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Timing Offset Correction and Channel Tap Estimation of OFDM Systems Over Frequency Selective Fading Channels

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Abstract: *In OFDM systems we need to perfectly time synchronize the receiver to the transmitter to maximize its BER performance. In the absence of timing synchronization the channel estimation can never be optimum. Both timing synchronization and optimum channel estimation are the essential requirements to maximize the performance. We present an algorithm that uses the cyclic prefix to determine the timing offset of the system. The same algorithm can be used to determine the number of channel taps and their variances also. Determination of number of channel taps helps us in minimizing the length of the cyclic prefix to be added and also in the channel estimation, to minimize the variance of the estimation error. The knowledge of the variances of the channel taps helps us in further reducing the estimation error. We present in this paper the timing offset corrected MMSE channel estimation and show by simulations the effectiveness of the method presented.*

Keywords: *OFDM, MMSE channel tap estimation, timing Offset, frequency selective channel.*

I. INTRODUCTION

Orthogonal frequency division multiplexing being the most efficient bandwidth conservative scheme, it is the basic frame work on which all modern communication systems are developed. To maximize the performance of OFDM based systems namely Orthogonal frequency division multiple access (OFDMA) and Single carrier frequency division multiple access (SCFDMA), we need to time synchronize the receiver to the received signal and estimate the channel with minimum possible error variance. Any non-zero timing offset reduces the BER performance of the system [1]-[4]. The determination of timing offset of the system which is randomly sampled is important to correct the timing offset and hence achieve timing synchronization.

Timing offset determination and elimination of inter-block interference caused due to a multipath channel requires cyclic prefix. But addition of cyclic prefix reduces the throughput of the system for a given bandwidth. To maximize the throughput of the system we need to add only the minimum required length which is equal to the channel delay spread. The channel delay spread depends on the number of channel taps. So determination of the number of channel taps in a given system becomes important. In this work we present the algorithm to determine the number of channel taps and estimate the timing offset of the system. Any channel estimation procedure assumes perfect timing synchronization. The estimated channel frequency response gives the stipulated BER performance only if the system is perfectly time synchronized. Any non-zero timing offset will reduce the BER performance of the system. We present in this work the method to determine the timing offset of the system so that it can be corrected. In [1]-[4], the effect of timing offset in multiuser OFDM based systems OFDMA and SCFDMA on the signal to interference ratio and the BER performance is presented in a detailed manner. In these papers ultimately the interference canceller is presented to improve the performance of the system. The interference cancellation requires the estimate of timing offset, number of channel taps and the channel frequency response. If the timing offset is more than a specific value each symbol will have interference from the other symbols of the current and the adjacent frames which makes the channel estimation to have more error variance. Methods present in the literature [5]-[8] on timing synchronization and channel estimation consider either timing synchronization or channel estimation. In [9], both are jointly considered and ML estimation is used. In pilot based channel estimation, knowing the number of channel taps we can fix the number of pilot symbols to be put, to reduce the error variance to the required value. More over the knowledge of the variance of the channel taps, we can further reduce the variance of the estimation error. In this case, as given in [10]-[11] we can use MMSE estimation which is most optimum. The rest of the work is organized as follows. Section II describes the system considered and the effect of timing offset, and Section III details the proposed algorithm to determine the number of channel taps, their variances and the timing offset. Pilot channel estimation is presented in Section IV. Simulation results are discussed in Section V, and conclusions are given in Section VI.

Notation: Vectors are denoted in bold-face letters. If z is a complex number, then z^* denotes its complex conjugate. If $\mathbf{x}$ is any vector with complex entries, then $\mathbf{x}^H$ denotes the vector with its entries being conjugates of those in $\mathbf{x}$. If X is a random variable, then $\mathbb{E}(X)$ and $var(X)$ denote the expectation and variance of X, respectively.

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If $\mathbf{x}$ and $\mathbf{y}$ are any two sequences of length N, then we denote the N-point circular convolution of $\mathbf{x}$ and $\mathbf{y}$ by $\mathbf{x} * \mathbf{y}$.

II. SYSTEM MODEL

Consider an OFDM system with N sub-carriers, transmitting the symbol frame independent symbols $\mathbf{X} = [X_0, X_1, \cdots, X_{N-1}]$. The output of the N-point IDFT unit in the transmitter is

$$x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(K) e^{\frac{j2\pi Kn}{N}}, \qquad 0 \le n \le N-1. \qquad (1)$$

After adding the cyclic prefix of N_g samples, the transmitted symbol frame is

$$x_t(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(K) e^{\frac{j2\pi Kn}{N}}, \quad -N_c \le n \le N-1. \qquad (2)$$

In this, with the normalized average power of each symbol X[k] to be unity, the average power of each sample x(n) is $\frac{1}{N}$. Since N-point IDFT is unitary transformation, the samples of x(n) are also independent.

Let $\mathbf{h} = [h_0, h_1, \ldots, h_{L-1}]$ be the L-tap Rayleigh fading channel impulse response, with each channel tap being i.i.d complex Gaussian with zero mean. Assuming perfect timing and frequency synchronization, the received signal after dropping the cyclic prefix can be represented as the N-point circular convolution of $\mathbf{x}$ and $\mathbf{h}$, given by

$$\mathbf{y} = \mathbf{x} * \mathbf{h} + \mathbf{z}, \qquad (3)$$

Where $*$ indicates the N-point circular convolution and $\mathbf{z}$ is the i.i.d additive white Gaussian noise (AWGN) with each entry having zero mean and variance σ_N^2. At the receiver, the K-th element of the N-point DFT unit is

$$Y(K) = X(K)H(K) + Z(K), \qquad K = 1, \ldots, N, \qquad (4)$$

where $H(K)$ and $X(K)$ are K-th elements of the N-point DFT of channel impulse response $\mathbf{h}$ and the data vector $\mathbf{x}$, respectively. Further, $Z(K)$ is the K-th element of the N-point From DFT of the noise vector $\mathbf{z}$. From (4), it

can be seen that, the symbol on each sub-carrier experiences flat fading, even though the channel is frequency selective. To achieve good BER performance, it is required to know the frequency response $\mathbf{H} = [H(1), \ldots, H(N)]$ and detect the symbols using the matched filter detector. The output of the matched filter detector is

$$R(K) = Y(K)H^*(K) \forall K = 1, \ldots, N. \qquad (5)$$

If the channel frequency response is known perfectly, the systems using BPSK, QPSK and 16-QAM modulation for different SNRs give standard maximum performances.

A. Effect of Timing Offset

If there is a timing offset in the system due to which the frame of samples selected are from $-n0$ to $-n0 + N - 1$ as shown in Fig. 1 corresponding to the frame arriving at the receiver through the first path provided the minimum length of cyclic prefix is $L - 1 + T0$ the received signal frame is either just the circularly shifted version or the circularly shifted version with interference from the adjacent frames [1]-[4]. The DFT output in that case is given by

$$Y[K] = X[K] e^{\frac{j2\pi K n_t}{N}} H[K] + I + Z \qquad (6)$$

If we multiply Y[K] with $H^*[K]$, the output contains phase rotated symbols added with noise and interference. If this is given as the input to the decision making device, the probability of error will be very high. So we should achieve perfect timing synchronization. Timing synchronization is achieved in two stages, first we determine the number of channel taps, adjust the cyclic prefix and time synchronize the system.

III. DETERMINATION OF NUMBER OF CHANNEL TAPS

Case 1: In this case we consider the system in which the number of channel taps L and the length of cyclic prefix N_c are such that $L \le N_c + 1$. When a frame of $N + N_c$ samples is transmitted through an L tap multi-path channel, the received sequence consists of $N + N_c + L$ samples as shown in the figure 1. In this the first and last $N_c + L - 1$ samples obtained by linearly convolving the cyclic prefix with the channel impulse response are correlated. For example assuming the channel output is sampled from the first sample in the cyclic prefix, for the system with $N_c = 3$ as the cyclic prefix and 3 channel taps, with

$$\begin{aligned}
S(0) &= h(0)x(N-3) \\
S(1) &= h(1)x(N-3) + h(0)x(N-2) \\
S(2) &= h(2)x(N-3) + h(1)x(N-2) + h(0)x(N-1) \\
S(3) &= h(2)x(N-2) + h(1)x(N-1) \\
S(4) &= h(2)x(N-1)
\end{aligned} \qquad (7)$$

The first 5 samples of the received sequence y(n) are

$$\begin{aligned}
y(0) &= S(0) + I_p(0) + Z(0) \\
y(1) &= S(1) + I_p(1) + Z(1) \\
y(2) &= S(2) + Z(2) \\
y(3) &= S(3) + I_C(3) + Z(3) \\
y(4) &= S(4) + I_C(4) + Z(4),
\end{aligned} \qquad (8)$$

and the last 5 samples are

$$\begin{aligned}
y(N-N_c) &= S(0) + I_C(N-N_C) + Z(N-N_C) \\
y(N-N_c+1) &= S(1) + I_C(N-N_c+1) + Z(N-N_c+1) \\
y(N-N_c+2) &= S(2) + Z(N-N_c+2) \\
y(N-N_c+3) &= S(3) + I_N(N-N_c+3) + Z(N-N_c+3) \\
y(N-N_c+4) &= S(4) + I_N(N-N_c+4) + Z(N-N_c+4),
\end{aligned} \qquad (9)$$

The values of these samples show the way in which the samples of y(n) are correlated through S(n). Using this correlation the number of channel taps can be determined, following the procedure given below.

Step-1: Initialize all zero vector of length N given by C and i=0.

Step-2: Starting at any random sample, $y(n_o)$ compute i^{th}

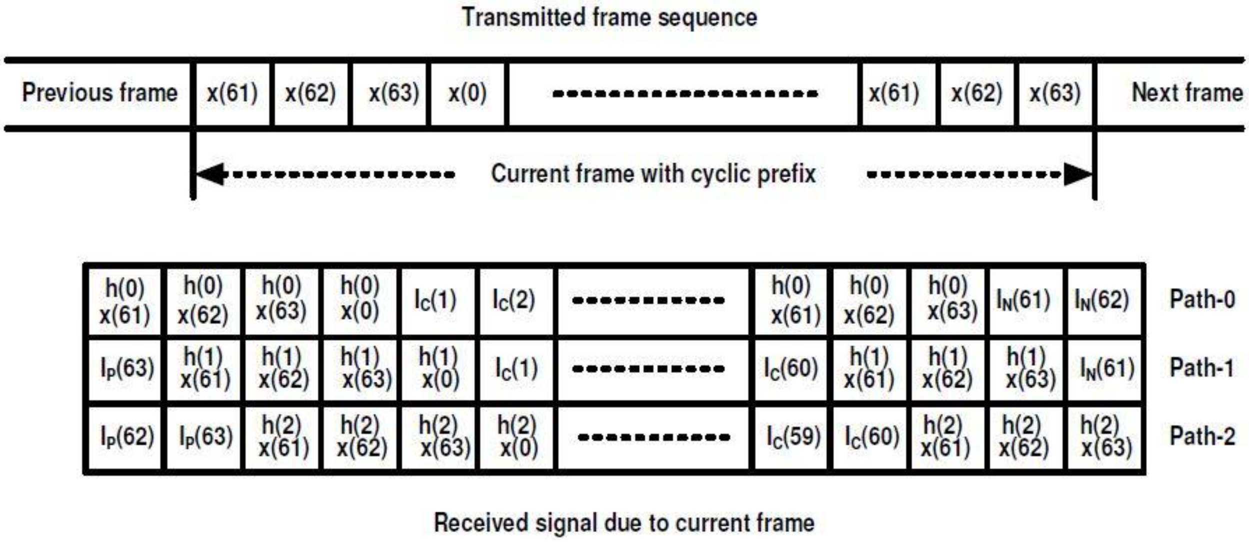

Fig.1 Received signal in the case of an OFDM system in Rayleigh fading channel, with $N = 64$, $N_c = 3$, $L = 3$, and BPSK modulation.

correlated frame of N samples $C(i)$

$$c_i(n) = y(n_o + iN + n) \times y^*(n_o + i(N+1) + n) \forall 0 \le n \le N-1, \quad (10)$$

where * represents complex conjugation and N is the number of sub carriers in the system.

Step-3: Using C(i), compute

$$C = C + C(i) \quad (11)$$

Increment i and repeat the steps 2 and 3, I-1 times where I is large.

Step-4: Compute $C_r = \frac{C}{I}$

In each of C_i, only $N_c + L - 1$ samples have non zero mean and all the other samples are of zero mean. depending on where we start sampling the channel (with or without timing offset), the samples with non-zero mean in addition contain noise term only or noise plus interference from the samples of previous or present or next frame as shown in the figure, for the example considered, given the received signal is sampled from a sample $n_x - 1$ samples before the first sample of a frame the values of $c_i(n_x)$ are

$$y_c(n_x) = |x(N-3)h(0)|^2 + I_0 + Z_0$$
$$y_c(n_x+1) = |x(N-2)h(0)|^2 + |x(N-3)h(1)|^2 + I_1 + Z_1$$
$$y_c(n_x+2) = |x(N-1)h(0)|^2 + |x(N-2)h(1)|^2 + |x(N-3)h(2)|^2 + I_2 + Z_2$$
$$y_c(n_x+3) = |x(N-2)h(2)|^2 + |x(N-1)h(1)|^2 + I_3 + Z_3$$
$$y_c(n_x+4) = |x(N-1)h(2)|^2 + I_4 + Z_4 \quad (12)$$

With P_x as the average power of the samples of x(n) and $\sigma_h^2(i)$ as the variance of i^{th} channel tap, the means of the samples of $y_c(n)$, which are the non-zero values of C_r are

$$C_r(n_x) = P_x \sigma_h^2(0)$$
$$C_r(n_x+1) = P_x(\sigma_h^2(0) + \sigma_h^2(1))$$
$$C_r(n_x+2) = P_x(\sigma_h^2(0) + \sigma_h^2(1) + \sigma_h^2(2))$$
$$C_r(n_x+3) = P_x(\sigma_h^2(1) + \sigma_h^2(2))$$
$$C_r(n_x+4) = P_x \sigma_h^2(2) \quad (13)$$

Simulations presented in this work support this, using which we can determine the number of channel taps.

Case 2: Consider the number of channel taps $L > N_c + 1$, then the number of correlated samples will remain at $N_c + L - 1$. In this case among the first and last $N_c + L - 1$ correlated samples, $L - 1 - N_c$ middle samples are of same maximum mean, the simulations also show this.

Case 3: If the number of channel taps is less than the N_c, $N_c - 1 - L$ middle samples are of same correlation. In any case looking at the samples of C_r it is possible to determine the number of channel taps. It is also possible to approximately determine the variances of the channel taps.

A. Variances of channel taps

Let the first L average correlation samples corresponding to the first $N_c + L - 1$ in the case of $L \le N_c$ be $C_r(n_x)$ to $C_r(n_x + L - 1)$, from (13), we can see that the variance of i^{th} channel tap is calculated using

$$\sigma_h^2(0) = \frac{C_r(n_x)}{P_x} \quad (14)$$

$$\sigma_h^2(i) = \frac{C_r(n_x+i) - C_r(n_x+i-1)}{P_x} \forall 1 \le i \le L-1$$

The number of nonzero samples in this is equal to $N_c + L$,

B. Timing Offset Estimation

In any frame, the correlation of the sample with index n=-1 is maximum. Irrespective of where we start sampling in a given frame, if we compute the vector C_{rd} , in which $C_{rd}(1) = C_r(1)$ and $C_{rd}(i) = C_r(i) - C_r(i-1) \forall i > 1$. In this the sample at which C_{rd} is negative corresponds to the first sample of the frame and the index of the sample gives the timing offset.

IV. PILOT BASED CHANNEL TAP ESTIMATION

Consider a multi path frequency selective channel with L channel taps. With L tap channel, the number of pilot symbols M should atleast be L. In this paper the channel taps are first estimated and by taking the DFT of the channel taps the frequency response coefficients are determined. Here given the timing offset of the system is known, if it is small so that the frequency response coefficients are only phase rotated with no interference term I in (6), the estimation is carried out after phase equalization. If the timing offset is more so that even the I term is present, then the timing offset is corrected and the channel taps are estimated.

Case 1: Perfectly time synchronized system with variances of the channel taps unknown: In this case from (4), with X(K)=1 on the set of pilot sub carriers, with indices Q_i for $0 \le i \le M$, the DFT outputs after phase equalization or timing offset correction are

$$Y(Q_i) = H(Q_i) + Z(Q_i), \tag{15}$$

where, with $H(K)$ is the N point DFT of the channel impulse response, $H(Q_i)$ are frequency responses of the channel for the sub carriers with index Q_i . Considering $M \ge L$ pilot sub carriers, with Y_Q as the vector of noise corrupted channel frequency responses and h as the channel impulse response, we model it as

$$Y_Q = D_P h + Z, \tag{16}$$

where Z is the partial noise vector and D_p is the partial DFT matrix of order $M \times L$ in which i^{th} row is $e^{\frac{-j2\pi Q_i n}{N}}$ for $0 \le n \le L-1$. Here with $\frac{N}{M}$ an integer, the indices of the pilot carriers are selected as

$$Q_i = K_0 + \frac{N}{M} i. \tag{17}$$

This ensures that the columns of D_P are orthogonal, which is essential for the optimum estimation of the channel impulse response. With this given that the covariance matrix of complex Gaussian noise vector Z is $C_{L \times L}$, the most efficient estimate of the channel impulse response given $()^H$
the Hermitian operation, is [10]

$$\hat{h} = (D_P^H C^{-1} D_P)^H H^H C^{-1} Y_Q \tag{18}$$

and the variances of the estimation error, given σ_n^2 as the variance of each noise sample, are the diagonal elements of

$$D_P^H C^{-1} D_P = \frac{\sigma_n^2}{M} I_{M \times M} \tag{19}$$

This says that knowing the number of channel taps L, allows us to select M, and increasing M reduces the variance of estimation error. The channel frequency response, given $D_{N \times N}$ the N-point DFT matrix is

$$\hat{H} = D\hat{h} \tag{20}$$

and the variance of estimation error of each $H(k)$ is $\frac{L\sigma_n^2}{M}$

Case 2: Perfectly time synchronized system with the knowledge of variances of the channel taps Given the variances of the channel taps $\sigma_h^2(i)$, the channel tap co-variance matrix is

$$R_h = \mathbb{E}(hh^H), \tag{21}$$

the noise co-variance matrix

$$R_z = \mathbb{E}(ZZ^H), \tag{22}$$

the MMSE estimate of the channel taps is as given in [11]

$$\hat{h} = R_h D_P^H [R_z + D_P R_h D_P^H]^{-1} Y_Q, \tag{23}$$

this gives the minimum mean square error with

$$MMSE = [R_h^{-1} + D_P^H R_z^{-1} D_P^H]^{-1}. \tag{24}$$

if σ_h^2 is the variance of each channel tap and σ_n^2 the variance of noise samples, the mean square error in estimation becomes $\dfrac{\sigma_n^2}{M + \dfrac{\sigma_n^2}{\sigma_h^2}}$ which is less than $\dfrac{\sigma_n^2}{M}$ which is achieved if only the number of channel taps is known. In this case we can see that the required variance of the estimation error can be obtained using lesser number of pilots, if we know the variance of the channel taps also.

V. RESULTS AND DISCUSSION

This section presents the simulation results to determine the number of channel taps, the amount of timing offset and the channel tap estimation in the presence of noise. In these simulations we consider 64 sub carrier OFDM with $N_c = 6$ and BPSK modulation. In all the simulations we took a timing offset of 35 samples. In each case we considered the channel taps to be of equal variance and the sum of variances of all the channel taps is equal to one. Figure (2) gives the average correlation of the samples of the received OFDM signal with 6 tap Rayleigh fading channel with each channel tap of variance $\frac{1}{6}$.With variance of each sample of x(n) being $\frac{1}{64}$, as expected the mean correlation of the first sample in cyclic prefix is $\frac{1}{6\times 64} = 2.6\times 10^{-3}$ and the next sample correlations are integer multiples of this value, reaching the value of $6\times\frac{1}{6\times 64} = 15.6\times 10^{-3}$., later they are decreasing. It can be observed that, the average correlation has $N_g + L - 1 = 11$ significant values in this case.

Figure (3) gives the graph of variances of the channel taps in the system with $N_c = 6$ and $L = 2$. The samples are computed using (15). It can be seen that each channel tap is of variance is close to 0.5, which we considered. In this case timing offset is the index of first sample that becomes negative which is 35 here.

Figure (4) gives the variance plot with $N_c = 6$ and $L = 12$. The samples are computed using (15). In this case we can see that it does not give the variances of all the taps but it says that the variance of each tap is close to $\frac{1}{12}$. In this case timing offset is the index of first sample that becomes zero which is also 35 here.

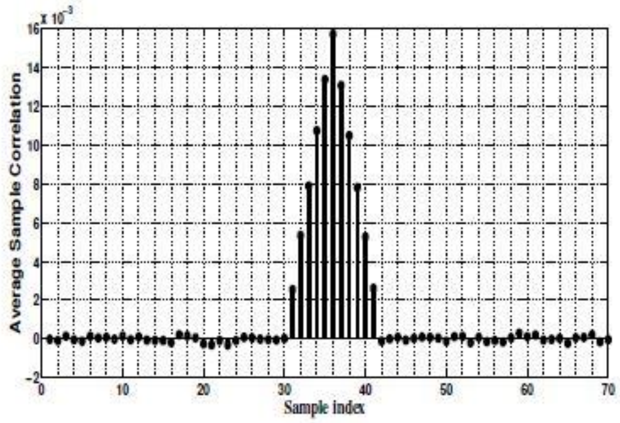

Fig 2.Average correlation of the an OFDM system in Rayleigh fading channel, with N=64, cyclic prefix of 6 samples, L=6, timing offset of 35 samples and BPSK modulation, for SNR 20 dB

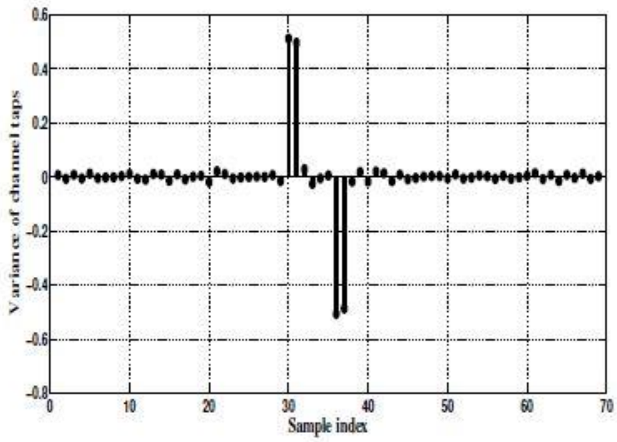

Fig 3.Channel tap variance of the an OFDM system in Rayleigh fading channel, with N=64, N_c=6, L=2, timing offset of 35 samples and BPSK modulation, for SNR 20 dB

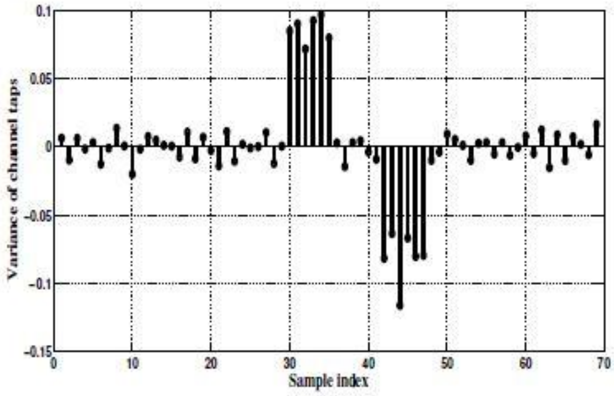

Fig 4.Channel tap variance of the an OFDM system in Rayleigh fading channel, with N=64, N_c=6, L=12, timing offset of 35 samples and BPSK modulation, for SNR 20 dB

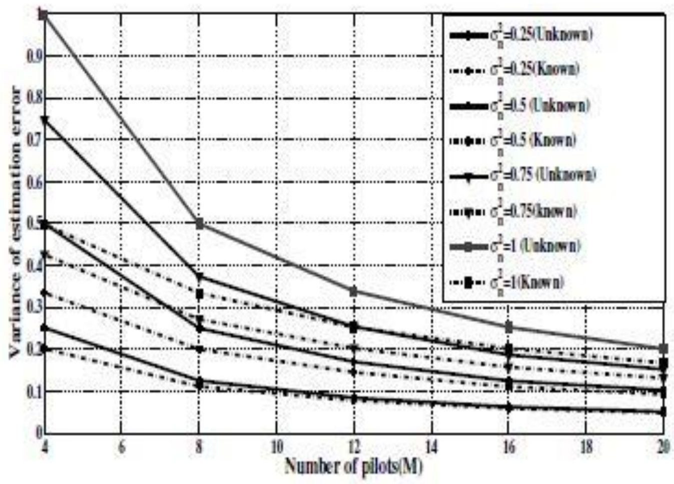

Fig 5.Variance of estimation error for 4 tap channel

VI. CONCLUSIONS

We presented an algorithm to determine the number of channel taps, their variances and the timing offset of the system, with no assumptions made. Used this information to estimate the channel taps using the most optimum MMSE estimation. Most importantly we showed that even when the interference on the pilot symbols is more, the method presented gives the method to estimate the channel taps with less error variance, so that this can be used in interference cancellation algorithms also.

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