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Some Derived Sequences of *k*-Jacobsthal and *k*-Jacobsthal Lucas

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Abstract:- ***In this article we present incredible associated some derived sequences of k - Jacobsthal and k - Jacobsthal Lucas numbers and we give Diophantine triples for (a,b,c) with the property $D(j^2_{k,n-1})$.***

Key words : derived k-Jacobsthal numbers, derived k-Jacobsthal Lucas numbers

I. INTRODUCTION

Fibonacci sequence is the very popular for its magnificent and remarkable properties. In recent times, some of the sequences were generalized for any positive real number *k*. The studies of *k*-Fibonacci sequence, *k*-Lucas sequence, *k*-Jacobhstal and *k*-Jacobhstal-Lucas sequences etc., can be found.

S. Uygun, and H.Eldogan, defined *k*-jacobsthal and *k*-jacobsthal lucas sequences $j_{k,n} = kj_{k,n-1} + 2j_{k,n-2}$, $j_{k,0} = 0, j_{k,1} = 1, n \geq 2$ and $\hat{\jmath}_{k,n} = k\hat{\jmath}_{k,n-1} + 2\hat{\jmath}_{k,n-2}$, $\hat{\jmath}_{k,0} = 2$, $\hat{\jmath}_{k,0} = k, n \geq 2$

Give some properties of the *k*-Jacobsthal and k-jacobsthal Lucas sequence. S.Vidhayalaxmi M.A.Gopalan and E.Premalatha provides observations on derived *k*-Fibonacci and derived *k*- Fibonacci Lucas sequences. Now , we bring in the some derived sequences of *k*-Jacobsthal and *k*-Jacobsthal Lucas numbers. Also give Diophantine triple for (*a*,*b*,*c*) with property $D(j^2_{k,n-1})$ and $D(\hat{\jmath}^2_{k,n-1})$.

Section I

Derived k-Jacobsthal Sequence

For any positive real number k, the derived *k*-Jacobsthal sequence $\{j_{k,n}\}_{n=1}^{\infty}$ is defined as $j_{k,0} = 0, j_{k,1} = 1$ and

$j_{k,n+1} = k\, j_{k,n} - 2\, j_{k,n-1}$ for $n \geq 2$ Binet form of for $j_{k,n}$ is

$$j_{k,n} = \frac{\alpha^n - \beta^n}{\alpha - \beta} \text{ where } \alpha + \beta = k, \alpha\beta = -2$$

Derived k-Jacobsthal Lucas Sequence

The derived *k*-Jacobsthal Lucas sequence $\{\hat{\jmath}_{k,n}\}_{n=1}^{\infty}$ is defined as $\hat{\jmath}_{k,0} = 2$, $\hat{\jmath}_{k,1} = k$ and

$\hat{\jmath}_{k,n+1} = k\, \hat{\jmath}_{k,n} - 2\, \hat{\jmath}_{k,n-1}$ for $n \geq 2$ Binet form of for $\hat{\jmath}_{k,n}$ is

$$\hat{\jmath}_{k,n} = \alpha^n + \beta^n \quad \text{where } \alpha + \beta = k, \alpha\beta = -2$$

Derived k-Jacobsthal facts are prearranged

1. $j_{k,2} = k$
2. $j_{k,3} = k^2 - 2$
3. $j_{k,4} = k^3 - 4k$
4. $j_{k,5} = k^4 - 6k^2 + 4$
5. $j_{k,6} = k^5 - 8k^3 + 12k$
6. $j_{k,7} = k^6 - 10k^4 + 24k^2 - 8$
7. $j_{k,8} = k^7 - 12k^5 + 40k^3 - 32k$
8. $j_{k,9} = k^8 - 14k^6 + 60k^4 - 80k^2 + 8$
9. $j_{k,10} = k^9 - 16k^7 + 84k^5 - 160k^3 + 72k$
10. $j_{k,11} = k^{10} + 18k^8 + 36k^6 - 280k^4 + 232k^2 - 16$
11. $j_{k,12} = k^{11} - 20k^9 + 68k^7 - 296k^5 + 352k^3 - 160k$
12. $j_{k,13} = k^{12} - 22k^{10} + 104k^8 - 101k^6 + 912k^4 - 624k^2 + 32$
13. $j_{k,14} = k^{13} - 24k^{11} + 149k^9 - 237k^7 + 1404k^5 - 1328k^3 + 152k$
14. $j_{k,15} = k^{14} - 26k^{12} + 193k^{10} - 445k^{10} + 1606k^6 - 3152k^4 + 1400k^2 - 64$

Derived k-Jacobsthal Lucas facts are prearranged

1. $\hat{\jmath}_{k,2} = k^2 - 4$
2. $\hat{\jmath}_{k,3} = k^3 - 6k$
3. $\hat{\jmath}_{k,4} = k^4 - 8k^2 + 8$
4. $\hat{\jmath}_{k,5} = k^5 - 10k^3 + 20k$
5. $\hat{\jmath}_{k,6} = k^6 - 12k^4 + 36k^2 - 16$
6. $\hat{\jmath}_{k,7} = k^7 - 14k^5 + 56k^3 - 56k$
7. $\hat{\jmath}_{k,8} = k^8 - 16k^6 + 80k^4 - 128k^2 + 32$
8. $\hat{\jmath}_{k,9} = k^9 - 18k^7 + 108k^5 - 240k^3 + 144k$
9. $\hat{\jmath}_{k,10} = k^{10} - 20k^8 + 140k^6 - 400k^4 + 400k^2 - 64$
10. $\hat{\jmath}_{k,11} = k^{11} - 22k^9 + 158k^7 - 508k^5 + 640k^3 - 208k$
11. $\hat{\jmath}_{k,12} = k^{12} - 24k^{10} + 198k^8 - 788k^6 + 1440k^4 - 1008k^2 + 128$
12. $\hat{\jmath}_{k,13} = k^{13} - 26k^{11} + 242k^9 - 1104k^7 + 2456k^5 - 2288k^3 + 544k$
13. $\hat{\jmath}_{k,14} = k^{14} - 28k^{12} + 290k^{10} - 1500k^8 + 4032k^6 - 5168k^4 + 2560k^2 - 256$
14. $\hat{\jmath}_{k,15} = k^{15} - 30k^{13} + 342k^{11} - 1984k^9 + 6240k^7 - 10080k^5 + 7136k^3 - 1344k$

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Properties of Derived k-Jacobsthal and k-Jacobsthal Lucas sequences

1. $j_{k,n+1} - j_{k,n-1} = \hat{\jmath}_{k,n}$
2. $j_{k,2n} = j_{k,n}\,\hat{\jmath}_{k,n}$
3. $\hat{\jmath}_{k,2n} - 2^{n+1} = (\hat{\jmath}_{k,n})^2$
4. $\hat{\jmath}_{k,3n} + 3\hat{\jmath}_{k,n} = (\hat{\jmath}_{k,n})^3$
5. $\hat{\jmath}_{k,2n} = (k^2-8)\hat{\jmath}^2_{k,n} + 2^{n+1}$
6. $\hat{\jmath}_{k,2n+1} = (k^2-8)j_{k,n}j_{k,n+1} + 2^n k$
7. $j_{k,n+1}j_{k,n-1} - \hat{\jmath}_{k,2n} = 2^{n-1}k$
8. $\hat{\jmath}^2_{k,n} - (k^2-8)j^2_{k,n} = 4.\, 2^n$
9. $\hat{\jmath}^4_{k,n} - (k^2-8)^2 j^4_{k,n} = 16.\, 2^n\hat{\jmath}_{k,2n}$
10. $\hat{\jmath}_{k,2n} = (k^2-8)j_{k,n+1}j_{k,n-1} + 2^n(k^2-4)$
11. $j_{k,m}\hat{\jmath}_{k,n} + j_{k,n}\hat{\jmath}_{k,m} = 2j_{k,m+n}$
12. $j_{k,m}\hat{\jmath}_{k,n} - j_{k,n}\hat{\jmath}_{k,m} = 2^{n+1}j_{k,m+n}$
13. $j_{k,m}\hat{\jmath}_{k,n} = j_{k,m+n} + 2^n j_{k,m-n}$
14. $\hat{\jmath}_{k,m+n}\hat{\jmath}_{m-n} = \hat{\jmath}_{k,2m} + 2\hat{\jmath}_{k,2n}$
15. $\hat{\jmath}_{k,m+n}j_{m-n} = j_{k,m+n} - 2^n\hat{\jmath}_{k,m-n}$

Section II & Results

The three distinct integers a, b, c such that "product of any two from the set added with $j^2_{k,n-1}$ is a perfect square".

Let $a = j_{k,n+1}$ and $b = \hat{\jmath}_{k,n}$

Let c be any non – zero integer.

Consider

$$ac + j^2_{k,n-1} = \beta^2$$

This becomes

$$j_{k,n+1}\,c + j^2_{k,n-1} = \beta^2 \quad (1)$$

as well

$$bc + j^2_{k,n-1} = \gamma^2$$

provides

$$\hat{\jmath}_{k,n}c + j^2_{k,n-1} = \gamma^2 \quad (2)$$

Using some algebra,

$$\hat{\jmath}_{k,n}\beta^2 - j_{k,n+1}\gamma^2 = j^2_{k,n-1}\,(\hat{\jmath}_{k,n} - j_{k,n+1}) \quad (3)$$

with the linear transformations

$$\beta = X + j_{k,n+1}T$$

$$\gamma = X + \hat{\jmath}_{k,n}T$$

and T = 1 we have

$X = j_{k,n-1} + j_{k,n+1}$ and $\beta = 2j_{k,n+1} + j_{k,n-1}$

From (1)

$c = 4(j_{k,n-1} + j_{k,n+1})$

hence (a, b, c) is the Diophatine triple with the property $D(j^2_{k,n-1})$.

n	k	a	b	c
1	1	1	1	4
2	1	3	5	16
3	1	5	7	24

Remark :

In the same way the identical mode to apply $D(\hat{\jmath}^2_{k,n-1})$ and to find the Diophantine triples.

Let $a = \hat{\jmath}_{k,n+1}$ and $b = j_{k,n}(k^2+8)$ $c = 4(\hat{\jmath}_{k,n-1} + \hat{\jmath}_{k,n+1})$

n	k	a	b	c
1	1	5	9	28
2	1	7	9	32
3	1	17	27	88

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