, 2009, pp. 5-11.
21. S. M. Nikol'skij. Elementy matematicheskogo analiza. Moscow, Nauka, 1981.
22. N. S. Piskunov. *Differencial'noe i integral'noe ischislenie*. Moscow: Nauka, 1985.
23. V. L. Lukashov. O raznosti mezhdu posledovatel'nymi prostymi chislami, *Zhurnal nauchnykh publikatsiy aspirantov i doktorantov*, 4(106), 2015, pp. 134-138.
24. R. M. Aron, J. A. Jaramillo, and T. Ransford. Smooth surjections without surjective restrictions, *Journal of Geometric Analysis*, 23(4), 2013, p. 2081.
25. R. M. Aron, J. A. Jaramillo, and E. le Donne. Smooth surjetction and surjective restrictions, *Annales Academiæ Scientiarum Fennicæ Mathematica*, 42, 2017, pp. 525-534.
26. H. M.. Le, A. Kang, Y. Yue, and P. Carr. Smooth imitation for online sequence prediction, in *ICML'16 Proceedings of the 33rd International Conference on International Conference on Machine Learning*, 48, 2016, pp. 680-688.
27. T. Kania, and M. Rmoutil. Restricting uniformly open surjectionsRestrictions des surjections uniformément ouvertes, *Comptes Rendus Mathematique*, 355(9), 2017, pp. 925-928.
28. A. K. Supriatha, E. Carnia, and M. Z. Ndii. Fibonacci numbers: A population dynamics perspective, *Heliyon*, 5(1), 2019, e01130.
29. A. G. Polyakova, M. P. Lpginov, E. V. Stelnikov, and N. V. Usova. Managerial decision support algorithm based on network analysis and big data. International, *Journal of Civil Engineering and Technology*, 10(2), pp. 291-300.