International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249-8958 (Online), Volume-8 Issue-6S3, September 2019

Evolution of Concentric Fuzzy Hypergraph for Inclusive Decision Making

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Abstract: Fuzzy hypergraphs plays a pivotal role in reaching concrete resolutions at times when the circumstances of making decisions is characterized by uncertainty. Decision making is an integrated process involving the dominance of cognition in the nature of decision making environment and the associated factors. The process of making managerial decisions centres on state of affairs, which can be tackled with the formulation of Concentric Fuzzy Hypergraph, an extended concept of fuzzy hypergraph to arrive at compatible and feasible pronouncements. The notion of Concentric Fuzzy Hypergraph is introduced and a comprehensive decision making model with the integration of Fuzzy Cognitive Maps is proposed. This research work outlines the conceptual framework of Concentric Fuzzy hypergraph and presents a comparative analysis of concentric fuzzy hypergraphic approach with Fuzzy Cognitive maps over fuzzy hypergraphic approach to justify the efficiency of the former. The idea of concentric fuzzy hypergraph will certainly open new vistas in the field of decision making.

Keywords: Concentric Fuzzy Hypergraph, Decision making, fuzzy hypergraphs, Fuzzy Cognitive Maps

I. INTRODUCTION

The concept of Fuzzy Hypergraph was devised by Kaufmann based on the conception of hypergraph by Berge. Hyung Lee-Kwang and Keon modified the definition of fuzzy hypergraph and presented the application of it. Fuzzy Hypergraphs are widely applied in decision making. Mathematical models based on Fuzzy Cognitive Maps (FCM) are also equally employed in making decisions by managerial people. The fore founder of FCM is Kosko who integrated the notion of Fuzzy to Cognitive Maps by Axelrod. FCM based decision making models involve several factors and they are represented as directed graphs with nodes as the factors and the edges as the association between the factors. The edges are assigned weights as -1,1and 0 if the association has negative, positive and null impact respectively. The most difficult task in FCM models is the selection of factors and it is purely based on the opinion of the experts. As the choice of the factors considered to study is highly subjective in nature, the choice of the factors considered to study varies and it is essential to consider all the factors. One of the merits of FCM models, is the assessment of inter relational impacts between the factors. To devise optimal decisions the confinement of factors is mandatory and it has to be carried out with systematic and scientific method. Nivetha.et.al formulated two FCM models with hypergraphic approach and fuzzy hypergraphic approach. Student's low academic performance appraisal model with hypergraphic approach in FCM was proposed by Nivetha and Pradeepa. This model is intended to determine the factors contributing to low academic performance and inter relational impacts. Initially 15 factors were considered for study, the concept of hypergraph was used and to determine 8 core factors. In hypergraphic approach the factors were grouped as hyperedges based on the expert's opinion, the restriction of 15 factors to 8 facilitated the decision making FCM model. Another FCM model was proposed by Nivetha and Pradeepa to prioritize the obstacles in building swipe and touch classroom environment using fuzzy hypergraphic approach. In this model 15 factors was initially taken into consideration and later it was confined to 6 factors. The factors were grouped as fuzzy hyperedges based on the expert's outlook, but the second model incorporated the weightage of the factors, which further facilitated the confinement of the factors. But in the FCM model with fuzzy hypergraphic approach, the expert's were not restricted in the formulation of fuzzy hyperedges. Each of the fuzzy hyperedge was the opinion of the experts with the factors and its weightage of their choice, suppose if an expert feels that all the factors considered for the study or the analysis plays a remarkable role and discriminates their contribution with weightage, then the representation of the fuzzy hypergraphs will take a new form as concentric fuzzy hypergraphs and this research work is evolved based on those circumstances. The paper is structured as follows: section 2 presents the basic definitions of concentric fuzzy hypergraph; section 3 encompasses the applications of concentric fuzzy hypergraph in FCM model; section 4 discusses the results and comprises of a comparative analysis between FCM models with concentric fuzzy hypergraph and FCM models with fuzzy hypergraphic approach and the last section concludes the paper.

II. CONCEPTUAL FRAMEWORK OF CONCENTRIC FUZZY HYPERGRAPH

This section presents the preliminaries of concentric fuzzy hypergraph.

2.1 Concentric Fuzzy Hypergraph

Lee has redefined the Kaufmann's definition of fuzzy hypergraph which has been extended to concentric fuzzy hypergraph by Nivetha and Pradeepa.

Manuscript published on 30 September 2019.
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A concentric fuzzy hypergraph C_H is defined as follows

C_H = (X, E)

X- finite set of vertex set

E- Concentric fuzzy hyper envelope – family of fuzzy sets of X

$E_j = \{(x_i, \mu_j(x_i)) / \mu_j(x_i) > 0 \text{ and } \forall x_i \in X\}$ j = 1,2,…m

Supp (E) = X = Supp (E_j) $\forall$ j = 1,2,…m

Example 2.1

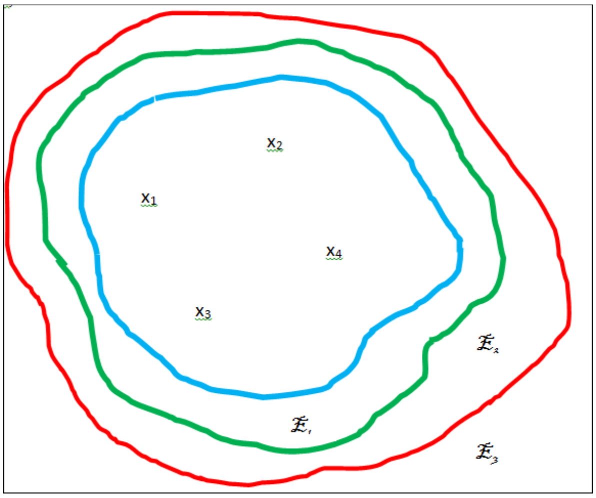

Fig 2.1 Concentric Fuzzy Hypergraph

To illustrate concentric fuzzy hypergraph, let C_H = (X, E), where

X = {x_1,x_2,x_3,x_4}

E_1 = { (x_1,0.4), (x_2,0.6), (x_3,0.5),(x_4,0.3)}

E_2 = { (x_1,0.3), (x_2,0.8), (x_3,0.4),(x_4,0.5)}

E_3 = { (x_1,0.6), (x_2,0.6), (x_3,0.7),(x_4,0.4)}

2.2 Strength of a concentric fuzzy hyper Envelope

The strength of a concentric fuzzy hyper envelope S_{Ei} is the minimum membership value of the vertices in the envelope E_i.

S_{E1}= 0.3, S_{E2}= 0.3S_{E3}= 0.4 (From Example 2.1)

2.3 Degree of Concentric fuzzy hyper Envelope and vertex of concentric fuzzy hypergraph

The degree of a concentric fuzzy hyper envelope is the summation of the membership values of all the vertices in an envelope.

D_{E1}= 1.8, D_{E2} = 2, D_{E3} =2.3 (From Example 2.1)

2.4. Degree of a vertex in a concentric fuzzy hypergraph

It is the max of the degree of a vertex in each of fuzzy hyper envelope.

$DC_{xi} = \max \{ \sum_k (\mu_j(x_k)) \; k \neq i, \; j = 1,2..m\}$

2.5 Adjacency between vertices and envelopes

The adjacency between the vertices and envelopes of a concentric fuzzy hypergraph is defined as

$A(x,y) = \max_j \min(\mu_j(x), \mu_j(y))$ j = 1,2..m

$B(E_j,E_k) = \max_j \min(\mu_j(x), \mu_k(x)) \; \forall x \in X$

2.6 α – cut hypergraph

According to Kaufmann ,the α – cut hypergraph is a hypergraph H at level α.

In example 2.1, if α = 0.5, then the α – cut hypergraph is represented in Fig. 2.2

E_1 ={x_2,x_3 }

E_2 ={x_2,x_4}

E_3 ={x_1,x_2,x_3}

Fig 2.2 α – cut hypergraph

2.7 α – cut hypergraph with envelope

Based on the notion of α – cut hypergraph, the α – cut hypergraph with envelope for α = 0.4 (say) is represented in Fig. 2.3.

$E_j = \{x_i / \mu_j(x_i) > \alpha, \forall x_i \in X\}$ j = 1,2,…m

E_1 = { x_1,x_2,x_3 }

E_2 = { x_2,x_3,x_4 }

E_3 = { x_1,x_2,x_3 }

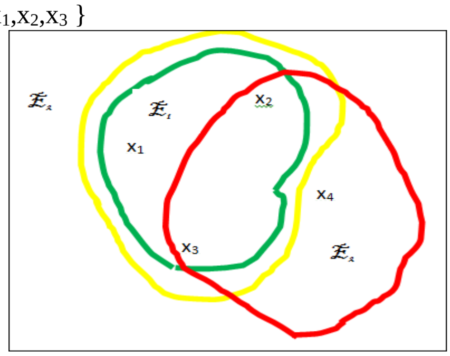

Fig.2.3 α – cut hypergraph with envelope

2.8 α – cut hypergraph with concentric envelope

The α – cut hypergraph with concentric envelope for α = 0.3 (say) is represented in Fig. 2.4

E_j= {x_1,x_2,x_3,x_4} j =1,2,3

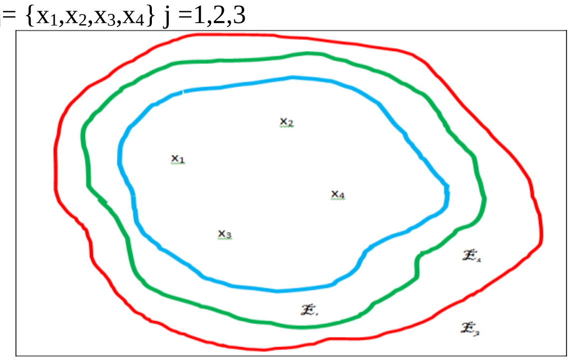

Fig.2.4 α – cut hypergraph with concentric envelope

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2.9 Incidence Matrix of Concentric Fuzzy Hypergraph

The association between the vertices and the envelopes are represented as incidence matrix. The incidence matrix M_{CH} for the concentric fuzzy hypergraph in Example 2.1 is

	E_1	E_2	E_3
x_1	0.4	0.3	0.6
x_2	0.6	0.8	0.6
x_3	0.5	0.4	0.7
x_4	0.3	0.5	0.4

2.10. Dual of concentric fuzzy hypergraph

The dual of the concentric fuzzy hypergraph D_{CH} has the vertex set as the envelope and vice versa. Fig 2.5 is the graphical representation of D_{CH} .The incidence matrix of D_{CH} for *C*H as in Example 2.1 is

	x_1	x_2	x_3	X_4
E_1	0.4	0.6	0.5	0.3
E_2	0.3	0.8	0.4	0.5
E_3	0.6	0.6	0.7	0.4

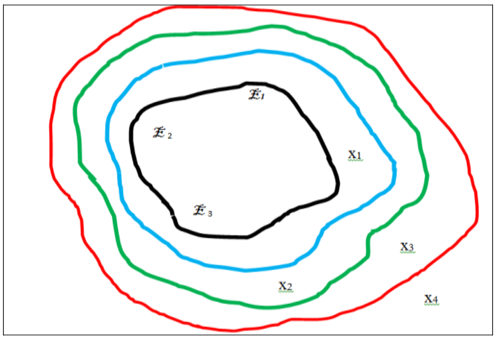

Fig 2.5. Dual of concentric fuzzy hypergraph

Here

X = {E_1, E2,E3 }

x_1= { (E_1, 0.4), (E_2,0.3), (E_3, 0.6)}

x_2 = { (E_1, 0.6), (E_2,0.8), (E_3, 0.6)}

x_3 = { (E_1, 0.5), (E_2,0.4), (E_3, 0.7)}

x_4 = { (E_1, 0.3), (E_2,0.5), (E_3, 0.4)}

2.11 α – cut of Dual Hypergraph

The α – cut of Dual hypergraph for α = 0.5 is represented in Fig. 2.6

x_1= {E_3 }

x_2 = {E_1, E_2E_3 }

x_3 = {E_1,E_3 }

x_4 = {E_2}

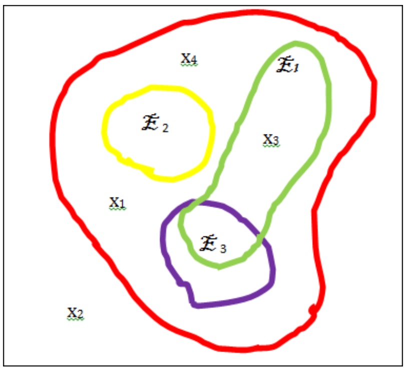

Fig.2.6 α – cut Dual hypergraph

2.12 α – cut of Dual Hypergraph with envelope

The α – cut of Dual Hypergraph with envelope for α = 0.4 is represented in Fig. 2.7

x_1= {E_1, E_3 }

x_2 = {E_1, E_2E_3 }

x_3 = {E_1, E_2E_3 }

x_4 = {E_2E_3 }

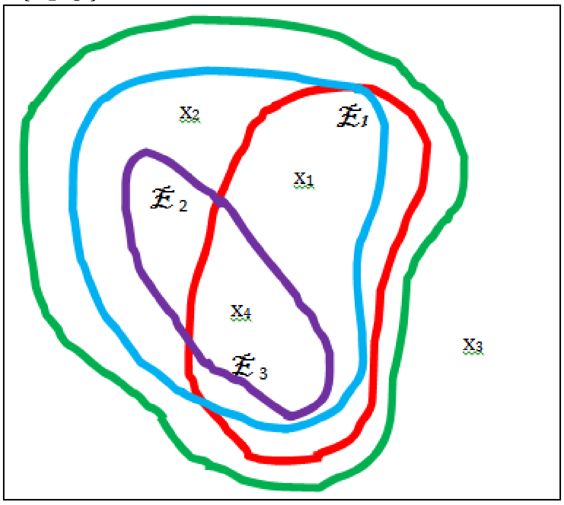

Fig. 2.7 α – cut of Dual Hypergraph with envelope

2.13 α – cut Dual hypergraph with concentric envelope

The α – cut of Dual Hypergraph with concentric envelope for α = 0.3 is represented in Fig. 2.8.

x_i = {E_1, E_2E_3 } i = 1,2,3,4

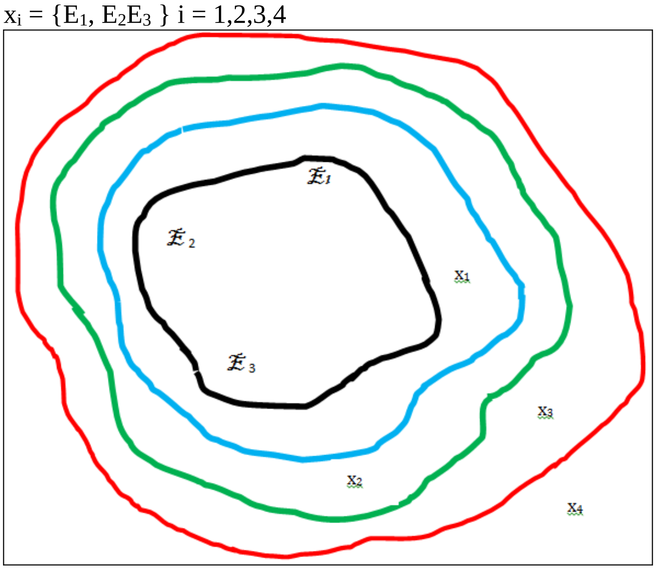

Fig. 2.8 α – cut of Dual Hypergraph with concentric envelope

III. FCM MODEL WITH CONCENTRIC FUZZY HYPERGRAPH

This section proposes a FCM model with the integration of concentric fuzzy hypergraph, an extension of the FCM model with fuzzy hypergraphic approach proposed by Nivetha and Pradeepa. The task of prioritization of the obstacles in building Swipe and Touch classroom environment is taken as the decision making problem with

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the same factors and the methodology differs only in the confinement of the factors using concentric fuzzy hypergraphic approach and inter relational impacts are determined by the usual induced FCM method.

Based on the linguistic questionnaire and the expert's opinion the following factors are considered as the obstacles in building Swipe and Touch classroom environment, and it is represented graphically in Fig.2.9

O1 Difficult to exercise control over the students

O2 Confrontation of the institutions with contemporary learning portals

O3 Crisis related to integrity and security of data

O4 Teacher student interface gets wide apart

O5 Demands huge capital investment

O6 Deficit in effective execution of the digital learning strategies

O7 Hardships in time management

O8 Routine practice of upgradation and replacement of technology

O9 Confinement of learning through digital modes makes it incomplete

O10 Development of overall personality will be shallow

O11 High rate of piracy of learning materials

O12 Lack of teachers with sound technical skills

O13 Complexity in software management

O14 Complications in accessibility

O15 The domain of social interaction becomes static

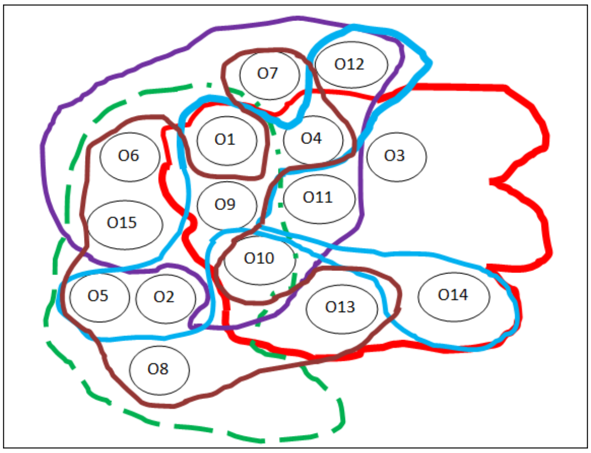

Fig.2.9 Fuzzy Hypergraphic representation of the factors based on Expert's opinion

In the above fuzzy hypergraphic representation, the fuzzy hyper edge set representing each expert's opinion focuses only on selective factors. But on profound analysis the above mentioned factors which are the obstacles make significant impact on setting up swipe and touch classroom environment. To make a wholistic representation of the factors and comprehensive decisions, the notion of concentric fuzzy hypergraph is incorporated into the FCM model. Fig.2.10 is the graphical representation of concentric fuzzy hypergraph

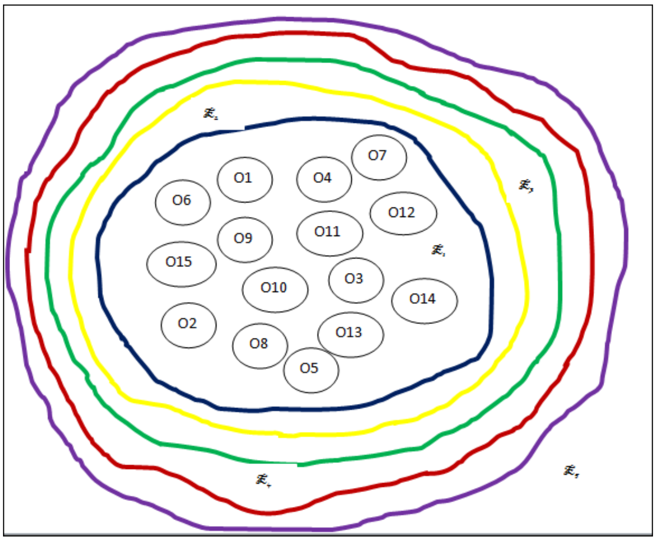

Fig.2.10 Concentric Fuzzy Hypergraphic representation of the factors based on Expert's opinion

The concentric fuzzy hyper envelopes of the concentric fuzzy hypergraph representing expert's opinion are presented below.

E_1,={(O1,0.7), (O2,0.3),(O3,0.6), (O4,0.8),(O5,0.5),(O6,0.4), (O7,0.5),(O8,0.4),(O9,0.6),(O10,0.8),(O11,0.8), (O12,0.4), (O13,0.5), (O14,0.5), (O15,0.6)}

E_2= {(O1,0.5),(O2,0.5), (O3,0.4),(O4,0.5),(O5,0.6),(O6,0.6) ,(O7,0.2),(O8,0.7),(O9,0.5),(O10,0.4), (O11,0.5), (O12,0.3),(O13,0.5),(O14,0.3),(O15,0.5)}

E_3 = {(O1,0.8),(O2,0.5), (O3,0.6),(O4,0.5),(O5,0.4),(O6,0.4),(O7,0.5),(O8,0.5),(O9,0.6),(O10,0.6),(O11,0.5),(O12,0.5),(O13,0.5),(O14,0.3),(O15,0.6)}

E_4={(O1,0.6),(O2,0.3),(O3,0.5),(O4,0.5),(O5,0.4),(O6,0.4),(O7,0.6),(O8,0.5),(O9,0.5),(O10,0.4),(O11,0.5),(O12,0.4),(O13,0.6),(O14,0.4),(O15,0.5)}

E_5 ={(O1,0.5),(O2,0.6),(O3,0.5),(O4,0.6),(O5,0.4),(O6,0.7),(O7,0.6),(O8,0.4), (O9,0.3),(O10,0.5),(O11,0.5),(O12,0.4),(O13,0.5),(O14,0.3),(O15,0.5)}

O1	O2	O3	O4	O5	O6	O7	O8	O9	O10	O11	O12	O13	O14	O15
3.1	2.5	**2.6**	**2.9**	2.3	**2.5**	2.4	2.4	**2.5**	**2.7**	**2.8**	2	**2.6**	1.8	**2.7**

The core factors are O1,O3,O4,O6,O9,O10,O11,O13,O15 are considered as F1,F2,F3,F4,F5,F6,F7,F8,F9 respectively and the FCM representation is as follows

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	F1	F2	F3	F4	F5	F6	F7	F8	F9
F1	0	0	1	0	0	1	0	0	1
F2	0	0	1	1	0	0	1	1	0
F3	1	0	0	1	1	1	0	0	1
F4	1	1	0	0	0	1	1	1	0
F5	0	0	0	0	0	1	0	0	1
F6	0	0	0	0	1	0	0	0	0
F7	0	1	0	1	0	0	0	0	0
F8	1	1	1	1	1	0	0	0	1
F9	0	0	1	1	1	1	0	0	0

By applying the same methodology as in the FCM model with fuzzy hypergraphic approach, the following triggering pattern is attained.

On Position of Factor	Triggering Pattern
(100000000)	F1→F3→F8→F8
(010000000)	F2→ F8→F8
(001000000)	F3→ F8→F8
(000100000)	F4→ F8→F8
(000010000)	F5→F9→ F8→F8
(000001000)	F6→F5→F9→ F8→F8
(000000100)	F7→F4→ F8→F8
(000000010)	F8→ F4→ F8→F8
(000000001)	F9→F8→F8

The inter relational impacts between the core factors are represented graphically

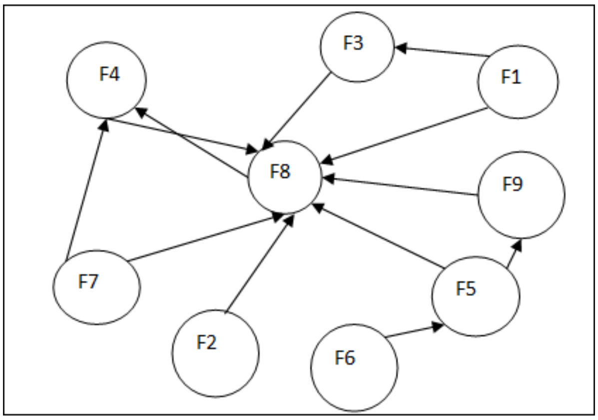

Fig.2.11 Graphical representation of the inter relational impacts between the core factors

IV. DISCUSSIONS & RESULTS

FCM model with concentric fuzzy hypergraphic approach has yielded interesting results. The core factors are evolved around certain important aspects such as digital strategies, development of personality, process of socialization and the outlook of the institution. The core factors have implications on the factor F8 which stands for Difficulty in software management, which is the most challenging task of the institution in implementing the swipe and touch classroom environment and all other factors also reflects the same. The results of FCM models with concentric fuzzy hypergraphic approach has favoured an environment of comprehensive decision making process, where as in the earlier proposed model with fuzzy hypergraphic approach, the aspect of teacher and learner phase was only focussed and the decision making was restrained only to one domain of obstacles, but in this proposed model the decision making process was subjected to several domains and the prime reason is the confinement of the factors with wholistic consideration of all the factors with discrimination by weightage.

FCM models with concentric fuzzy hypergraphic approach are highly workable and consistent as these models are embedded with the unique feature of assimilating all the factors pertaining to decision making. Comprehensive decisions with FCM models can be arrived easily only with concentric fuzzy hyper graphs. FCM models with hypergraphic and fuzzy hypergraphic approaches are also beneficial models to decision makers but the proposed model with concentric fuzzy hypergraphic approach stands high in comparison with the earlier two models.

V. CONCLUSION

This paper introduces the innovative concept of concentric fuzzy hypergraph to the field of research for making comprehensive decisions in integration with FCM models. This research work has presented the conceptual framework of concentric fuzzy hypergraphs and has also demonstrated the efficiency of the proposed model over the earlier models in terms of reliability and compatibility through illustration. This work will certainly unlock the gates of new research ideas and will surely lay a foundation of new beginning of such novel concepts in the field of fuzzy hypergraph. The proposed model can also be extended based on the needs of the decision makers.

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