International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249-8958 (Online), Volume-8 Issue-6, August 2019

A Road Map of Optimization Problem Solving Techniques, Nomenclature and Algorithms

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Abstract**: *Optimization problems are different from other mathematical problems in that they are able to discover solutions which are ideal or near ideal in accordance to the goals. Problems are not solved in one step, but we follow different sequence of steps to reach the solution. The steps could be to define problems, construct and solve models and evaluate and implement solutions. This paper presents an overall outlook of how a problem of optimization type can be solved.***

Keywords : A Constraints, Optimality criteria, Taxonomy, Optimization algorithms

I. INTRODUCTION

Optimization refers to discovery of the solution which is best from among an prevailing set of reasonable solutions. It can be to maximize or minimize a function with any possible constraints [1] [17][18] . There were many optimization problems which were solved earlier which was before the calculus of variations was invented. Later the mathematical analysis that is the basis of calculus of variation was created by I. Newton (1660s) and G.W. von Leibniz (1670s). J.L .Lagrange introduced the method of optimization for constrained problems. The gradient method to solve unconstrained problems was used by Cauchy in 1847[2]. In 1947, Simplex method was proposed. In 1984, N. Karmakar introduced the polynomial time algorithm which began a boom of interior point optimization methods.

2. Steps for solving an optimization problem

Following are the steps:

Building a Prototype:

The initial and foremost step in problem solving using optimization is to construct a prototype. A model / prototype will explain in mathematical terms the three important concepts of an optimization problem namely, the objective, the variables, and the constraints of the problem [3].

The performance of a system is an example of an Objective of an optimization problem. It is thus a quantitative measure that we want to minimize or maximize. For example in a business problem the objective may be to maximize gains or minimize loss.

Revised Manuscript Received on August 30, 2019.
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In order to achieve maximization or minimization of the objective function, we need to analyze and find out the values of some unknowns, which are otherwise termed as variables [4].

We find the values of variables which are bounded by certain restrictive conditions. These restrictive conditions which bound the value of variables are called the constraints. [5]. For example, in the task of optimizing a business problem, there may be time constraints, cost constraints, labor constraints and so on.

Determining the Problem Type: In this we determine in which category of optimization, model belongs to. A list is provided for the various optimization problem types, with some basic information in Table.1.

Selecting Software: There are different categories of Optimization problems. Each problem requires a different software based on the type of the problem [6]. The software for solving optimization problems is 1) Solver Software and 2) Modeling Software. If you need to find answer to a precise example of a model, we use Solver software. The foremost goal of modeling software is to help users, frame optimization prototypes and evaluate their solutions [7].

Commercial vs. Open Source Solvers: The solvers may be commercial solvers and open source solvers. Commercial solvers come under a cost and are more reliable than open source solvers. For open source solvers, the source code is freely available.

Table 1. Optimization Problem Types

Optimization Problem Types	Description
Continuous Optimization	Optimization problems that can take real values ;
Discrete Optimization	Optimization problems which take discrete values
Constrained Optimization	Where variables are bounded by constraints belong to constrained optimization types. The other subcategories are (e.g., linear, nonlinear, convex) as per the nature of the constraints.
***Unconstrained** Optimization*	In Unconstrained optimization problems, we use penalty terms in place of constraints, in the objective function.
None, One or Many Objectives	When there is no objective or multiple objective functions, such problems belong to this category.

Deterministic Optimization	In deterministic optimization, we know the data for the given problem accurately. This is far from real.
Stochastic Optimization	When the model itself has built in uncertainty it is called stochastic optimization. These models make assumptions on probability distributions governing the data.

2.1 Classification of Optimization

In decision making of an organization or business, there are different levels: Strategic decisions which define the long term operations and the operational level which are decisions taken on daily basis. Optimization methods may be applied to any of these planning levels [8]. However, it is difficult to apply optimization for making operational decisions, since we have to take into consideration multiple and varied criteria in the optimization model [9][19][20]. Another hurdle is that actual requirements may be different from the routine working of the optimization method. Finally, the users may have distrust or misapprehension of automated optimization systems [10].

A better alternative is to apply hybrid optimization approach, where the decision maker will also be part of the optimization process. Thu optimization methods in turn end up as effective decision tools. There are numerous approaches discovered in the last century. It is difficult to provide taxonomy of optimization because many of the subfields have multiple contents. One perspective, focused mainly on the subfields of deterministic optimization with only one objective function is described in the Figure.1.

2.2 Optimization Algorithms

An algorithm which repeatedly executes various sequence of steps to reach at an ideal solution, is an Optimization algorithm.

Optimization algorithms, are also used to assess design tradeoffs, to find patterns in data and so many more. A reasonable number of papers have tried to compare effectiveness of diverse optimization algorithms [11][12][13].

List of optimization algorithms is described in the table2 below:

Table 2: List of Optimization Algorithms

Optimization Algorithms	Description
Deterministic Algorithms	They use precise rules for moving one solution to other. They are used extensively in engineering design problems.
Stochastic Algorithms	The stochastic algorithms follow probabilistic translation rules. No single optimization algorithm which will work in all optimization problems equals efficiently.
Single-variable optimization algorithms i) Direct methods ii) Gradient based methods	Direct methods will not use any derivative information of the objective function; only objective function values are used to monitor the search process. Gradient-based methods use derivative information (first and/ or second order) to help the search process.
Multi- variable optimization algorithms	These algorithms attempt to search for the optimum point in multiple dimensions. If Gradient information is used, these algorithms are called direct techniques and gradient-based techniques otherwise.
Constrained optimization algorithms	These algorithms are mostly used in engineering optimization problems.
Specialized optimization algorithms	Two of these algorithms - integer programming and geometric programming - are often used in engineering design problems. Integer programming methods can solve optimization problems with integer design variables. Geometric programming methods have the objective functions and constraints written in a special form.
Non-traditional optimization algorithms	There are two algorithms which are nontraditional, these are: *a) Genetic algorithms and b) Simulated annealing.*
Other optimization algorithms	Algorithms for Quadratic Programming, Augmented Lagrangian Methods, Broyden's Method , Central Cutting Plane Methods, Difference Approximations , Discretization Methods, Feasible Sequential Quadratic Programming ,Gauss-Newton Method , Gradient Projection Methods, Homotopy Methods , Hybrid Methods, Interior-Point Methods, KKT Reduction Methods, Large Scale Methods, Levenberg-Marquardt Method, Line Search Methods, Newton Methods, Nonlinear Conjugate Gradient Method, Nonlinear Simplex Method, Quasi-Newton Methods, Reduced-Gradient Methods, SQP Reduction Methods, Sequential Quadratic Programming, Simplex Method, Tensor Methods, Truncated Newton Methods, Trust-Region Methods.
Combinatorial optimization	Optimization problems where the set of feasible solutions is discrete. • Greedy randomized adaptive search procedure (GRASP): • Hungarian method: a combinatorial optimization algorithm.
Ellipsoid method	It is an algorithm used to solve convex optimization problems
Evolutionary computation	Optimization inspired by biological mechanisms of evolution: Gene expression programming, Memetic algorithm, *Swarm intelligence*: Ant colony optimization, Bees algorithm, Particle swarm.
Local search	A meta-heuristic designed to solve computationally hard optimization problems: climbing, Abu.

3. Optimization Application and related Fields

The different areas of Application of optimization and the related fields of implementation are provided in the Table 3 below:

Table 3: Optimization Application and related Fields

Optimization Applications	Important fields of implementation
• Scheduling airlines, trains, buses etc. • Assignment problems. • Facility location (deciding the most suitable location for the new facilities such as a warehouse, factory or fire station). • Network flows (managing the flow of water from reservoirs). • Health service (information and supply chain management for health services). • Game theory (identifying, understanding and developing the strategies adopted by companies).	• Assignment problem • Black box analysis • Decision analysis • Dynamic programming • Inventory theory • Linear programming • Mathematical optimization • Queuing theory • Stochastic process • Systems analysis • Behavioral operations research • Business engineering • Business process management
• Stakeholder based approaches including metagame analysis and drama theory morphological analysis and various forms of influence diagrams. • Travelling sales operation • cognitive mapping • Medical electronics • Robustness analysis. • May be applied to military, medical, public administration, charitable groups, political groups or community groups. • Determining optimal prices, in pricing science. • Power systems and Energy conservation Intrusion detection and deadlock prevention • Machine Learning and intelligent computing • Automation and Control System • Bioinformatics • Interactive Learning system • Software code and Query optimization • Critical Path Methods (CPM) and Project Evaluation Techniques (PERT). • Load Balancing • Software Architecture optimization • Consumption Behavior • Search Engine Optimization • Optimal (minimum time) trajectories for space missions	• Database normalization • Econometrics • Engineering management • Geographic information system • Industrial engineering • Industrial organization • Managerial economics • Military simulation • Modeling and simulation • Reliability engineering • Scheduling • Scientific management • Search-based software engineering • Simulation • System dynamics • System safety • Systems theory • Big data and Business Analytics • Data mining • Financial engineering • Forecasting • Game theory • Graph theory • Industrial engineering • Logistics • Mathematical modeling • Probability and statistics • Project management • Policy analysis • Social network/Transportation • Supply chain management

4. Procedure for Optimal Problem Formulation

With a priori problem knowledge alternative solutions are created and by comparing these solutions a rough optimal design is achieved [14]. The steps are outlined in Figure 2.

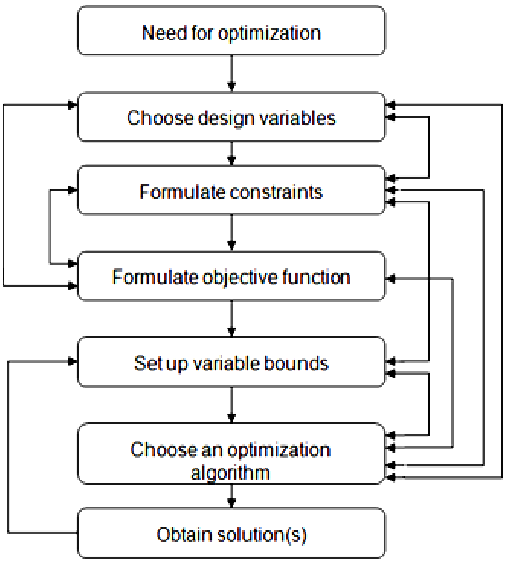

Figure 2 Procedure for the Solving Optimization Problem

The General format of a Non Linear Problem may be written [15] as depicted below:

Column vector $x = (x_1, x_2 \ldots x_N)^T$ *denotes design variables*, the objective function as a scalar quantity $f(x)$, J inequality constraints as $g_i(X) >= 0$ and K equality constraints as $h_k(x) = 0$, we write the NLP problem:

Minimize $f(x)$ Subject to,

$g_j(x) \geq 0 \qquad j = 1,2,3,\ldots J;$

$h_k(x) = 0 \qquad k = 1,2,3,\ldots K;$

$x_i^{(L)} \leq x_i \leq x_i^{(U)} \qquad I = 1, 2,3,.. N;$

4.1 Optimality criteria

We assume that on reaching optimum solution, some characteristic will be attained at such optimum which is known as optimality criteria. [16] [17]

The various types of Optima are as depicted in below figures:

Local and Global Optima

A global optimum is an optimum of the whole domain X while a local optimum is only an optimum of one of its subsets.

Local Maximum: A (local) maximum χ? X of an objective function f : X ? R is an input element with f(χ) = f(x) for all x neighboring χ

if X ? R, we can write:

χ: ? ε > 0 : f(χ) = f(x) ? x ? X, | x - χ| < ε

Local Minimum: A (local) minimum χ? X of an objective function f : X ? R is an input element with f(χ) = f(x) for all x neighboring χ

if X ? R, we can write:

χ: ? ε > 0 : f(χ) = f(x) ? x ? X, | x - χ| < ε

Figure 3: Local Maximum and Local Minimum

Local Optimum: A (local) optimum xi* ? X of an objective function f : X ? R is either a local maximum or a local minimum (or both)

Global Maximum: A global maximum χ? X of an objective function f : X ? R is an input elemen with f(χ)= f(x) ? x ? X

Global Minimum: A global minimum χ? X of an objective function f : X ? R is an input element with f(χ)= f(x) ? x ? X

Global Optimum: A global optimum x* ? X of an objective function f : X ? R is either a global maximum or global minimum (or both)

Figure 4: Local and Global Optima

The following figures give a clear pictorial comparison of local maximum and local minimum and also Global Maxima with Local maxima

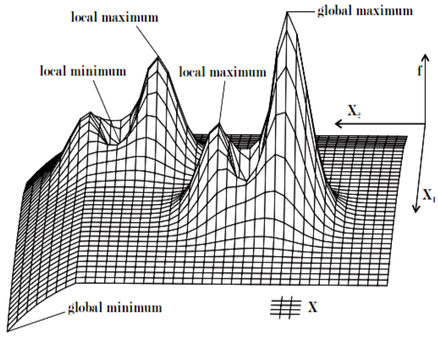

Figure 5: Local Maximum and Local Minimum

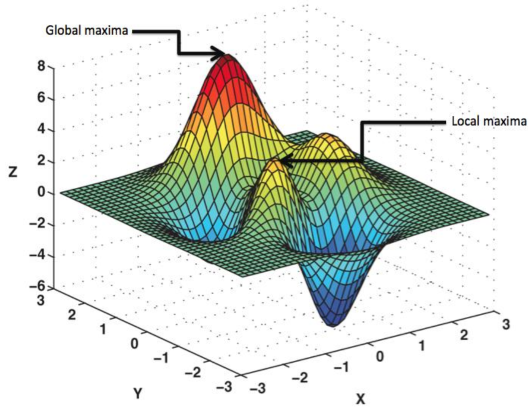

Figure 6: Local and Global maxima

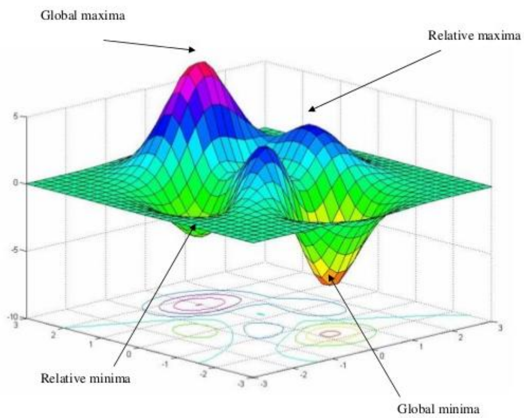

Figure 7: Relative Minimum and maximum

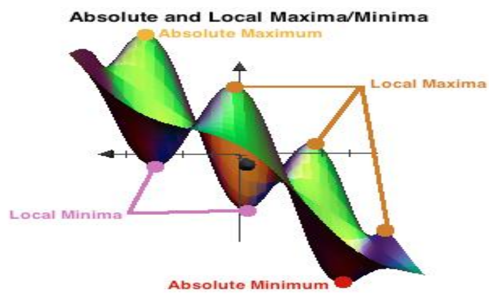

Figure 8: Absolute Maxima/Minima

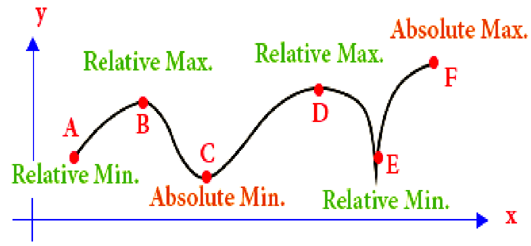

Figure 9: Relative Maxima/Minima

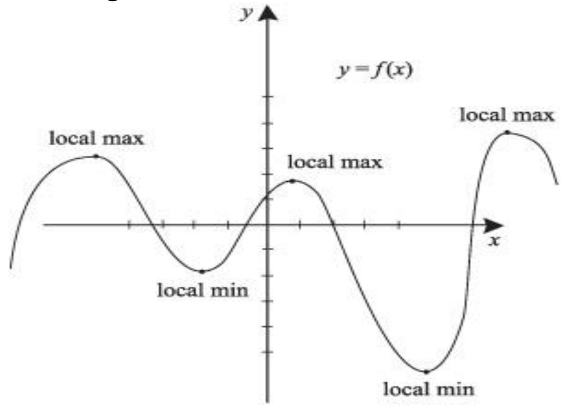

Figure 10: Local Maxima / Minima

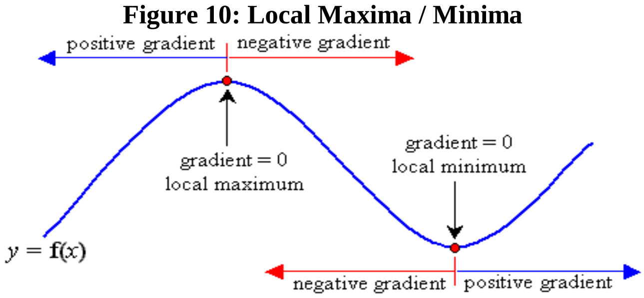

Figure 11: Local Maximum / Minimum

5. Conclusion

The paper is an overview on Optimization techniques used in the various facets of business and a classification of optimization algorithms. The results obtained have been arranged in tables so as to provide a quick and easy insight for researchers in the research field of optimization. The various implications of the cross analysis provided by the optimization techniques and their myriad application areas, can be used as inputs for further research. The different research sub areas can also be explored. The review presented in this paper facilitates for easy and fast information transfer.

To summarize, the belief is that the outcomes of this systematic review and also the presentation of the taxonomy itself will become useful in evolving and mediating new methodologies.

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