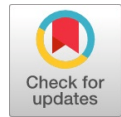


Research of Periodic Orbits and Chaos in Current Controlled Boost Converter

Jayant J. Mane, K. Vadirajacharya



Abstract:—This paper investigates the periodic orbits and produced chaos produced in a current controlled boost converter using theory of one-dimensional piecewise-smooth maps, simulation and hardware. Initially the one-dimensional linear piecewise-smooth map with suitable parameters which can be used to represent the boost converter is derived. To achieve simulation results the converter is mathematically modelled such that the multi-dimensional system is converted into single-dimensional non-linear. The inductor current is taken as a state variable. Therefore, the effect of parameter change has to be observed and the qualitative change has to be observed and investigated. In this paper we choose to change current reference. The periodic orbits produced in the current are shown with the help of MATLAB Simulink model and the phase portrait is also plotted. Further the same results are validated with the help of hardware.

Index Terms—Border collision bifurcation, discontinuous map, periodic orbits

I. INTRODUCTION

THE piecewise-smooth functions and bifurcation theorems are applicable in many systems like impact oscillator, relay feedback system, dc-dc converter, piecewise linear continuous map and boundary equilibrium bifurcation. Switched dynamical system exhibit this kind of phenomenon mainly due to their nonlinear components and switching phenomenon. Power electronic devices exhibit most of the nonlinear properties in the dynamical system. DC-DC Boost converter will have one set of equations when the switch is open and another set of equations when the switch is closed.

Initially chaos theory was proposed by Henri Poincare. In 1880s he found that there exists non periodic orbits which do not approach any fixed points, neither do not increase. In 1898 JAMES HADAMARD had published a study on chaos and it was called as HADAMARDS BILLIARDS. He showed the unstability of trajectories with a positive Lyapunov experiment. George David Birkhoff, Andrey Nikolaevich, Mary Lucy Cartwright and John Edensor Littlewood and Stephen Smale has extended the study on differential equations. This theory was formalized only after mid-century. This theory needs a lot of

calculations which can't be done by hand. An electronic computer is required for this purpose which had played an important role in the development of chaos theory. On November 27 in 1961 a student of Kyoto university, Yoshisuke Ueda noticed the Randomly Transition phenomenon but his reports came out in 1970 due to some reasons. Edward Lorenz was a pioneer of this theory. His interest in chaos came out suddenly when he was working on the weather predictions. In 1963 recurring patterns in every scale of data

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were found by Benoit Mandelbrot. He invented Naoh effect and Joseph effect which means any sudden change can occur in a system and persistence in a value can occur for a while respectively. First symposium was conducted in New York academy of sciences in 1971. Many scientists like Robert shaw, Robert May, Edward Lorenz and David Ruelle have attended it. They have discovered the universality in chaos. It had led the penetration of chaos in different fields. In 1979 Albert J. Lochaber and Pierre Hohenberg gave a result on bifurcation which leads to the chaos and turbulence. In 1986 New York Academy of Sciences held first important conference on chaos. James Gleick published a book called Making a New Science in which there are general principles about Chaos.

Border collision bifurcation became main focus of interest in 1990's. International journal bifurcation and chaos was published by Helena E. Nusse and James A. Yorke in 1995. They have observed the bifurcation in one dimensional maps. They have considered them as piecewise continuous and they only depend on one parameter. Bifurcation curves calculation by map replacement was done by Viktor Avrutin, Michael Schanz and Laura Gardini. Complex bifurcation with piecewise linear scalar map with a single discontinuity was completely studied by N. N. Leonov 50 years ago. He used an efficient technique for the calculation of bifurcation curves. His work is not only limited to this he also extended that for calculation border collision bifurcation curves for periodic orbits and complex symbolic sequences by using the correct periodic orbits with low period. Iryana Sushko, Victor and Laura Gardini had worked on bifurcation structure and its applications on border collision bifurcation. They have collected the results regarding the dynamics of one dimensional map. Scientists like Adelai El Aroudi, M. Debbat Roberto Giral, Gennaro Oliver, Luis Benadero, Eliezer Toribio have worked on bifurcation in DC-DC switching converters.

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* Correspondence Author (s)

Jayant J. Mane, Department of Electrical Engineering, DBATU, Lonere, Maharashtra, India.

K. Vadirajacharya, Department of Electrical Engineering, DBATU, Lonere, Maharashtra, India.

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They have provided information about bifurcation and chaos. Any multiple DC-DC switching converters are modelled by considering a general Poincare maps.

II. PIECE WISE SMOOTH FUNCTIONS

Piece wise smooth functions are defined as functions that can be divided into many pieces whereas each piece of function should be continuous and derivative. Each piece will be defined by a particular function. Therefore, total system is defined by a different set of functions. Piece wise smooth functions are defined by their degree of smoothness (DOS). Piece wise smooth maps are used in the dynamics of many systems. So to understand the properties of those dynamic systems there is a necessity of understanding these piece wise smooth maps. Due to the state depending and time depending switching non- linear behaviour is seen in the dynamic systems. The dynamics of switching circuits can be expressed in 1-D smooth piece wise maps Mainly periodic variations in piece wise smooth maps exists due to noise or due to switching actions. Bifurcations like period doubling, saddle node, homo clinic tangencies are exhibited by the piece wise smooth maps. Because of the variation in parameters the multipliers of a fixed point get changed.

Mathematically piece wise smooth maps are shown as:

- $f_1(x,p)$ for $x \in R_1$
- $f_2(x,p)$ for $x \in R_2$
- $f_3(x,p)$ for $x \in R_3$
- $x = f(x,p) = ..$
- $f(x,p)$ for $x \in R_n$

Here p =system parameter
R=region of state space

Therefore, system is divided by the n discrete events The study of the complex behaviour of dynamics and systems via mathematical modelling is very often used in science and technology. Originally, systems were classified as smooth dynamical systems for which the related mathematical theory has been well developed. However, in the present scenario, it has been observed that many systems exist whose internal processes are classified into non-smooth or only piece-wise smooth. Some of the examples of a piece-wise smooth curve include: heartbeats, bouncing, slipping, switching, Poincare' maps, rocking blocks, friction, Chua circuit etc.

III. ONE DIMENSIONAL MAPS

One Dimensional maps (sometimes called iterated maps or difference equations or recursion relations) are mathematical systems that model a single variable as it evolves over discrete steps in time.

One-Dimensional Maps observed in:

1. Modelling natural phenomena such as electronics, dynamics etc.
2. Chaos Example.

Consider the system $x_{n+1} = f(x_n)$, where x_n is the current state of the system, x_{n+1} is the next state, and f is a smooth function mapping the real line to the real line. A point x_0 such that $x_0 = f(x_0)$ is called a fixed point.

Let's assume for the above system $x_0 = x_n$, Then $x_{n+1} = f(x_n) = f(x_0) = x_0 = x_n$

Therefore, the state of the system remains fixed. Thus, to find a fixed point of a given one-dimensional map we just set $x_n = f(x_n)$ and solve for x_n .

A. One Dimensional Maps in Boost Converter

DC- DC converters under current mode control give piece-wise smooth and piece-wise monotonic maps.

Assumptions:

1. The output voltage is constant because of the capacitor across the load.

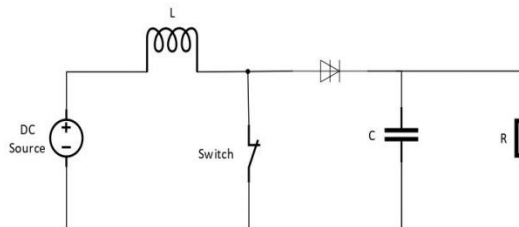


Fig. 1. Power Circuit of Current Controlled mode DC-DC Boost Converter

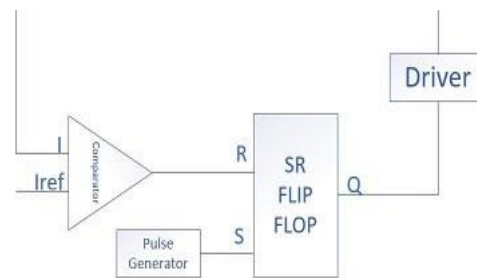


Fig. 2. Control Circuit of Current Controlled mode DC-DC Boost Converter

2. The switching period is short enough ($T = RC \ll 1$) for the inductor current to be essentially linear during switching. The Boost converter circuit is shown in fig.1. When the inductor current i reaches a reference value i_{ref} , the switch opens. The switch is again closed at the arrival of the next clock pulse from a free running clock of period T . Clock pulses during the on period are ignored.

The only state variable is the inductor current i . The value of the current at the $(n + 1)^{th}$ clock pulse is given in terms of that at the n^{th} clock pulse.

Consider V_{in} is the input voltage and V_{out} is the output voltage. Time Period is T .

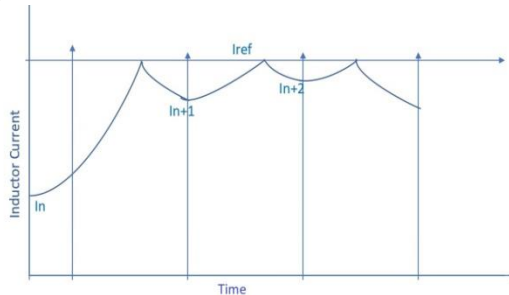


Fig. 3. Inductor Current Waveform

i_n is the initial current and i_{n+1} is the next state current. The borderline between the two cases is given by the value of i_n for which the inductor current reaches i_{ref} exactly at the arrival of the next clock pulse, i.e.,

$$i_{border} = i_{ref} - \frac{V_{in}T}{L}$$

We obtain the Maps as:

for

$$i_n \leq i_{border},$$

$$i_{n+1} = i_n + \frac{V_{in}T}{L} \dots (1)$$

and when

$$i_{n+1}$$

$$i_n \geq i_{border},$$

$$= i_{ref} \left(1 + \frac{V_{out} - V_{in}}{V_{in}} \right) - \frac{(V_{out} - V_{in})T}{L} - \frac{V_{out} - V_{in}}{V_{in}} i_n \dots (2)$$

In the above equations 1 and 2 only the current through inductor (i) is state variable. So, these are one dimensional mapping equations of current controlled mode DC-DC Boost converter.

IV. SIMULATION OF CURRENT CONTROLLED BOOST CONVERTER IN MATLAB SIMULINK TO CHECK THE ORBITS IN THE CURRENT I_L

A Matlab Simulink Model was developed using the system equations. The structure of the Simulink model is given in Figure4. The specifications are same as above.

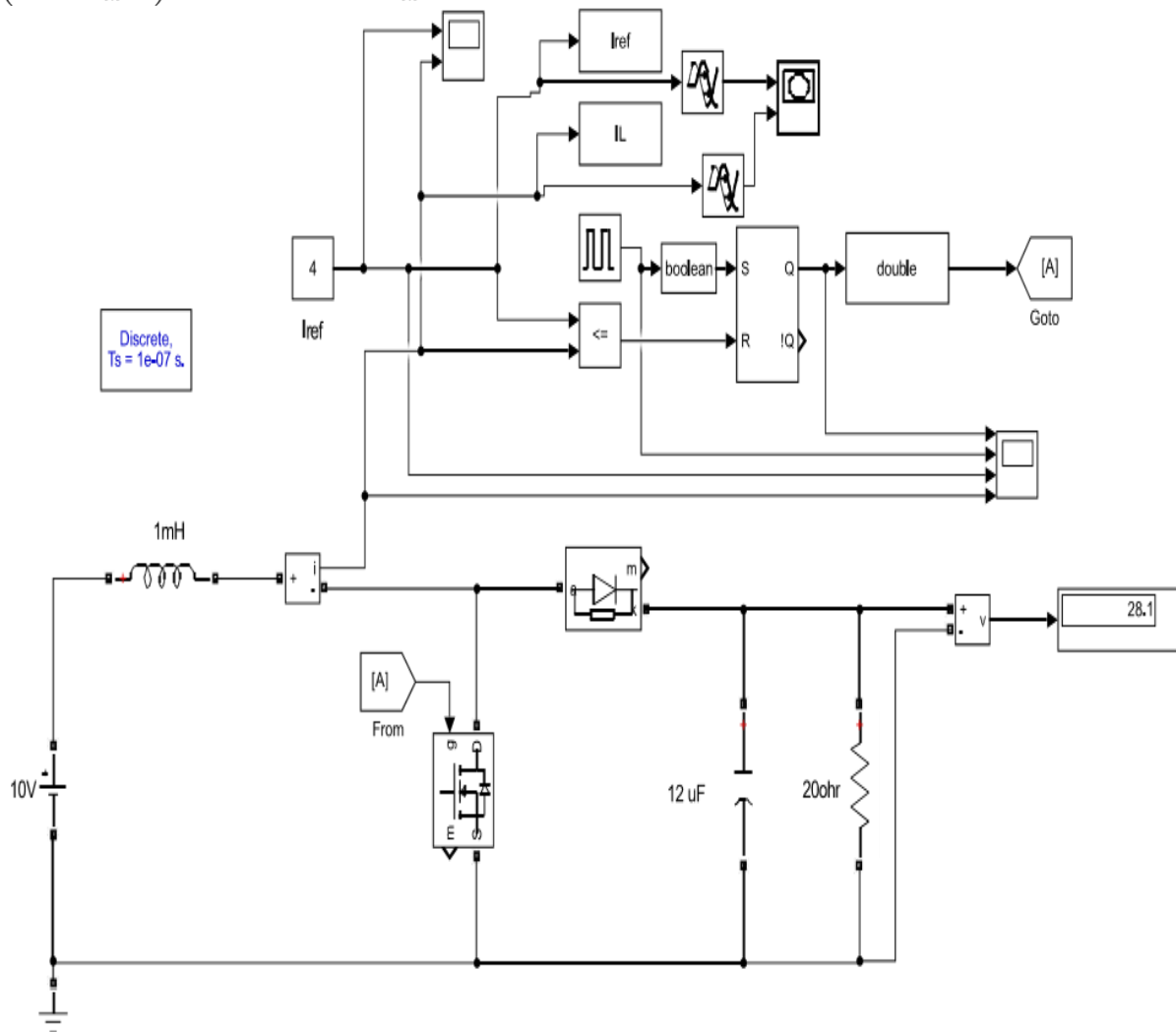


Fig. 4. Matlab Simulink model of current controlled boost converter

Simulated inductor current wave forms relating the inductor current I_L to the capacitor voltage V_C and clock pulses are shown for two different values of I_{ref} and input voltage

$$V_{in} = 10V.$$

All the results are in good agreement with the theoretically obtained bifurcation diagram corresponding to these three cases clearly demonstrate period-2, period-3, and chaotic behaviour (Figure 5, 6, 7).

The Figure5 shows the results for period-2 operation with $I_{ref} = 2A$. The two values of turn on I_L currents are

approximately 1.16A and 1.95A. The Figure6 shows the results for period-3 operation with $I_{ref} = 2.5A$. The two values of turn on I_L currents are 1.4A, 1.6A and 2.45A which is also depicted using the bifurcation plot. The Figure7 shows results for chaotic behaviour with $I_{ref} = 3.5A$. As expected, the value of I_L at turn on is unpredictable. After observing the orbits from the current waveform now the bifurcation diagram is plotted by continuously varying the current reference from 0.5A to 4.5A.

The bifurcation diagram confirms the behaviour of the circuit.

V. HARDWARE IMPLEMENTATION OF CURRENT CONTROLLED BOOST CONVERTER& RESULTS

In order to verify the results obtained by bifurcation curves, an experimental converter using the parameters (above mentioned) was built. The circuit is built using discrete components. The circuit has been divided into 5 modules, as shown.

- Module I - Main circuit
- Module II - Current Sensor and Comparator
- Module III - Clock Pulse Generator
- Module IV - Gate Pulse Optimizer
- Module V - Gate Driver Circuit

A 20kHz clock signal is obtained using an Arduino Mega board consisting of ATmega2560 microcontroller. The rising edge of the clock signal drives the set input of SR flip flop constructed using IC SN74LS279A. The inductor current I_L , is sensed by a current sensor shunt of 0.5Ω , the voltage drops across the shunt is then fed to comparator which compares this value to a given fixed value of voltage or reference. The output of the comparator is consequently fed to Reset pin of SR flip flop. The output of the flip flop is then applied to the gate driver circuit constructed using IC 2110. The output from the Gate circuitry is given to the switch, which then triggers the MOSFET ON. The rest of the procedure for obtaining results has been explained previously.

Deciding cut-off frequency of low pass filter by observing FFT of inductor current.

$$F_c = 35kHz$$

Deciding minimum necessary frequency bandwidth, turn on turn off time of switching components by measuring the smallest ON OFF Time intervals when system is in chaos.

$$\text{Minimum On Off Time interval} = 9.4 - 10\mu \text{ sec.}$$

Maximum Turn On Turn Off time combined = 300 - 400 μ sec.

$$F(BW) = 50 - 60kHz$$

Capacitor rating was decided by measuring the simulated ripple current and ripple voltage while circuit was in chaos operation.

$$I_{(ripple\ peak-peak)} = 1.2A - 1.5A$$

$$V_{(ripple\ peak-peak)} = 25V - 26V$$

$$F_{(ripple)} = 4.42kHz - 20kHz$$

VI. CONCLUSIONS

This paper investigated the periodic orbits and produced chaos produced in a current controlled boost converter using theory of one-dimensional piecewise-smooth maps, simulation and hardware. Initially the one-dimensional linear piecewise-smooth map with suitable parameters which can be used to represent the boost converter was derived. To achieve simulation results the converter was mathematically modelled converting the multi-dimensional system into single-dimensional non-linear. Therefore, the effect of parameter I_{ref} change has been observed. The qualitative change has to be observed and investigated. The periodic orbits produced in the current are validated with the help of simulation and hardware.

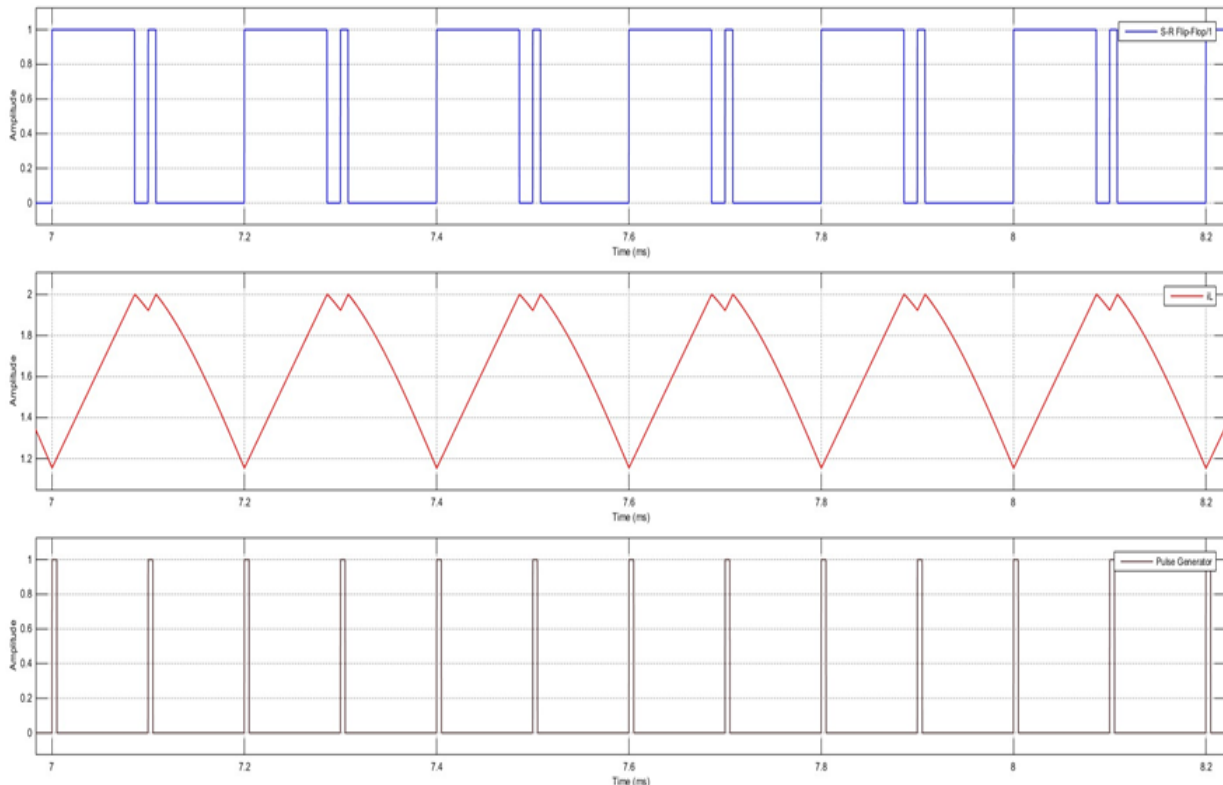


Fig. 5. Period 2 waveforms: Iref = 2A

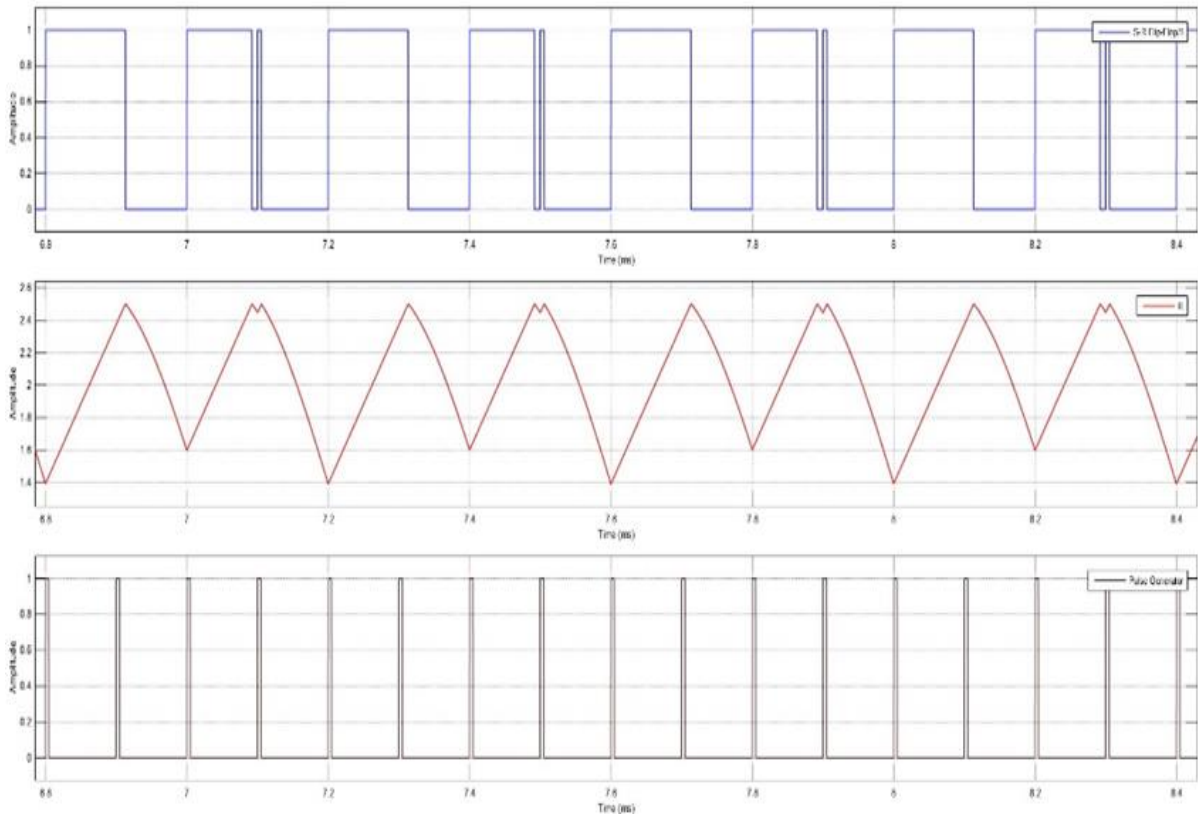


Fig. 6. Period 3 waveforms: $I_{ref} = 2.5A$

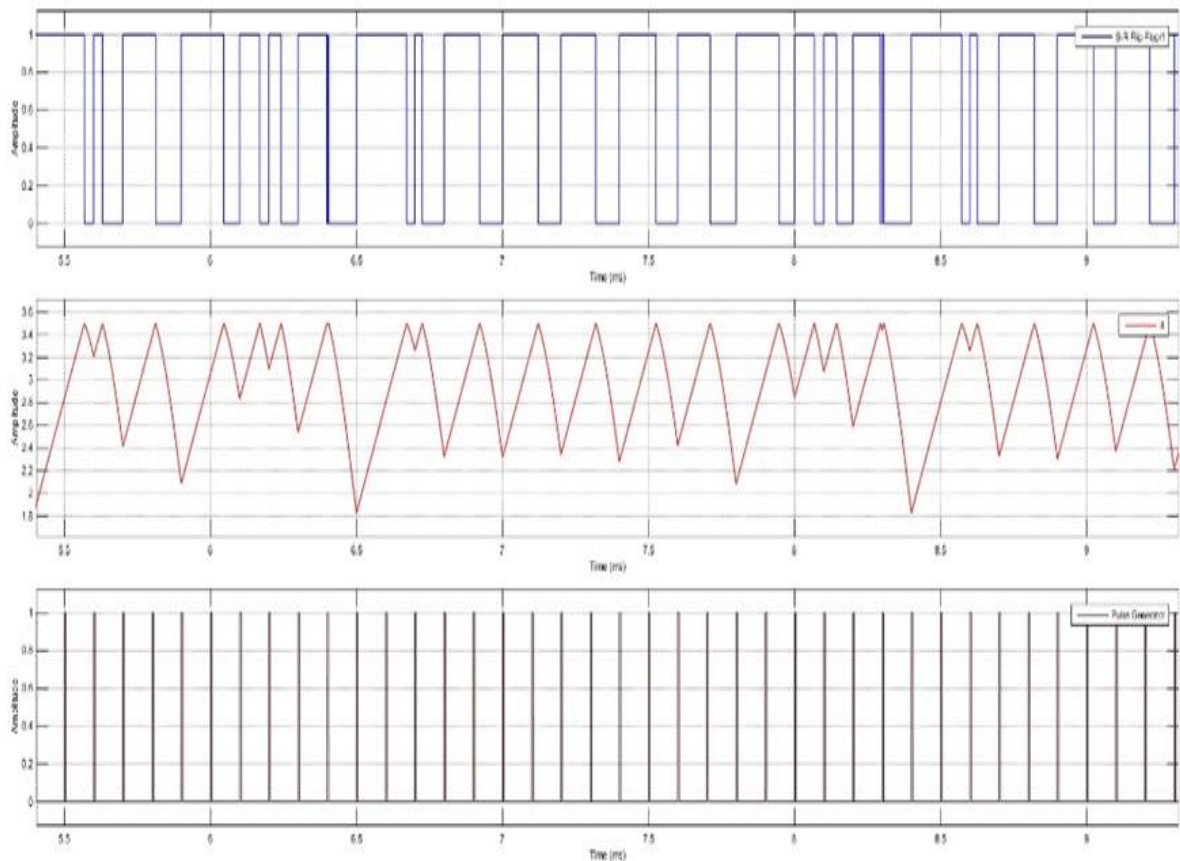


Fig. 7. Chaos waveform: $I_{ref} = 3.5A$

Table I

Components used to built the hardware of current controlled boost converter along with specifications.

Components	Specifications
Module I (Main Circuit)	
R	20 Ω
L	1mH
C	12 μ F , Parallel combination of (10+1+1) μ F, 10 μ F FlowESR capacitor C4AF3BW5100A3MK, 1 μ F FlowESR capacitor C4AF3BU4100A1YK
Diode	6A10 ($I_f = 6A, V_f = 1V, V_r = 1000V$)
DC supply	10V
MOSFET	IRFZ44N ($V_{DSS} = 55V$) ($R_{DS(on)} = 17.5m\Omega, I_D = 49A$)
R_{snub}	5 Ω
C_{snub}	680pF
Module II (Current Sensor and Comparator)	
Current Sensor giving input to amplifier with gain 2 built using CA3130)	0.5 (Ω) shunt
Module III (Clock Pulse Generator)	
Arduino Nano	ATmega 2560
Module IV (Gate Pulse Optimizer)	
NOT gate	7404
Comparator	LM393
Op-Amps	LM358 and CA3130
SR-Flip Flop	SN74LS279A (Quadruple SR Latches)
Module V (Gate Driver Circuit)	
Gate Driver IC	IR2110
R	10 Ω , 1k Ω , 10k Ω
C	100 μ F, 100nF(ceramic)
DC supply	12V

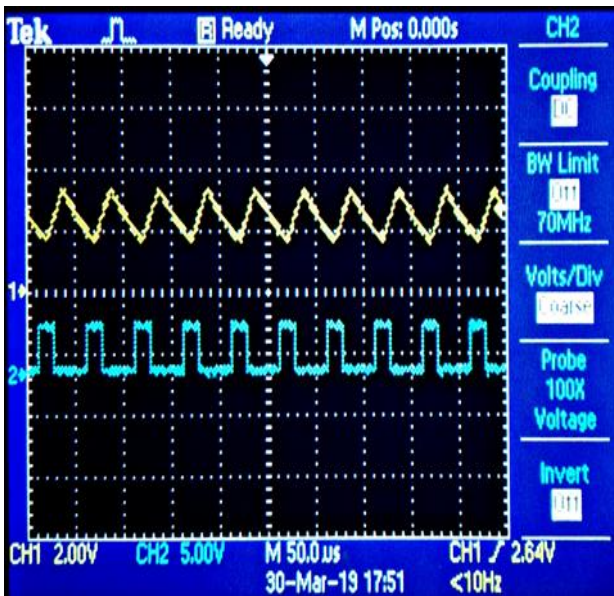


Fig. 8.Period-1 waveform. Channel 1 is showing inductor current and Channel 2 is S-R flip-flop output

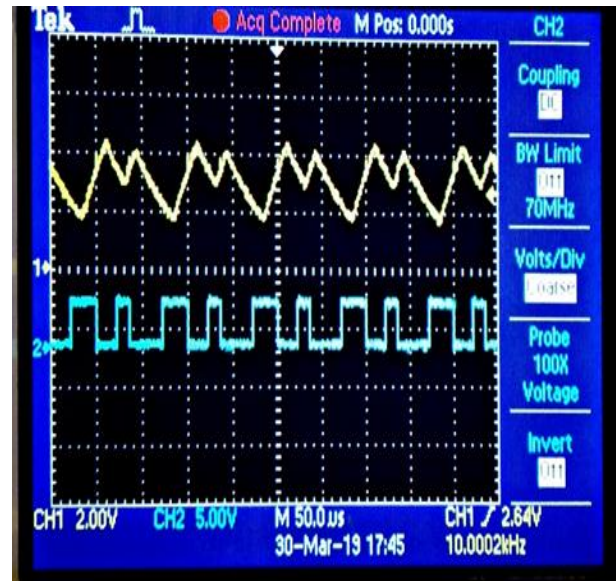


Fig. 9.Period-2 waveform. Channel 1 is showing inductor current and Channel 2 is S-R flip-flop output

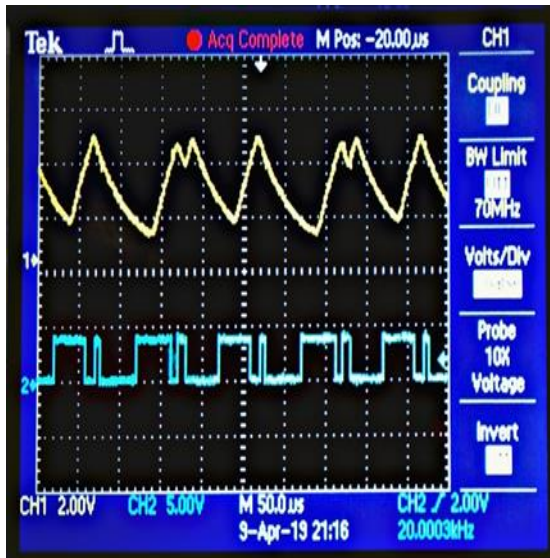


Fig. 10.Period-3 waveform. Channel1 is showing inductor current and Channel 2 is S-R flip-flop output

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