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Inventory Model of Deteriorating Product with Discontinuous Demand

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***Abstract*: In this paper, an inventory model has been developed for deteriorating items with discontinuous demand rate. It is assumed that the deterioration rate is constant and demand rate is quadratic function of time. Shortages are not allowed.**

***Keywords* : Deterioration, demand and quadratic.**

I. INTRODUCTION

Several authors developed different inventory models. Some of them have taken the stock dependent demand while the others the time dependent demand. In the development of the inventory models the deterioration of the product is an important factor. Deterioration rate is assumed constant or variable. In most of the inventory models discussed so far the demand rate is assumed to be continuous in most of the inventory models discussed so far. It is assumed as constant or current stock dependent or time dependent.

Covert, R.P. & Philip, G.C. (1973) developed an inventory model in which they considered Weibull distribution deterioration rate of items. **Hari Kishan & Mishra, P.N.** (1997) developed an inventory model in which they considered exponential demand, constant deterioration with shortage. **Jain, S. & Kumar, M.** (2007) developed an inventory model in which they considered the stock dependent demand rate with shortage

The demand is not always continuous in practice. For example the shop opens for certain period in a day in which demand exists. Then the shop is closed and the demand does not exist. But the deterioration of items continues. In this way it is a realistic factor that that demand may be discontinuous. So it is fruit full to develop such a model. **Hari Kishan, Saroha and Megha Rani** (2014) developed an inventory model in which they considered discontinuous demand and constant deterioration. They assumed linear demand rate.

In this paper, an inventory model has been discussed for deteriorating items with discontinuous demand rate. It is assumed that the deterioration rate is constant and demand rate is quadratic function of time. Shortages are not allowed.

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II. THE ASSUMPTIONS AND THE NOTATIONS

In this paper, the following assumptions are considered:

(i) Replenishment is instantaneous.
(ii) Demand rate is quadratic function of time, i.e. it is taken of the form $at^2 + bt + c$.
(iii) Deterioration is constant.
(iv) Shortages are not allowed.

In this paper, the following notations are used:

(i) Q=The current stock level.
(ii) Q:=The initial stock level
(iii) C= The set up cost for one cycle.
(iv) $c_i =$ Inventory carrying cost per unit of item and per unit time.
(v) $c_d =$ The deterioration cost per unit of item
(vi) $\theta =$ The deterioration rate.
(vii) $T =$ The cycle time.
(viii) $T_1 =$ The time upto which the demand exists.
(ix) $K =$ The total average cost of the inventory system.

III. 3 The Mathematical Model:

It is assumed that Q is the starting stock level and q is the stock level at any time t. It is considered that the demand exists upto the time T_1. After that due to the closing of the shop the demand becomes zero. Now due to the stock of items the deterioration of items continues upto time T. In this period the stock level reduces to Q_1 at time T. The inventory model is shown in the following figure 1:

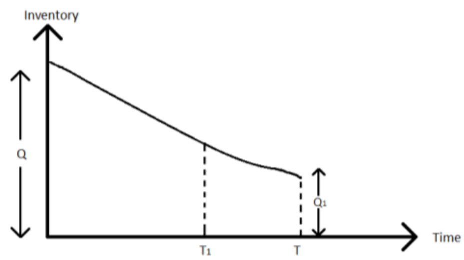

Figure 1

Mathematically, this inventory model may be represented by the following differential equations

$$\frac{dq}{dt} + \theta q = -(at^2 + bt + c), \qquad 0 \le t \le T_1 \qquad \ldots(1)$$

$$\frac{dq}{dt} + \theta q = 0. \qquad T_1 \le t \le T \qquad \ldots(2)$$

The corresponding boundary conditions are given by

$$q(0) = Q \text{ and } q(T) = Q_1 \qquad \ldots(3)$$

IV. ANALYSIS

Solving equation (1), we get

$$qe^{\theta t} = -(at^2+bt+c)\frac{e^{\theta t}}{\theta} + (2at-2a+b)\frac{e^{\theta t}}{\theta^2} + c_1.$$

Using boundary condition (3) in the above expression, we get

$$q = -(at^2+bt+c)\frac{1}{\theta} + (2at-2a+b)\frac{1}{\theta^2} + \left(Q+\frac{c}{\theta}+\frac{(2a-b)}{\theta^2}\right)e^{-\theta t}.$$

$$0 \le t \le T_1 \qquad \ldots(4)$$

Solving equation (2) and using boundary condition (3), we get

$$q = Q_1 e^{\theta(T-t)}. \qquad T_1 \le t \le T \qquad \ldots(5)$$

The carrying cost in the cycle $[0, T]$ is given by

$$C_C = c_i \int_0^T q dt$$

$$= c_i\left[\int_0^{T_1} q dt + \int_{T_1}^T q dt\right]$$

$$= c_i\left[\int_0^{T_1} -(at^2+bt+c)\frac{1}{\theta} + (2at-2a+b)\frac{1}{\theta^2} + \left(Q+\frac{c}{\theta}+\frac{(2a-b)}{\theta^2}\right)e^{-\theta t}dt + \int_{T_1}^T Q_1 e^{\theta(T-t)}dt\right]$$

$$= c_i\left[-\left(\frac{aT_i^3}{3}+\frac{bT_i^2}{2}+cT_1\right)\frac{1}{\theta} + (aT_i^2-2aT_1+bT_1)\frac{1}{\theta^2} + \left(Q+\frac{c}{\theta}+\frac{(2a-b)}{\theta^2}\right)\frac{(1-e^{\theta T_1})}{\theta} + \frac{Q_1}{\theta}\left(e^{\theta(T-T_1)}-1\right)\right] \qquad \ldots(6)$$

The deteriorating cost is given by

$$D_C = c_d \int_0^T \theta q dt$$

$$= c_d\left[\int_0^{T_1} \theta q dt + \int_{T_1}^T \theta q dt\right]$$

$$= c_d\left[-\left(\frac{aT_i^3}{3}+\frac{bT_i^2}{2}+cT_1\right) + (aT_i^2-2aT_1+bT_1)\frac{1}{\theta} + \left(Q\theta+\frac{c}{\theta}+\frac{(2a-b)}{\theta^2}\right)(1-e^{\theta T_1}) + Q_1\left(e^{\theta(T-T_1)}-1\right)\right] \qquad \ldots(7)$$

The average cost in the cycle $[0, T]$ is given by

$K =$ Average setup cost + average inventory carrying cost +average deterioration cost

$$= \frac{C}{T} + \frac{c_i}{T}\left[-\left(\frac{aT_i^3}{3}+\frac{bT_i^2}{2}+cT_1\right)\frac{1}{\theta} + (aT_i^2-2aT_1+bT_1)\frac{1}{\theta^2} + \left(Q+\frac{c}{\theta}+\frac{(2a-b)}{\theta^2}\right)\frac{(1-e^{\theta T_1})}{\theta} + \frac{Q_1}{\theta}\left(e^{\theta(T-T_1)}-1\right)\right] + \frac{c_d}{T}\left[-\left(\frac{aT_i^3}{3}+\frac{bT_i^2}{2}+cT_1\right) + (aT_i^2-2aT_1+bT_1)\frac{1}{\theta} + \left(Q\theta+\frac{c}{\theta}+\frac{(2a-b)}{\theta^2}\right)(1-e^{\theta T_1}) + Q_1\left(e^{\theta(T-T_1)}-1\right)\right]$$

By the application of the second approximation in θ, the above expression becomes

$$K = \frac{C}{T} + \frac{c_i+c_d\theta}{T}\left[-\left(\frac{aT_1^3}{3}+\frac{bT_1^2}{2}+cT_1\right)\frac{1}{\theta} + (aT_1^2-2aT_1+bT_1)\frac{1}{\theta^2} + \left(Q+\frac{c}{\theta}+\frac{(2a-b)}{\theta^2}\right)\left(T_1-\frac{\theta T_1^2}{2}\right) + Q_1\left((T-T_1)+\theta(T-T_1)^2\right)\right] \qquad \ldots(8)$$

Now the following cases are considered:

Case 1: When Q and T are fixed and T_1 is variable:

In this case, for minimum value of K, we have

$$\frac{dK}{dT_1} = 0,$$

or

$$\left[-(aT_1^2+bT_1+c)\frac{1}{\theta} + (2aT_1-2a+b)\frac{1}{\theta^2} + \left(Q+\frac{c}{\theta}+\frac{(2a-b)}{\theta^2}\right)(1-\theta T_1) - Q_1\left(1+\theta(T-T_1)\right)\right] = 0. \qquad \ldots(9)$$

$$\frac{d^2K}{dT_1^2} = \frac{c_i+c_d\theta}{T}\left[-(2aT_1+b)\frac{1}{\theta} + (2a)\frac{1}{\theta^2} - \theta\left(Q+\frac{c}{\theta}+\frac{(2a-b)}{\theta^2}\right) + \theta Q_1\right]. \qquad \ldots(10)$$

Now K will be minimum for the value of T_1 obtained from (9) which provides $\frac{d^2K}{dT_1^2} > 0$.

Case 2: When Q and T_1 are fixed and T is variable:

In this case, for minimum value of K, we have

$$\frac{dK}{dT} = 0,$$

or

$$-c + (c_i+c_d\theta)\left[\left(\frac{aT_1^3}{3}+\frac{bT_1^2}{2}+cT_1\right)\frac{1}{\theta} - (aT_1^2-2aT_1+bT_1)\frac{1}{\theta^2} - \left(Q+\frac{c}{\theta}+\frac{(2a-b)}{\theta^2}\right)\left(T_1-\frac{\theta T_1^2}{2}\right) - Q_1\left(T_1+\theta(T^2-T_1^2)\right)\right] = 0. \qquad \ldots(11)$$

$$\frac{d^2K}{dT^2} = \frac{C}{T^3} + \frac{c_i+c_d\theta}{T^3}\left[-\left(\frac{aT_1^3}{3}+\frac{bT_1^2}{2}+cT_1\right)\frac{1}{\theta} + (aT_1^2-2aT_1+bT_1)\frac{1}{\theta^2} + \left(Q+\frac{c}{\theta}+\frac{(2a-b)}{\theta^2}\right)\left(T_1-\frac{\theta T_1^2}{2}\right) + 2Q_1(T_1-\theta T_1^2)\right] \qquad \ldots(12)$$

Now K will be minimum for the value of T_1 obtained from (11) which provides $\frac{d^2K}{dT^2} > 0$.

Case 3: When Q is fixed and T and T_1 are variables:

In this case, for minimum value of K, we have

$$\frac{\partial K}{\partial T_1} = 0 \text{ and } \frac{\partial K}{\partial T} = 0, \qquad \ldots(13)$$

provided $\frac{\partial^2 K}{\partial T_1^2} > 0$, $\frac{\partial^2 K}{\partial T^2} > 0$ and

$$\left(\frac{\partial^2 K}{\partial T_1^2}\right)\left(\frac{\partial^2 K}{\partial T^2}\right) - \left(\frac{\partial^2 K}{\partial T_1 \partial T}\right)^2 > 0. \qquad \ldots(14)$$

From (13), we have

$$\left[-(aT_1^2+bT_1+c)\frac{1}{\theta}+(2aT_1-2a+b)\frac{1}{\theta^2}+\left(Q+\frac{c}{\theta}+\frac{(2a-b)}{\theta^2}\right)(1-\theta T_1)-Q_1\big(1+\theta(T-T_1)\big)\right]=0. \quad \ldots(15)$$

$$-c+(c_i+c_d\theta)\left[\left(\frac{aT_1^3}{3}+\frac{bT_1^2}{2}+cT_1\right)\frac{1}{\theta}-(aT_1^2-2aT_1+bT_1)\frac{1}{\theta^2}-\left(Q+\frac{c}{\theta}+\frac{(2a-b)}{\theta^2}\right)\left(T_1-\frac{\theta T_1^2}{2}\right)-Q_1\big(T_1+\theta(T^2-T_1^2)\big)\right]=0. \quad \ldots(16)$$

The solution of simultaneous equations (15) and (16) that satisfies relations (14) will give the optimal solution. These may be represented by T_1^*, T^* and K^*.

V. NUMERICAL EXAMPLE:

Consider Q=1000, Q_1=200, θ=0.001, a=15, b=10, c=5, c_i=\$3, c_d=\$3 and C=\$5000.
Using equations (4) to (16), we get the optimal solution using the software MATHEMATICA. These may be represented by $T_1^* = 0.00001$, $T^* = 5600.83$ and $K^* = \$3964.018$.

VI. CONCLUDING REMARKS:

In this paper, an inventory model of deteriorating items has been discussed. Here the deterioration rate is taken as constant and the demand rate is quadratic function of time for one part of the cycle and zero in the other part of the cycle. This work can further be extended for other forms of demand and deterioration.

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