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# Minimization of Bit Error Rate in Polar Codes for Achieving Channel Capacity

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***Abstract- Polar codes are designed to achieve Shannon's theoretical limit for channel capacity with low complexity constructive approach. polar codes invented by E Arikan with the exceptional phenomenon by considering the generator matrix instead of parity bits with the information bits. As the block length N increases the sequential decoding paths are increases this may cause a reduction in SNR and increases the BER, this will occupy more channel bandwidth and consumes more power to transmit the signal. To notice the issues, we proposed a more constructive approach for error-free polar codes design up to 6 Gbps with proposed priority enabled reliability sequence (PERS), bit channels and CRC aided polar codes. This approach outperforms as compared with earlier ones.***

***Keywords: Bit error rate, polar codes, SNR, polarization, channel capacity, CRC, PERS.***

## I. INTRODUCTION

Polar code construction for recursive approach by considering generator matrix $G_N$ where information bits and generator matrix are together generates the code words of different block length.

$$G_N = F_N = F_2^{\otimes \log_2 N} \quad (1)$$

Where $F_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

For N=4 $G_N$ is

$$G_4 = F_2^{\otimes 2} = \begin{bmatrix} F_2 & 0 \\ F_2 & F_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Non-systematic Polar Code Encoding as

$$x = uG_N \quad (2)$$

Where $G_N = B_N F_N$, $B_N$ is bit-reversal permutation matrix
Information rate for binary symmetric channel is

$$I(W^-) + I(W^+) = I(U_1U_2; Y_1Y_2) = I(X_1; Y_1) + I(X_2; Y_2) = 2I(W) \quad (3)$$

Where 'W' is Channel and 'I' is information It holds

$$I(W^-) \leq I(W) \leq I(W^+) \quad (4)$$

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For N=4 block length polar code construction is shown in figure.1. where $U_0, \ldots U_3$ are information bits $y_0 \ldots . y_3$ are output bits $V_0 \ldots . V_3$ are the modulo 2 bits and $x_0, x_1 \ldots . . x_7$ are channel inputs to W.
In figure .1. the polar code constructed for four bits block length in which input and output bits are equal but the bits at output are modulo two operational bits these are known as synthetic channel inputs can be applied for three types of channels known as BEC,BSC and AWGN and these sequential source coding of output bits are given to channel source which could face several hardships as noise, interference etc. to maintain SNR for a desired output from the channel coding applications which can be received as input to the receiver[1]. The output of the receiver could give accurate, reliable information same as the input bits. To achieve channel capacity[3], in this work varied levels are applied for low power which is slightly more than the noise signal, the purpose of this is to transmit very low power from the transmitting side, even such case information transferred got better results, over the noisy channels, because of this method we could able to save power and that can be used for other applications or can be used same device for longer time.

## II. POLARIZATION

The bit channels took in this work are used to synthesize the polarized channels i.e. good and bad channels. Each bit channels carried a sequence of codewords for length equal to N. The bit channels are constructed in a sequence equal to block length N, and the encoded bits will further be polarized based on "priority enabled reliability sequence (PERS)" to categorize which are bad and good channels. Based on the reliability sequence few channels are active and few channels are placed as inactive due to the frozen bits [2] in this work bad channels considered as Frozen paths and these are in-active paths. The proposed methodology for attaining channel capacity is possible with help of polarized bit channels [14]. Which given accurate information at decoder output up to 6 Gbps [10].

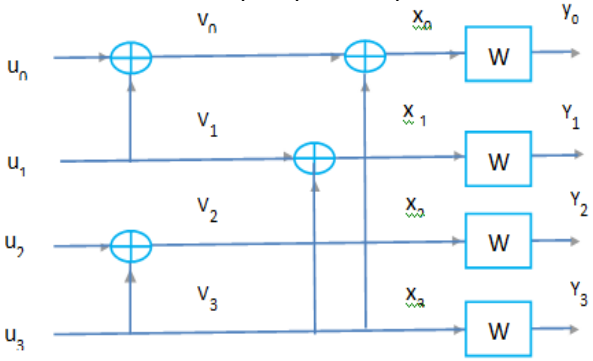


**Figure.1. Polar code construction for N = 4**

The block length in this work is n=10 each block again creates 1024-bit channels these bit channels can send in each sequence 100 to 1000's of bits which could process large amount of data for multiple applications.
When compared with existing codes this work achieved better performance [3].

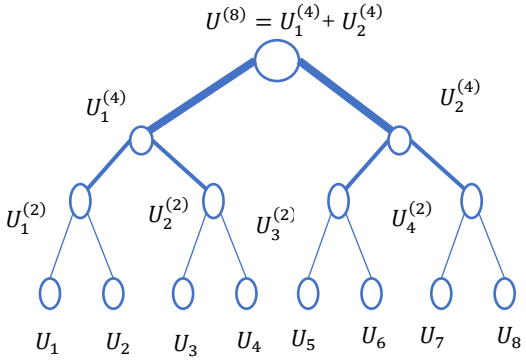


**Figure.2. Polar code tree construction structure for N = 8**

In figure 2 proposed tree construction for (8,4) polar code input information bits are noted as $U_1, U_2, .. U_8$ at depth of the tree. Top of the tree same 8 bits are achieved with modulo operation from each twigs of the tree node.

**Table.1. Latency and power comparison [12-13]**

| Decoder Name | Block length N | Rate | Latency | Mbps |
|---|---|---|---|---|
| Improved SC decoding | 1024 | $1/2$ | 24 | 14 |
| SCL and SCS | 1024 | $1/2$ | 17 | 21 |
| Hybrid Decoding | 1024 | $1/2$ | 27 | 12 |
| CRC-Aided Decoding | 1024 | $1/2$ | 18 | 19 |
| Belief Propagation Decoding | 1024 | $1/2$ | 19 | 16 |
| MAP decoding | 1024 | $1/2$ | 21 | 12 |
| CRC + SC List (This work) | 1024 | $1/2$ | 15 | 24 |

From table.2.the proposed Polar code CRC plus SC list decoder obtained lowest latency compared to existing decoders in literature, which indicates minimum latency faster is the data computational space time. The proposed decoder has given highest data processed through the channel proposed which obtained 24 Mbps, for a block length N=1024, and each channel block sent variations in input bits for computing experimental values such as number of bits in errors, channel capacity, latency and information transferred for various block length.

$$G_8 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^{\otimes 3}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$[U_1 \ U_2 \ U_3 \ U_4 \ U_5 \ U_6 \ U_7 \ U_8 \ ]G_4$$
$$= [U_1 \ + U_2 + \ U_3 \ + U_4 \ + U_5 \ + U_6 \ + U_7 + U_8 \ , U_2 + U_4 \ + U_6 \ + U_8 \ , U_3 + U_4 + U_7 + U_8 \ , U_4 + U_8 \ , U_5 + U_6 + U_7 + U_8 \ , U_6 + U_8 \ , \ U_7 + U_8 \ , \ U_8 \ ] \quad (5)$$

## III. BIT CHANNELS

Bit channels are used to transfer the information bits for BPSK modulated polarized channels.

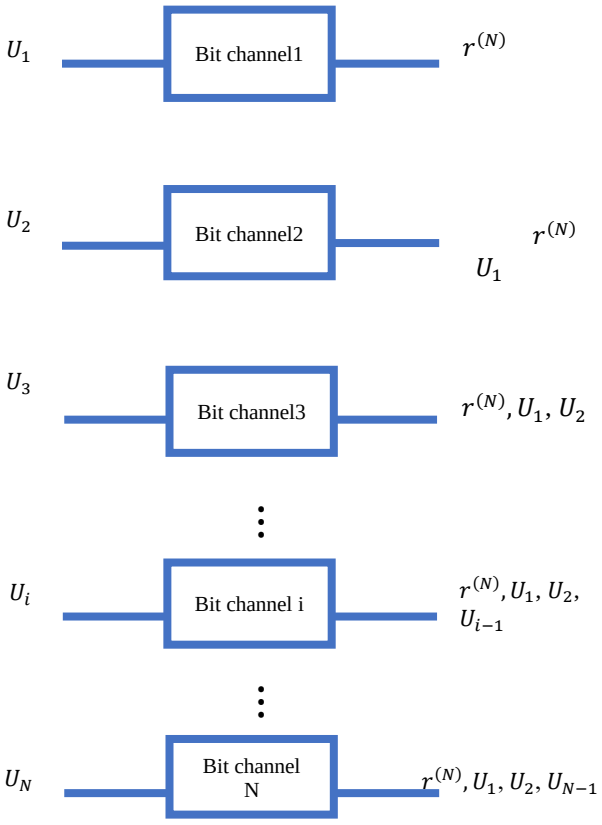


**Figure.3. Bit Channels for polarization**

The bit channels are mapped using BPSK signals 0's as 1 and 1's as -1. All information bits are mapped and performance evaluated for each decoder for better reliable [15] performance.

Reliability sequence [5] for N=8 is 1 2 3 5 4 6 7 8, reliability sequence for N=16 is 1 2 3 5 9 4 6 10 7 11 13 8 12 14 15 16, reliability sequence for N=32 is 1 2 3 5 9 17 4 6 10 7 18 11 19 13 21 25 8 12 20 14 15 22 27 26 23 29 16 24 28 30 31 32 similarly we got for N=1024.

**(A) (8,4) Polar Code**

Where N = 8 and K = 4 with generator matrix 8 input bits and 8 output bits formed in tree structure given, top node is depth 0 and bottom node is depth 3 for block length eight. The bit incoming from depth3 to depth2 are added and similarly added at top node where we could get 8 bits as output bits. At each node incoming bits are information bits or frozen bits [6] are added to successive node computation incoming bits. [ 8,9]

## IV. CRC AIDED POLAR CODE DECODERS

The reliability of a SC decoding rate [7] path $v_1^i$ can be measured using APP

$$P_N^{(i)}\left(v_1^i/y_1^N\right) = \frac{W_N^{(i)}\left(y_1^N,\ v_1^{i-1}/v_i\right)}{2P(y_1^N)} \quad (6)$$

$$\sum_{v_1^i \varepsilon \{0,1\}^i} P_N^{(i)}\left(v_1^i/y_1^N\right) = 1 \quad (7)$$

Belief Propagation Decoding

$$\mathrm{LLR(x)} = \log\frac{\mathrm{P}(\mathrm{x}=0/\mathrm{y})}{\mathrm{P}(\mathrm{x}=1/\mathrm{y})} \quad (8)$$

The log-likelihood ratio (LLR) of a binary variable is

$$L(x) = \log\left(\frac{P(x=0)}{P(x=1)}\right) = \log(P(x=0)) - \log(P(x=1))$$

If $P(x=0) > P(x=1), L(x) >$ 0 *and a larger* variation results in a larger LLR

If $P(x=1) > P(x=0), L(x) <$ 0 *and a larger* variation results in a larger(negative) LLR. [ 11-12]

$$Z(W) = \sum_y \sqrt{W(y/0)W(y/1)} \quad (9)$$

$$U^{(8)} = U_1^{(4)} + U_2^{(4)}$$

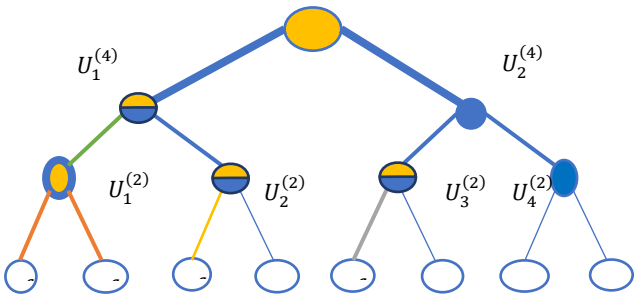


**Figure.4. (8, 4) Polar Codes with message 'm' and frozen 'f' bits**

**Table.2. Comparison of work with existing method**

| Algorithm | Technology ??? | ?? | Code | Latency ??? | Power(mw) |
|---|---|---|---|---|---|
| CRC+PC (Existing) | 45 | 1024 | (1024, 512) | 1.0 | 214 |
| CRC +List+PC (this work) | 45 | 1024 | (1024, 512) | 0.65 | 191 |

In this proposed work latency and power utilized achieved is better when compared with the CRC plus polar codes, which given 191 mw power as compared with existing 214 mw.

## V. RESULT AND ANALYSIS

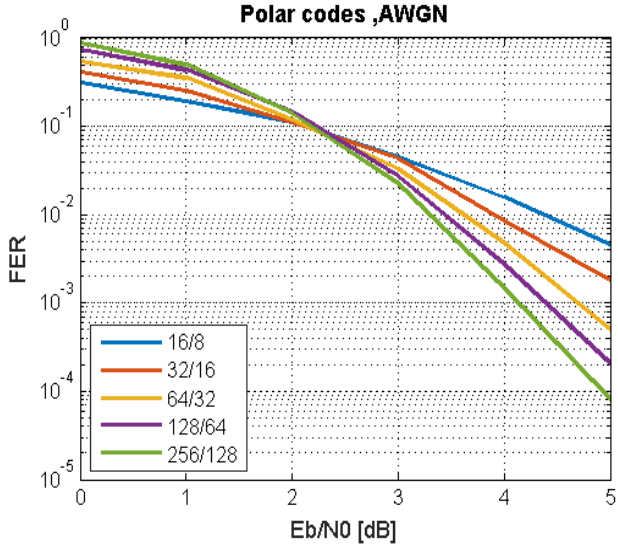


**Figure.5. FER and *E_b / N_0* on AWGN Channel**

**In figure 5. The frame error rate (FER) are presented.**

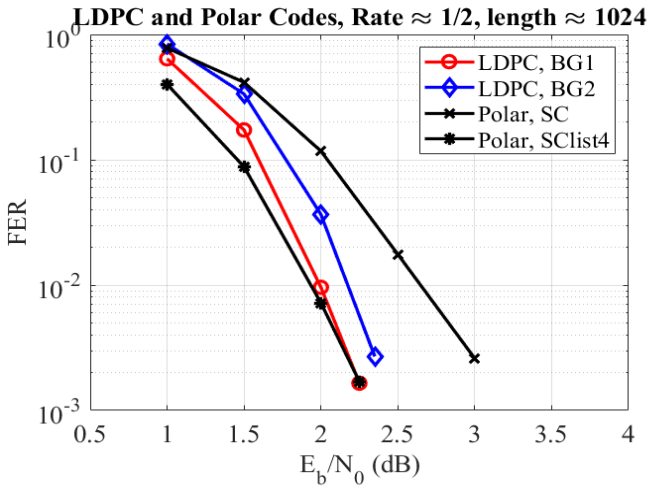


**Figure. 6. Polar Code and LDPC code performance**

In figure.6. In this figure.6. LDPC and polar codes comparison, Polar codes performed better than LDPC.

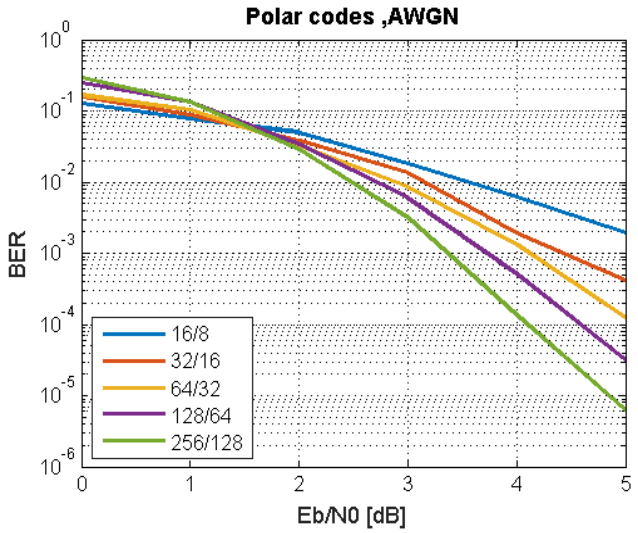


**Figure.7. BER and $E\_b / N\_0$ on AWGN Channel**

In figure .7. more the block length performance is better,256/128 code rate gives better SNR and lower BER.

## VI.CONCLUSION

Polar code decoders performed better than LDPC codes in terms of BER, transmission power is minimized and latency made at lower level for faster data transfer applications. Data processing rate is increased than the existing algorithms 1Gbps in this work it achieved 6Gbps, latency minimized to 0.65 $\mu s$ from existing 1 $\mu s$ and power reduced to 191 mw in our work from 214 mw.

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