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Solving Multi-Objective Probabilistic Factional Programming Problem

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Abstract*:*This paper presents the solution methodology of a multi-objective probabilistic fractional programming problem. In the proposed model the parameters in the constraints coefficient and the right-hand sides of the constraints follow continuous random variables having known distribution. Since the programming problem consists of random variables, multi-objective function and fractional objective function, it is lengthy, time-consuming and clumsy to solve the proposed programming problem using analytical methods. Stochastic simulation-based genetic algorithm approach is directly applied to solve multi-objective probabilistic non-linear fractional programming problem involving beta distribution and chi-square distribution. In the proposed method, it is not necessary to find the deterministic equivalent of a probabilistic programming problem and applying any traditional methods of fractional programming problem. The stochastic simulation-based genetic algorithm is coded by Code block C++ 16.01 compiler. A set of Pareto optimal solutions are generated for a multi objective probabilistic non-linear fractional programming problem. A numerical example and case study on inventory problem are presented to validate the proposed method.

Keywords: Continuous distribution, Fractional programming problem, Multi-objective programming problem, Probabilistic programming problem, stochastic simulation based genetic algorithm

I. INTRODUCTION

Fractional programming problem is an optimization model where the objective function is the ratio of functions such as output/input, profit/capital, return/cost, doctor/patient, etc., subject to constraints. Some application areas of fractional programing problem are agricultural planning problem, production planning problem, healthcare, and hospital planning problem, financial planning, etc.

A fractional programing problem is said to be linear programming if the function to be optimized is the ratio of linear functions. If one or both of the two functions are non-linear functions, then the fractional programming is called non-linear fractional programming (NLFP) problem. Most of the world decision-making problems have uncertainty in objective functions and constraints due to incomplete information or vague information or uncertain information. Such type of uncertainties are addressed in or via fuzzy programming problem or stochastic programming problem. The mathematical models are said to be stochastic NLFP problem if some or all parameters of NLFP problem are considered as random variables. In this case, the random variables follow either continuous distribution or discrete distributions having known probability density or probability mass function respectively. There are two well-known mathematical models for solving stochastic programming problems. These are chance-constrained and two-stage mathematical programming. Two or more non-linear fractional objective functions which are conflicting each other is called multi-objective non-linear fractional programming (MNLFP) problem. If the parameters of the MNLFP problem are characterized by random variables, it is known as multi-objective probabilistic non-linear (MPNLFP). Like single objective probabilistic NLFP problem, MPNLFP problem do not have a single optimal solution that optimizes all objective functions simultaneously.

In this paper a set of Pareto optimal solutions that attains the prioritized multi-objective as closely as possible is generated. NLFP problem methods are categorized as methods based on the change of variables, direct method, and parametric methods. In these methods, decision-maker has to obtain the deterministic equivalent of parametric mathematical modeling. These methods are tedious and time taking. To overcome this problem, stochastic simulation-based genetic algorithm (GA) is applied to solve the proposed problem. In the proposed method, no need of finding the deterministic equivalent of a probabilistic programming problem and applying classical methods of MPNLFP problem.

GA is one of the heuristic methods in the evolutionary algorithm which depends on the concept of natural selection, developed by John Holland in 1975. GA works on population of individuals which are called chromosomes or strings. The algorithm is based on four operators called selection, crossover, mutation, and elitism. For a given generation, a set of the population is generated. The individual in the population are solutions for the given problem. By the idea of

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principle of survival of the fittest, novel population is produced fromexisting population by selection operator. The fitter the individuals are the greater

the probability of being selected in to the new population. Then, the population undergoes the operation crossover and mutation. In the crossover, two-parent chromosomes are selected randomly according to the fixed crossover probability which produces two-child chromosomes bearing the best characteristic of the parent chromosomes. In mutation, the new individuals are created by some random change in the gene (bit) of the chromosome which depends on mutation probability. Once the process of selection, crossover, and mutation are completed, elitism can be applied for further improvement in the population for the next generation. In Elitism, the initial population of a particular generation undergoes mingling, sorting and then the best half is chosen from the population. In this way, the best chromosomes survive and moves to the next generation and the weaker onside off. As the generation increases, the best-fitted individual survives over time and once the terminating criteria is reached, the algorithm stop and give the best individual in terms of a solution to the given problem.

This paper is structured in 6 Sections followed by references. The first section presents an introduction to MPNLFP problem. The second section describes related literatures. MPNLF programming problem. Finally, examples are presented in Section 5 flowed by conclusion in Section 6.

II. LITERATURE SURVEY

The common difficulty in real-life decision-making problem is determining the exact values of parameters. Some or all parameters are characterized by uncertainty which is difficult to forecast. One of the most known mathematical programming problem under uncertainty is a stochastic programming problem. Chance constrained programming is a part of a stochastic programming problem which is used to obtain the solutions for mathematical models having probabilistic constraint with some probability violation. It was initially developed by Charnes and Cooper [1]. To find the optimal solution of probabilistic programming problems, researchers have found its deterministic equivalent. Biswal et al. [2] found the deterministic equivalent of probabilistic programming where the coefficients in the constraints which follow exponential distributions. Charnes and Cooper [3] presented chance- constrained programming problem that involves independent normal random variables. Some other researchers presented multi-objective probabilistic mathematical programming problems. Hulsurkar et al. [4] solved multi-objective probabilistic mathematical programming using fuzzyprogramming method. Charles et al. [5] presented multi-objective probabilistic mathematical programming problems by considering the parameters in the right side of constraints follow generalized continuous distributions. Javaid et al. [6] presented multi-objective probabilistic programming where the right hand side parameters follow Weibull distribution.

Zhou et al. [7] presented multi-objective programming problems under uncertainty environment by considering random variables as normally distribute. But, finding the deterministic of probabilistic constraints is not easy due to the convexity of the feasible region. To avoid this drawback, Iwamura and Liu [8] proposed a genetic algorithm for probabilistic programming problem including probabilistic goal programming, probabilistic multi objective programming problem. Following this, Iwamura and Liu [9] used GA to solve probabilistic goal programming. Later Jana and Biswal [10] proposed stochastic simulation based GA to solve single objective probabilistic programming problem. Wang et al. [11] proposed a genetic algorithm based on non-linear probabilistic programming for portfolio selection problem. Poojari and Varghese [12] presented a complete framework consisting of GA to probabilistic programming problems. Initially, Dinkelbach [13] presented the parametric approach for linear and non-linear fractional programming. Ibaraki et al [14] presented parametric programming technique of quadratic fractional programming. Chenet al. [15]presented the application of non-linear fractional programming in economic power dispatch. Roy [16] presented GA to solve single-objective fractional programming problems. Mishra [17] presented non dominated solution of fractional programming problem having multi-objective functions. Sameeullah et al. [18] used GA for solving the linear fractional programming problem. In fractional programming problem, if some of the data are characterized by a random variable, it called a probabilistic fractional programming problem. Solution technique for probabilistic fractional programming has seen in [19], [20]. Charles and Dutta [21] recognized a solution method to solve multi-objective probabilistic fractional programming problem by finding its deterministic equivalent. In all the above methods the deterministic of programming problems is required, which is difficult to find the deterministic equivalent. Udhayakumar et al. [22] presented a stochastic simulation-based genetic GA for probabilistic fractional programming. They combined the parametric method developed by Charles and Dutta [23] and stochastic simulation GA to solve single-objective probabilistic linear fractional programming problem.

In this paper, stochastic simulation-based GA, will be applied to solve MPNLFP problem, where the parameters in the constraint coefficient and right-hand side parameter follow the continuous distribution.

III. FORMULATION OF MPNLFP PROBLEM

A MPNLFP problem is expressed as:

$max: Z_q = \frac{N_q(x_j)}{D_q(x_j)}, Q = 1,2, \dots Q$ (1)

Subject to $\mathrm{P}\left(\sum_{j=1}^{n} a_{ij}\, x_{j \le b_i}\right) \ge \alpha_{i,} i = 1,2, \dots, m$ (2)

$x_j \ge 0, j = 1,2,3 \dots . n$ (3)

$0 \le \alpha_{i,} \le 1$

(4)

Where $N_q(x_j)$ and $D_q(x_j)$ are non-linear function of x_j, $a_{ij} \in R$ are constraint coefficients, "P" indicates probability, $\alpha_{ij,}$ is the given probability at which the i^{th} constraint violations are admitted.

In this paper, we assume that the constraint coefficient and right-hand side parameters follow beta distribution and chi-square distribution respectively.

A random variable x has:

i. Beta distribution having two shape parameters α and β, if it's probability density function is defined as

$$f(x) = \begin{cases} \dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \text{if } 0 \le x \le 1 \\ 0, \quad \text{other wise} \end{cases}$$

Where Γ is gamma function.

ii. Chi-square distribution with parameter k , if it's density probability function is defined as:

$$f(x) = \begin{cases} \dfrac{x^{\frac{k}{2}-1} e^{-\frac{x}{2}}}{2^{\frac{k}{2}}\Gamma(\frac{k}{2})}, \text{if } x \le 0 \\ 0, \quad \text{other wise} \end{cases}$$

Beta distributed and Chi-square distributed random numbers can be generated using Code Block 16.01 C++ compiler as follow:

A random numbers having beta distribution with two parameters α and β are generated as.

1) Generate from $G(\alpha, 1)$, where G is gamma distribution
2) Generate n_2, from $G(\beta, 1)$.
3) Return ${}^{n_1}/_{n_1+n_2}$

If k is the degrees of freedom of chi-square distribution then the random number with chi-square distributed can be generated as follows.

a) Generate y_j, $j = 1,2,\ldots,k$ from $N(0,1)$, where N is standard normal distribution.
b) Return $y_1{}^2 + y_2{}^2 + y_3{}^2 + \cdots + y_k{}^2$.

IV. SOLUTION PROCEDURE FOR MPNLFP PROBLEM

The mathematical programming problem (1)-(4) is solved by using stochastic simulation based GA directly, without finding the deterministic equivalent of probabilistic programming problem.

Moreover, no need of using the traditional methods of fractional programming problems to solve fractional programming. The procedures of stochastic simulation based GA are described as follows.

Stochastic simulation is used to handle the probabilistic constraints of the given programming problem. The basic algorithms of stochastic simulation-based GA are discussed as follows.

The notationst,p(t), p_c and p_m stands for number of generation, population size, cross over probability and mutation probability respectively.

Step 0: Fix the parameterst, p_c andp_m.

Step 1: Initialize p(t) and generate random variables

Step 2: Lower bounds and upper bounds of decision variables are identified by the decision maker based on the decision making situation.

Step 3: Evaluate the value of objective functions.

Step 4: Check the probability criteria of the constraints. If the criteria is fulfilled then proceed to step 5, otherwise proceed to step1.

Step 5: Selection operator is applied.

Step 6: Crossover operator is applied.

Step 7: Mutation operator is applied.

Step 8: Again evaluate the value of objective functions.

Step 9: The probability criteria of the constraints are again checked. If the criteria is fulfilled then proceed to step 10, otherwise proceed to step1.

Step 10: Apply elitism operator.

Step 11: When termination criteria is reached the present population gives the best solution. Else go to step 5.

Stochastic simulation based GA flow chart is given by Fig 1.

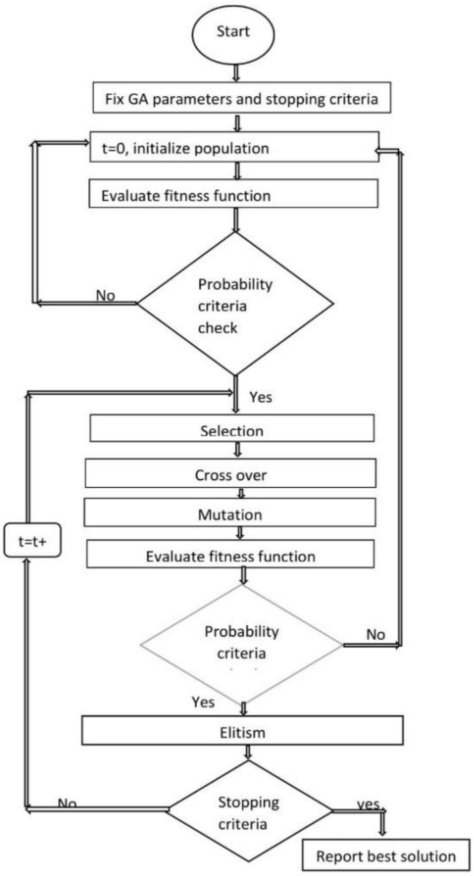

Fig 1: Flow chart for stochastic simulation based GA

V. EXAMPLES

In this section we consider two examples namely one numerical example and application on inventory model.

A. Numerical example

Consider the following example.

$$max: Z_1 = \frac{2x_1 + 3x_2 + x^2{}_3}{2x^2{}_1 + 3} \quad (5)$$

(6)

$$xmax: Z_1 = \frac{3x_1 + 4x_2}{5x^2{}_2 + x^2{}_3 + 4}$$

$$P(-x_1 - a_1 x_2 + a_2 x_3 \leq b_1) \geq 0.85 \quad (7)$$

$$P(x_1 - x_2 - x_3 \leq b_2) \geq 0.65 \quad (8)$$

$$P(a_3 x_1 - a_4 x_2 - x_3 \leq b_3) \geq 0.45 \quad (9)$$

$$0 \leq x_i \leq 3, i = 1,2,3 \quad (10)$$

Where $a_1, a_2,\ a_3$,and a_4 follow beta distribution with two parameters α and β in which their values are given by Table I.

Table I: The values of the parameters for beta distributed random variables

Parameters	a_1	a_2	a_3	a_4
A	2	3	4	9
B	4	6	4	10

In addition b_1, b_2, b_3 follow chi-square distribution with parameter value k : k_{b_1} = 3, k_{b_2} = 4 and k_{b_3} = 5.

The stochastic simulation based GA is coded by using Code Block 16.01 C++ programming language. In the algorithm, we use binary tournament selection, one point cross over and bitwise mutation and stopping criteria is maximum generation.

The parameters of GA are defined as follow: $p(t) = 200, pc = 0.8, pm = 0.01$ and $t = 100$.

The Pareto optimal solutions are obtained using stochastic simulation based GA using parameters $p(t), pc, pm, and\ t$ and given distribution parameters.

The non-dominated solutions of the above example are given by Table II.

Table II: Non dominated solutions and values of Z1 and Z1 for the numerical example

x_1	x_2	x_3	Z_1	Z_2
0.184751	2.89736	1.21701	1.73735	2.31595
0	3	2.24927	2.3432	2.28404
2.90909	2.90616	2.95308	1.01447	2.32428
0.539589	2.28739	2.77713	2.37816	2.2834
0	3	2.90909	2.91047	2.25
0.0909091	1.40469	2.92962	2.15715	2.31673
0.0058651	2.43695	2.90909	2.63087	2.25071
3	0.123167	0	0.3033095	2.329

The set of all Pareto optimal solutions which are called Pareto frontier points are expressed by Fig 2.

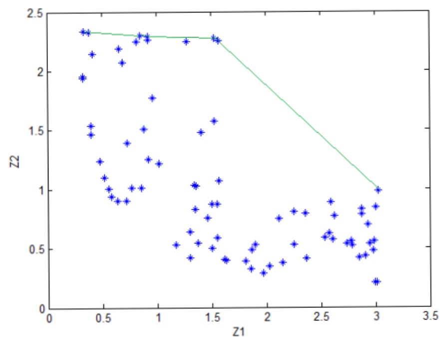

Fig 1: Pareto frontier of Z_1 and Z_2 for numerical example

B. Application on inventory problem

We consider the practical application of MNLPP problem for inventory problem.

Inventory deals with materials and materials management that is how to store materials for subsequent use. Inventory essentially answers two important questions, namely how much to order? And when to order?

So inventory control or management deals with these two aspects. There two broad category of inventory model are called single period inventory model and multi period inventory model. Single period inventory model assumes that the planning horizon is single period and the decision is made at once. Whereas multi period inventory model assumes the planning horizon is multi period and the decision is made more than once.

The 4 various important costs associated with this model are discussed as below.

a) Item Cost of item: it is the unit cost associated with each item.

b) Ordering cost: it is a costs earned every time you place an order. It is independent of the quantity being ordered. Some components of ordering cost are cost of people (salary), transportation cost.

c) Carrying cost (holding cost): Some components of caring costs are warehouse and power cost, security cost, inventory risk costs due to shipment error or damage in the storage.

a) d) Shortage cost (Back ordering cost): These costs occur when inventories are out of stock by different reasons such as disrupted production, emergency shipments, Customer loyalty and reputation.

In this paper we focus on single item inventory model with continuous demand, instantaneous replenishment.

Suppose that a stock manager is interested to sell refrigerator in his super market for 2 years. The item is produced by another producer. The item is produced from different company having fixed purchasing price. The manager needs to maximizing the ratio of profit to total cost (order cost + holding cost +back order cost) and the ratio of total quantity to back ordering quantity subject to limited inventory level and stock holding capacity. Hence the manager wants to know the number of ordered quantity, number of inventory item in the stock and back order item to optimize the objective functions. Due to some uncertainty factors in the minimum inventory level and area of item in the stock which are characterized by random variables that follow continuous distribution, the problem becomes probabilistic inventory problem. For example the minimum inventory level of the item is a random variable which follows Chi-square distribution which has one parameter known n and the area of each item is a random variable following beta distribution having two parameters α and β. To construct the multi-objective fractional probabilistic inventory model, consider the following notations and assumptions:

Notations:

Q: order quantity of an item.

B: back order level of item.

I inventory level of item.

D: demand of item.

p: Profit of 1 unit of inventory item rupees/unit.

s: Profit of 1 unit of back order item rupees/unit.

c: Back ordering cost of item rupees/unit.

c0: order cost of item rupees/unit.

h: Inventory cost of item rupees/unit.

a: The area required for 1 itemm^2.

I_{min}: Minimum inventory level in the stock.

R: Total storage space of stock.

$\frac{D}{Q}$: Number of order per year.

$\frac{D}{Q}c_0$: Total order cost rupees per year.

$\frac{Q}{2}$: Average inventory.

$\frac{Q}{2}c$: Total back order cost rupees per year.

$\frac{Q}{2}h$: Total holding cost rupees per year.

Assumptions

1) The quantity Q is optimal at the beginning of the period.
2) At the end of the season the remind items are used.
3) No lost sales penalty cost.
4) The storage space for each item cannot exceed from the total space of stock R.
5) The total inventory level must exceed from minimum inventory level in the stock$I_{\min}$.

The fractional objective functions and the probabilistic constraints are given by:

1. Maximize total profit of quantities per total order cost:

$$max: Z_1 = \frac{(\mathrm{pI} + \mathrm{sB})}{\frac{D}{I+B}c_0 + \frac{hI^2}{2(I+B)} + \frac{cB^2}{2(I+B)}}$$

2. Maximize item quantity to back ordering item .

$$max: Z_2 = \frac{B^2 + I^2}{I+B}$$

The objective functions are optimized subject to the following two probabilistic constraints.

a. the total inventory level in the stock is must exceed from the minimum stock level

$$P\left(\frac{I^2}{2(I+B)} \geq \mathrm{I_{min}}\right) \geq \eta$$

b. the storage space for each item cannot exceed from the total space of stock R

$$P(a(I + B) \leq R) \geq \rho$$

Where "P" is probability, η and ρ are probability levels for which$0 < \eta < 1, 0 < \rho < 1$.

Thus the single-item probabilistic inventory model under inventory level and storage space limitation having fixed probability η and ρ is given by:

$$max: Z_1 = \frac{(\mathrm{pI} + \mathrm{sB})}{\frac{D}{I+B}c_0 + \frac{hI^2}{2(I+B)} + \frac{cB^2}{2(I+B)}} \quad (11)$$

$$max: Z_2 = \frac{B^2 + I^2}{I+B} \quad (12)$$

Subject to $P\left(\frac{I^2}{2(I+B)} \geq \mathrm{I_{min}}\right) \geq \eta$ (13)

$P(a(I + B) \leq R) \geq \rho$ (14)

$10 \leq B, Q \leq 400$ (15)

Suppose that the storage space of the stock is 400 $m^2, \eta = 0.35, \rho = 0.65$ and the other different parameter values of the system are given by Table III.

Table III: Multi-item data values

Item	D	c_0	C	H	S	P	$\eta_{I_{min}}$	α_{a_i}	β_{a_i}
Refrigerator	1000	400	25	20	800	1000	5	2	4

Substituting all the data given in Table III to (11)-(15) and solving the resulting multi-objective fractional probabilistic inventory model using stochastic simulation-based GA, We get the following results.

Result and Discussion

The Pareto optimal solutions are obtained using stochastic simulation based GA using parameters p(t) = 200,pc = 0.8,pm = 0.01 and t = 100 and distribution parameters given above. The stochastic simulation-based GA is coded by using C++.We obtain the following non dominated optimal solutions of the inventory model.

Table IV shows some values of inventory level (I) and backorder level (B) which are non-dominated solutions.

Table IV: Non- dominated solutions and values of Z_1 and Z_2 for the application.

I	B	z_1	z_2
377	233	56.3409	3.61801
388	316	66.797	2.50761
393	358	71.3953	2.20509
396	379	73.5883	2.09172
398	389	74.6621	2.04681
398	396	75.2297	2.2.01013
399	398	75.5102	2.2.00503
207	17	11.1492	149.262

The Pareto frontier points are expressed by Fig 3.

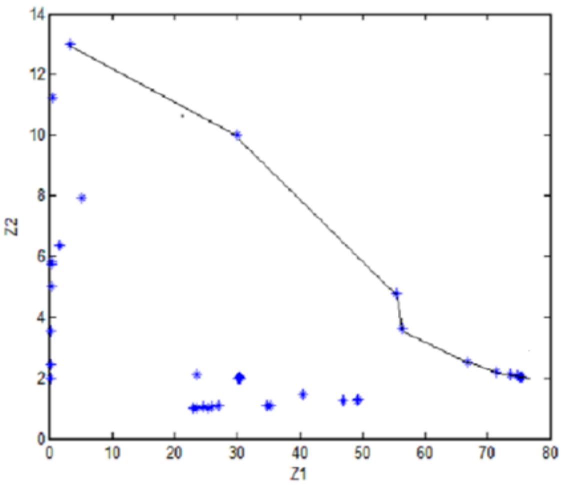

Fig 2 :Pareto frontier of Z_1 and Z_2 for the application

VI. CONCLUSIONS

In conclusion, MPNLFP problem has been handled by stochastic simulation-based GA. The deterministic of probabilistic constraint is not needed. The other benefit of stochastic simulation based GA is, no need of using methods of fractional programming like variable change method and parametric approach method to reduce the NLFP problem into non-linear programming problem. A numerical example and case study are given by considering some parameters as continuous random variables following beta and chi-square distributions. This problem can be extended to multi-objective bi-level fractional programming having discrete random variables.

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