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Research on CS-MRI Reconstruction with WHT as Sparsifying Transform

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Abstract--- In this paper an efficient method for the reconstruction of Magnetic Resonance Image (MRI) from the compressively sampled MR k-space. Compressive Sensing (CS) gives an efficient structure for getting back the signal or image from lesser measurements than that are really necessary according to the Nyquist criterion. The Walsh Hadamard transform is used as the sparsifying transform. In the proposed work radial and Cartesian sampling patterns are applied on k-space to collect minimum samples and MR image is recovered by taking Inverse Fourier transform of the k-space data . The Qualitative and quantitative analysis of the reconstructed images depict that the performance of Walsh Hadamard Transform as sparsifying transform gives better result in comparison with DFT. Experiments conducted on the MR Images of brain and knee show that proposed method gene-rates good quality images.

Keywords--- CS-MRI, MR Reconstruction, Compressed Sampling, Undersampling, Sampling Trajectory, Wlash Hadamard Transform, Sparsifying Transform.

1. INTRODUCTION

Magnetic Resonance Imaging is a non-invasive imaging scheme that generates images of internal organs of the human body. It is an effective non-invasive imaging modality used in the diagnosis of wide range of diseases [1]. Excellent soft tissue contrast of MRI enables good visualization of tissues and organs within the patient body. MRI works on the principle of Nuclear Magnetic Resonance (NMR). During data acquisition phase, the echo signals are stored in k-space. The collected crude k-space data is transformed into an image by suitable reconstruction method which is then visualized by medical practitioners for diagnostic and analysis purposes. Main drawback of MRI is that it is inherently a slow process [2]. This is because the k-space samples are acquired sequentially in time. Therefore various methods are developed to reduce the scan time by collecting lesser number of samples and to reconstruct the image using minimum number of samples. This process speeds up the scanning process.

The traditional approach of reconstructing signals or images from measured data follows the well-known Shannon-Nyquist's sampling theorem, which states that the sampling rate must be twice the highest frequency. Similarly, the fundamental theorem of linear algebra suggests that the number of collected samples (measurements) of a discrete finite-dimensional signal should be at least as large as its length (its dimension) in order to ensure reconstruction.

The idea of CS, which was developed in recent years by, Donoho [3],Candes [4]and Lustig et al., [5] applied it on MRI. In recent times, applying the theory of Compressive Sensing or compressive sampling in MR imaging is of great interest. Compressive sampling reduces the data collection time which in turn reduces patient's exposure to magnetic field [5]. In traditional sampling methods the images are sampled at Nyquist's sampling rate which is twice the largest frequency present in the signal of interest [4]. CS-MRI enables superior quality image reconstruction from undersampled k-space data. It is done by evaluating constrained minimization difficulties by means of nonlinear optimization techniques by utilizing the sparsity of images in a particular sparsifying transform. Using this strategy it is possible to reconstruct MRI from reduced measurements, thus minimize the imaging time.

CS technique makes use of sparse nature of the signal in some domain and performs the reconstruction of complete signal / image from comparatively a small number of samples. Due to the high correlated feature of MRI data, the image reconstruction from partial k-space using CS approach will have an effect on the time taken for MRI of patient[15]. In MRI k-space, energy distribution of image information is concentrated more at central region than at peripheral region. Since energy allocation is non uniform, a random undersampling technique will result in low frequency aliasing. Therefore variable density undersampling pattern is used which selects more samples from the central region of k-space and lesser samples from the outer peripheral region [7]. Such optimum sampling pattern should consider the energy distribution of k-space [14]. An adaptive learning sparsifying transform appraoch(dictionary based) is also used for MRI reconstrcution from higly undersampled MR k-space[13]. In this paper we demonstrate that the Walsh Hadamard transform can also be used as the sparsifying transform in CS MRI. The proposed work is compared with the standard DFT technique. Total Variation is also used as a penalty to suppress noise in the reconstruction. Performance analysis of CS-MRI reconstruction for different sampling percentages is carried out in terms of PSNR, MSE and SSIM. The efficiency of the proposed method is established in the quality of reconstructed MR image.

2. METHODOLOGY

2.1 Proposed method

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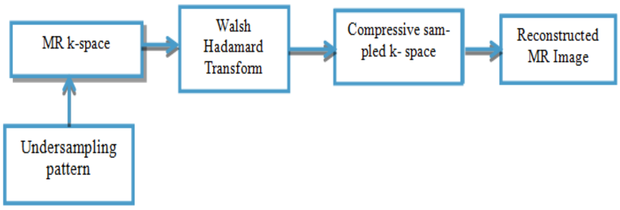


Figure 1: Block diagram of the proposed method

The block diagram of the proposed work is as shown in Figure.1. The undersampling pattern is applied on the generated k-space. The Walsh Hadamard Transform (WHT) is used as sparsifying transform. The Cartesian or Radial sampling patterns are used to collect samples from the k-space. The sampling patterns used for compressive sampling are as shown in Figure.2. The MR image is reconstructed using the reconstruction standard algorithm. The performance of the WHT as sparsifying transform is compared with the standard DFT.

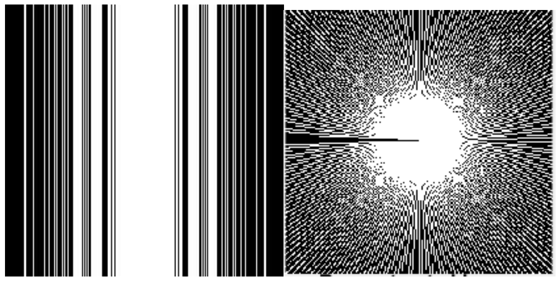
2.2 MR k-space

The raw MR data collected from the scanning machine is stored in k-space. The raw data is in spatial domain which will be subjected to Fourier transform to generate the MR image. Each slice contain information about the frequency, phase and intensity of the signal. The final image is reconstructed by applying Inverse Fourier Transform on the k-space. The central region of k-space holdcontrast information whereas the peripheral region contains spatial resolution[9]. The sampling pattern is designed in such a way to collect more samples from the central region than the peripheral. Hence the quality of reconstruction will not suffer even with lesser number of k-space samples. Undersampling the k-space will reduce the time consumed for imaging thereby minimizing the discomfort caused due to long scanning time.

2.3 Compressive Sampling of the k-space

CS takes the advantage of sparsity or compressibility of real world signals. Natural signals have concise characterization when expressed using proper basis Ψ . Here Ψ denotes the linear operator responsible for the transformation of the data from pixel representation to any other selected representation. Incoherence deals with the idea about duality of the signal in time and frequency domain. This leads to faster measurements with lesser memory requirement. Undersampling patterns are developed in such a way to preserve more information at the center than at the periphery of the k-space since majority of the information in k-space image resides at the center. Sparsity and Incoherence are the two conditions to be satisfied to apply CS on MRI.

For a good Signal to Noise Ratio (SNR) and a smaller Mean Squared Error (MSE) more samples must be collected near the origin of the k-space. The variable-density sampling scheme targets to collect larger number of samples near the center of the k-space[10]. Figure. 2 shows the Cartesian pattern for heavy sampling of central k-space lines and lesser selecting lines near the edges.



(a) (b)

Figure 2: (a) Cartesian undersampling pattern (b) Radial Undersampling Pattern

The significant information from the k-space is acquired by applying the sampling patterns as shown in Figure.2(a) and (b). The undersampled data obtained from Cartesian sampling as well as radial sampling pattern[8] are used for the reconstruction of MR image. The white pixels in the sampling pattern correspond to the locations from where the samples are collected. The sampling patterns shown in Figure. 2(a) and (b) collect more samples near the center of k-space (the density of white lines is more) and less samples collected from the peripheral regions(black lines are more).The number of k-space lines that are need to be undersampled from the fully acquired k-space data is known as sampling percentage [14]. The quality of reconstruction is analyzed for lesser sampling percentages.

2.4 Walsh Hadamard Transform

Walsh Hadamard Transform (WHT) is a fast transformation method which makes use of only subtraction and addition operations. The energy compaction characteristics in a WHT are low. Pattern matching can be efficiently performed in WHT domain. In signal processing application WHT is non-sinusoidal, orthogonal transformation technique that splits a signal into a basis function set. These decomposed basis functions are called as Walsh functions and these functions are rectangular or square waves which has the values of +1 or –1. WHT are also referred as Hadamard (Walsh, or Walsh-Fourier transforms)[11]. WHT makes use of basis functions and provides piece-wise constant discrete image band limited approximation.

$$Y_n = \frac{1}{N}\sum_{i=0}^{N-1} x_i WAL(n.i), n = 1,2, \dots, N-1 \ (1)$$

$$x_i = \sum_{n=0}^{N-1} y_n WAL(n,i), i = 1,2, \dots, N-1 \ (2)$$

where $i = 1,2, \dots, N-1$ and $WAL(n.i)$ are Walsh functions, N is the scaling factor. The Walsh matrix is symmetric. The forward and inverse transformations are equal operation except for the scaling factor of 1/N. The equation (1) represents the Forward WHT Transform and the equation (2) represents the Inverse WHT respectively.

2.5 CS Reconstruction

The general form of data acquirement with incomplete samples for CS-MRI is given as:

$$b = Rx \ (3)$$

where$x$ is the reconstructed image, b is the acquired k-space measurement and R is the undersampled Fourier Transform operator which directly relies on the k-space undersampling scheme.

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Let Ψ denote the linear operator that transforms the data from pixel depiction into the selected depiction. x can be correctly reconstructed from portion of k-space by evaluating the minimization problem which is given as[5]:

$$minimize\ \|\Psi\ x\|_1\ s.t.\ \|Rx - b\|_2 < \epsilon (4)$$

where ϵ is a statistic that describes the magnitude of the error defined as the noise variance or the maximum allowable error in the approximation. Here the l_1 norm $\|x\|_1 = \sum_i |x_i|$. Minimizing the objective function $\|\Psi\ x\|_1$ promotes the sparsity of the images.

2.6 Total Variation Minimization

Reconstruction from sub-Nyquist acquisition results in artifacts which are due to undersampling of high spatial frequencies. This can be resolved by smoothing filters. But smoothing results in blurring of image due to the loss of edge details.

To generate the images with suitable quality from sub-Nyquist acquisition a technique called Total Variation (TV) minimization can be used which is done by minimizing the pixel by pixel finite differences in both horizontal and vertical directions across an image. Total Variation minimization procedure is used to improve the quality of the reconstructed image[12].

The non-linear convex program is used to reconstruct the MR image using the down sampled k-space is applied along with TV minimization as given in equation(3) below:

$$minimize\{\|\Psi x\ \|_1 + \alpha.TV(x)\}\ such\ that \|F_u x - y\|_2 < \varepsilon (5)$$

where x is the reconstructed image,y is the deliberate k-space data, F_u is the under sampled Fourier transform, Ψ is the sparsifying transform, $TV(.)$ is the total-variation and ε controls the fidelity of their construction, α is the regularization parameter. The equation (5) finds the sparsest of every single solution to the underdetermined linear system by limiting the l_1-norm of the measured data. The threshold parameter ε is usually set below the expected noise.

3. EXPERIMENTAL RESULTS AND DISCUSSIONS

CS provides an efficient way to acquire and reconstruct natural images from a limited number of linear projection measurements leading to sub-Nyquist sampling rates. Compressive sensing is achieved by applying the sampling trajectory on the k-space. In this the proposed variable density sampling is applied to MRI image acquisition and reconstruction. This algorithm is implemented in MATLAB 2015a and the MRI image to be measured is of size 256 x 256.The MR Image used for the experiment is obtained from “MR_tip” database.

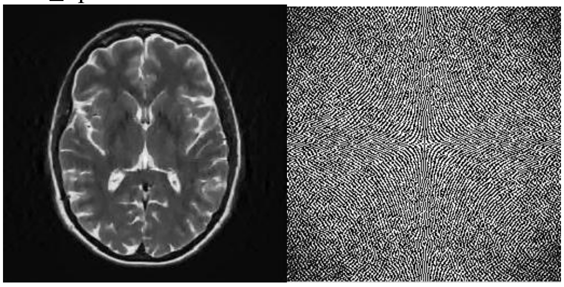

Figure 3: (a) Original MR Brain Image (b) k-space

Figure 3(a) shows the original MR image of brain.The k-space generated is shown in Figure 3(b).The sampling pattern is masked with the k-space of input MR image to compressively sample the k-space. Thereby only few important samples are acquired.

The reconstruction of the MR image from k-space is performed by taking IDFT of the k-space samples. Since the k-space is undersampled nonlinear optimization technique is used to reconstruct the image.

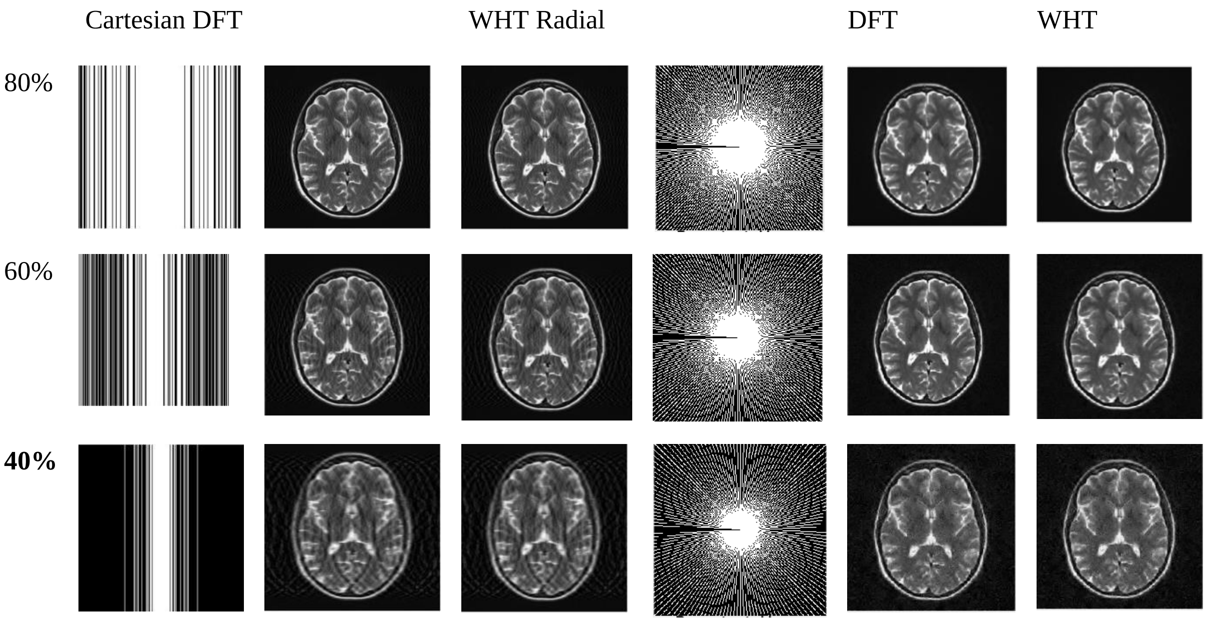

Figure 4: MR Image Reconstructed using DFT and WHT for Cartesian and Radial Sampling Pattern

The under-sampled data obtained from Cartesian sampling pattern are used for the reconstruction process using DFT and WHT technique. The simulation results are as shown in Figure 4. As the sampling percentage reduces reconstruction suffers from artifacts in the reconstructed image. To reduce the artifacts total variation minimization technique is applied.

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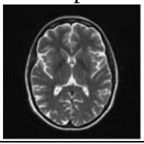
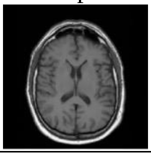
Table I: Numerical analysis of MR image Reconstruction using WHT as sparsifying transform

MRI	Cartesian sampling without TV				Cartesian sampling with TV		
	Sampling Percentages	**PSNR**	**MSE**	**SSIM**	**PSNR**	**MSE**	**SSIM**
Brain of patient 1	80	45.38	1.88	0.9893	45.91	1.66	0.9914
	70	43.16	3.14	0.9846	43.65	2.80	0.9876
	60	40.18	6.23	0.9716	40.75	5.46	0.9781
	50	35.71	17.42	0.9262	36.36	15.01	0.9462
	40	31.61	44.8	0.8449	32.29	38.29	0.8873
Knee	80	40.17	6.24	0.9616	40.67	5.57	0.9727
	70	38.20	9.82	0.9447	38.62	8.93	0.9597
	60	36.29	15.26	0.9183	36.69	13.92	0.9402
	50	34.06	25.48	0.8714	34.48	23.17	0.9081
	40	32.02	40.76	0.8120	32.45	36.95	0.8651
Brain of patient 2	80	45.88	1.67	0.9878	43.36	1.50	0.9902
	70	43.99	2.59	0.9835	44.46	2.32	0.9867
	60	41.58	4.51	0.9742	42.11	4.0	0.9801
	50	38.19	9.85	0.9443	38.82	8.5	0.9594
	40	34.92	20.90	0.8920	35.62	17.80	0.9220

The numerical analysis carried on the set of data which consist of MR data of brain of patient 1 and brain MRI data for patient 2 and knee MR data. Table I. displays the PSNR, MSE and SSIM measured on the reconstructed MR Image when Cartesian undersampling pattern is applied on the k-space to collect samples and WHT is used as sparsifying transform. With the sampling percentage of 40, appreciable values generated for PSNR, MSE and SSIM. SSIM at 40% sampling percentage is 0.8873 for brain image of patient 1, 0.8651 for knee and 0.9220 for brain image of patient 2. This means the quality of the reconstructed image is almost similar to the image reconstructed using maximum number of samples. Total variation minimization reduces the noise in the reconstruction, improving the quality of reconstructed image.

Table II: Numerical analysis of MR image Reconstruction using WHT as sparsifying transform

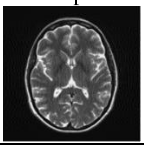
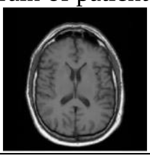
MRI	Radial sampling without TV				Radial sampling with TV		
	Sampling Percentages	**PSNR**	**MSE**	**SSIM**	**PSNR**	**MSE**	**SSIM**
Brain of patient 1	80	44.32	2.40	0.9874	44.82	2.41	0.9899
	70	41.52	4.57	0.9788	42.04	4.06	0.9832
	60	37.62	11.24	0.9504	38.25	9.72	0.9630
	50	33.12	31.68	0.8789	33.81	27.04	0.9125
	40	28.85	84.70	0.7633	29.53	72.39	0.8273
Knee	80	39.01	8.15	0.9528	39.45	7.37	0.9657
	70	36.97	13.06	0.9292	37.36	11.92	0.9479
	60	34.96	20.71	0.8917	35.37	18.86	0.9218
	50	32.83	33.85	0.8380	33.24	30.77	0.8830
	40	30.70	55.29	0.7679	31.12	50.15	0.8332
Brain of patient 2	80	44.76	2.16	0.9859	45.24	1.94	0.9884
	70	42.36	3.77	0.9775	42.89	3.33	0.9824
	60	39.73	6.9	0.9603	40.32	6.03	0.9702
	50	36.27	15.34	0.9185	36.93	13.16	0.9407
	40	32.17	39.39	0.8188	32.90	33.29	0.8695

Table II gives the numerical analysis for the application of proposed method on the same set of MR images with the radial sampling pattern is applied on the k-space to collect undersampled data and WHT is used as sparsifying transform. Qualitative and quantitative analysis on the proposed method reveals that the radial sampling pattern is inferior to the Cartesian sampling pattern. The PSNR, MSE and SSIM values for the MR image reconstructed using 40% samples of the k-space show that the k-space sparsified using WHT and radial sampling even with the combination of TV minimization performs poor compared to the former method.

For different set of MR data reconstructed image suffers from poor values of PSNR, MSE and SSIM for smaller sampling percentages with radial sampling pattern. Image quality improves by combining TV minimization with WHT.

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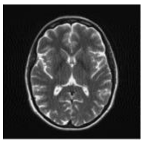
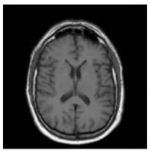
Table III: Comparative results of MR image Reconstruction using DFT TV and WHT TV

MRI	DFT TV				WHT TV		
	Sampling Percentages	**PSNR**	**MSE**	**SSIM**	**PSNR**	**MSE**	**SSIM**
Brain of patient 1 	80	42.18	10.96	0.9828	45.91	1.66	0.9914
	70	40.29	12.46	0.9810	43.65	2.80	0.9876
	60	36.70	14.01	0.9803	40.75	5.46	0.9781
	50	34.46	23.42	0.9791	36.36	15.01	0.9462
	40	32.12	24.19	0.9677	32.29	38.29	0.8873
Knee 	80	43.03	4.87	0.9854	40.67	5.57	0.9727
	70	40.25	6.52	0.9823	38.62	8.93	0.9597
	60	38.68	8.88	0.9739	36.69	13.92	0.9402
	50	35.45	18.68	0.9668	34.48	23.17	0.9081
	40	37.14	12.65	0.9667	32.45	36.95	0.8651
Brain of patient 2 	80	47.01	1.25	0.9889	43.36	1.50	0.9902
	70	46.84	1.32	0.9873	44.46	2.32	0.9867
	60	46.58	1.44	0.9830	42.11	4.0	0.9801
	50	44.63	2.26	0.9820	38.82	8.5	0.9594
	40	39.29	7.71	0.9755	35.62	17.80	0.9220

The proposed work is compared with the standard method used for reconstruction. The numerical analysis performed on the DFT TV and WHT TV methods are given in Table III. As the sampling percentage reduces the image quality in terms of SSIM remains still near 1(standard SSIM value between 0 to 1). Therefore visual quality of reconstructed image is good even at lesser sampling percentages. PSNR decreases slightly with no effect on reconstruction quality. The values obtained in this experiment for different MR images depict that WHT combined with TV performs well compared to DFT technique. Further it can be observed that application of Cartesian sampling mask for k-space data collection results in better MR image than by using radial sampling pattern.

Brain of patient 1

Knee

Brain of patient 2

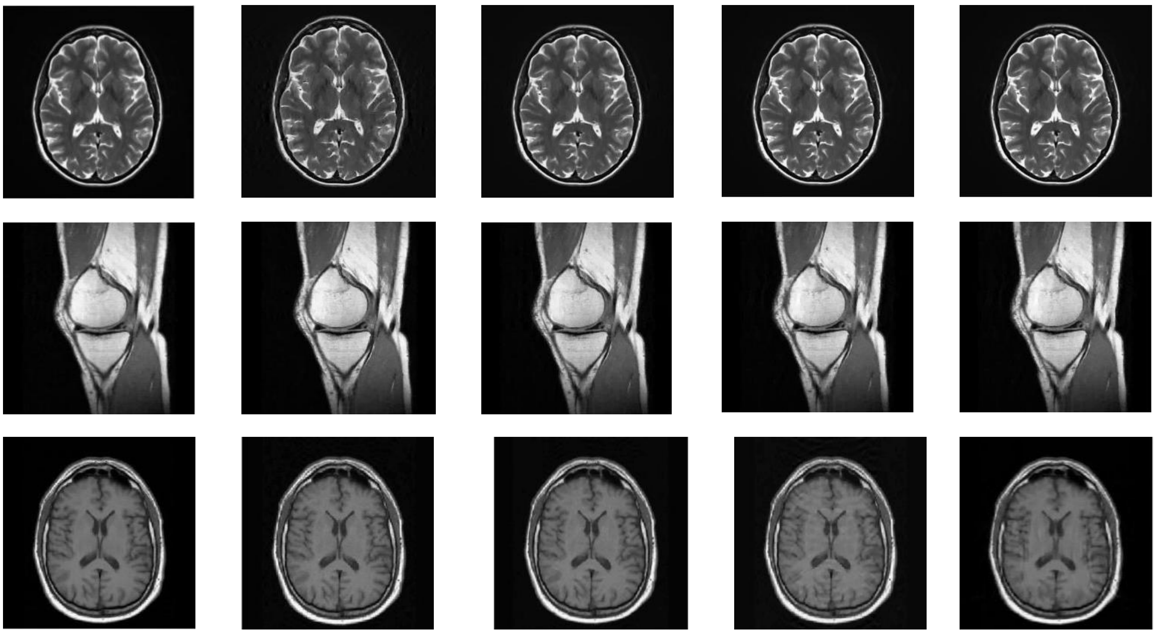


Figure 5: The reconstructed MRI of brain of patient1, knee, brain of patient2 using Cartesian sampling pattern for DFT for different scan percentage with TV for 80, 70, 60,50,40 percent sampling respectively

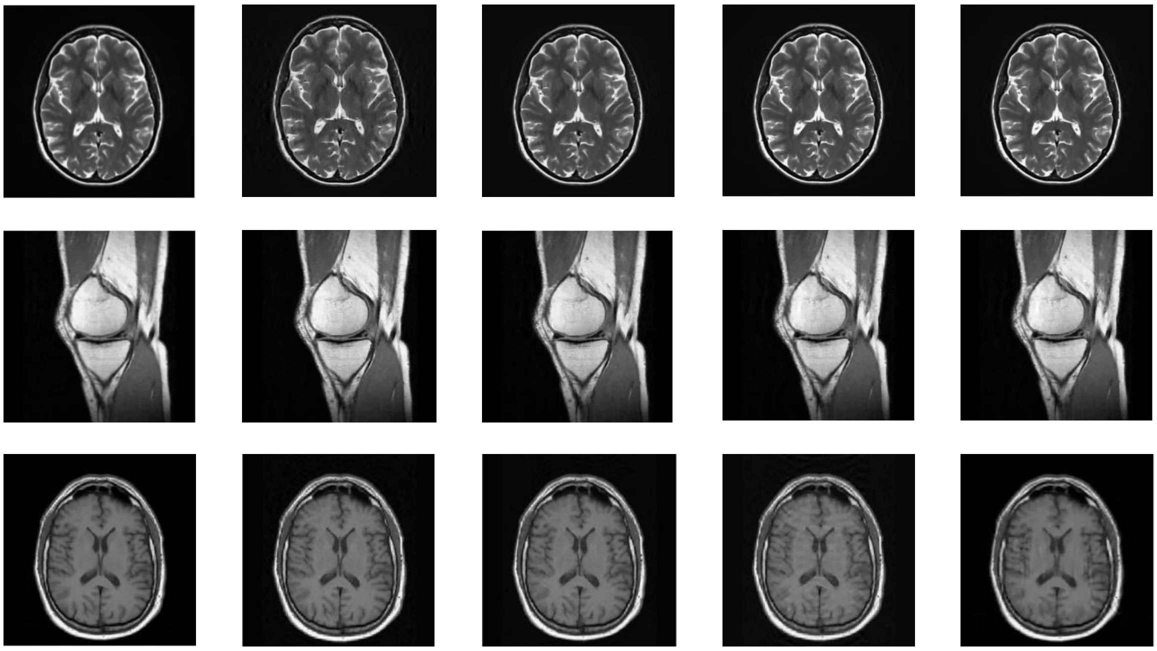


Figure 6: The reconstructed MRI of brain of patient1, knee, brain of patient 2 using Cartesian sampling pattern for WHT for different scan percentage with TV for 80, 70, 60,50,40 percent sampling respectively

Figure 6.shows the reconstructed images at different sampling percentages for three types of MR images. The image quality is substantially good at 40% sampling retaining all the features as in original image. The proposed work can reconstruct MR images with considerably good quality with the lesser k-space samples in comparison with the existing method which uses DFT as the sparsifying transform. Collecting lesser samples speeds up the MR process and hence the time of exposure of patient to the magnetic field reduces.

4. CONCLUSION

In the proposed work the MRI is reconstructed from the very few k-space coefficients which are sparsified usingWalsh Hadamard Transform and the reconstructed image is compared with the standard method which uses DFT. The k-space is compressively sampled using Cartesian and using Radial patterns. For different sampling percentages the quality of the reconstructed MR image is analyzed in terms of PSNR, MSE and SSIM. The experimentation performed on different MR images demonstrates that the proposed method produces a faithful recovery even with lesser samples. The reconstruction with the proposed method is compared with the standard method which uses DFT, reveals that WHT can also be used as an effective sparsifying transform. Further the experimentation can be conducted with lesser sampling percentages. Smaller the number of k-space samples the smaller will be the time the patient spends for the MR scanning process.

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