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Finite Dimensional Imprecise Numbers

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ABSTRACT--- We discuss difference between the definition of fuzzy number and imprecise number in term of complement. Later using the definition of imprecise number we study the multi-dimensional imprecise numbers. Various classical set theory properties under the intersection and unions are discussed in terms of multi-dimensional imprecise numbers.

Keywords: Indicator Function, Imprecise Numbers, Multi-dimensional imprecise numbers, Membership Value

I. INTRODUCTION

To define a set of membership containing all the elements lies between the real number 0 and 1, fuzzy set is formed in the field of mathematics. The term was first used by Electrical Engineer Lofti A Zadeh in the year 1965. Later the fuzzy set is found that it can also construct a membership function including all their elements. Those membership functions are either trapezoidal shape, bell shape or triangular shape etc. Such special type of fuzzy set is known as fuzzy number. It has many applications in the field of science and Technology. As for example, fuzzy number range is used in the washing machine, sensor technology etc.

In the case of properties study, it is found that fuzzy set/fuzzy number doesn't satisfies the universal laws of classical set. It is due to the complement of fuzzy set defined in terms of membership function form. Later Baruah [2], [3] has defined complement definition of fuzzy number/fuzzy set in terms of membership function and reference function. This new definition of fuzzy number is known as imprecise number.

In this article all the classical set theory properties will be discussed with the help of the definition of imprecise number in terms of multi-dimensional imprecise form.

II. IMPRECISE NUMBERS

N^{th} dimensional imprecise number is expressed in the n^{th} co-ordinate geometry system containing n^{th} number of different faces.

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Here, each and every faces has already membership function in such a way that full membership along the X_1-axis, the X_2 -axis …….Xn axes respectively is considered as a membership value one.

A. *Definition:* The N^{th} dimensional imprecise number

$$N_{X_1X_2.......X_n} = [(\alpha_{x_1},\alpha_{x_2} \dots \dots, \alpha_{x_n}); (\beta_{x_1},\beta_{x_2} \dots \dots, \beta_{x_n}); (\gamma_{x_1},\gamma_{x_2} \dots \dots \dots \dots, \gamma_{x_n})]$$

is divided into sub intervals with a partial element is presence in both the intervals. Where all the points in this interval are elements of Cartesian product $X_1 \times X_2 \times \dots \times X_n$ of n^{th} sets and $X_1, X_2, ...X_n$ are individually imprecise numbers.

In particular, when n=1, then,

$N_X = [\alpha_x; \beta_x; \gamma_x]$……………………………………….(1)

One-dimensional imprecise number.

When, n=2, then

$$N_{X_1X_2} = [(\alpha_{x_1},\alpha_{x_2}); (\beta_{x_1},\beta_{x_2}); (\gamma_{x_1},\gamma_{x_2})]$$

………………..(2)

is the two-dimensional imprecise number.

So, on.

B. *Definition:* Indicator function of the N^{th} -dimensional imprecise number

$$N_{X_1X_2.......X_n} = [(\alpha_{x_1},\alpha_{x_2} \dots \dots, \alpha_{x_n}); (\beta_{x_1},\beta_{x_2} \dots \dots, \beta_{x_n}); (\gamma_{x_1},\gamma_{x_2} \dots \dots \dots \dots, \gamma_{x_n})]$$

is represented by

$$\mu_{N_{X_1X_2.......X_n}}(x_1, x_2, \dots \dots, x_n) = \begin{cases} \mu^1{}_{X_1X_2.......X_n}(x_1, x_2, \dots, x_n); (\alpha_{x_1},\alpha_{x_2}, \dots, \alpha_{x_n}) \le (x_1, x_2 \dots, x_n) \le (\beta_{x_1},\beta_{x_2} \dots, \beta_{x_n}) \\ \mu^2{}_{X_1X_2.......X_n}(x_1, x_2, \dots, x_n); (\beta_{x_1},\beta_{x_2} \dots, \beta_{x_n}) \le (x_1, x_2, \dots, x_n) \le (\gamma_{x_1},\gamma_{x_2} \dots, \gamma_{x_n}) \\ 0 ; \qquad otherwise \end{cases}$$

……………………………..(3)

Such that

$\mu^1{}_{X_1X_2.......X_n}(\alpha_{x_1},\alpha_{x_2} \dots \dots, \alpha_{x_n}) = \mu^2{}_{X_1X_2.......X_n}(\gamma_{x_1},\gamma_{x_2} \dots \dots \dots \dots, \gamma_{x_n}) = (0,0, \dots \dots,0)$ and $\mu^1{}_{X_1X_2.......X_n}(\beta_{x_1},\beta_{x_2} \dots \dots, \beta_{x_n}) = \mu^2{}_{X_1X_2.......X_n}(\beta_{x_1},\beta_{x_2} \dots \dots, \beta_{x_n})$.

Where $\mu^1{}_{X_1X_2.......X_n}(\alpha_{x_1},\alpha_{x_2} \dots \dots, \alpha_{x_n})$ is non-decreasing function over the interval $[(\alpha_{x_1},\alpha_{x_2} \dots \dots, \alpha_{x_n}), (\beta_{x_1},\beta_{x_2} \dots \dots, \beta_{x_n})]$ and $\mu^2{}_{X_1X_2.......X_n}(x_1, x_2, \dots \dots, x_n)$ is non-increasing over the interval $[(\beta_{x_1},\beta_{x_2} \dots \dots, \beta_{x_n}), (\beta_{x_1},\beta_{x_2} \dots . \beta_{x_n})]$ respectively. Then,

Case I: N^{th}-dimensional normal imprecise number if

$\mu^1{}_{X_1X_2.......X_n}(\alpha_{x_1},\alpha_{x_2} \dots \dots, \alpha_{x_n}) = \mu^2{}_{X_1X_2.......X_n}(\gamma_{x_1},\gamma_{x_2} \dots \dots \dots \dots, \gamma_{x_n}) = (0,0, \dots \dots,0)$ and $\mu^1{}_{X_1X_2.......X_n}(\beta_{x_1},\beta_{x_2} \dots \dots, \beta_{x_n}) = \mu^2{}_{X_1X_2.......X_n}(\beta_{x_1},\beta_{x_2} \dots \dots, \beta_{x_n}) = (1,1,1, \dots \dots,1)$

…………………(4)

Case II: N[th] -dimensional subnormal imprecise number if

$$\mu^1{}_{X_1X_2\ldots\ldots X_n}(\alpha_{x_1},\alpha_{x_2}\ldots\ldots,\alpha_{x_n}) = \mu^2{}_{X_1X_2\ldots\ldots X_n}(\gamma_{x_1},\gamma_{x_2}\ldots\ldots\ldots\ldots,\gamma_{x_n}) = (0,0,\ldots\ldots,0)$$

and

$$\mu^1{}_{X_1X_2\ldots\ldots X_n}(\beta_{x_1},\beta_{x_2}\ldots\ldots,\beta_{x_n}) = \mu^2{}_{X_1X_2\ldots\ldots X_n}(\beta_{x_1},\beta_{x_2}\ldots\ldots,\beta_{x_n}) \neq (1,1,1,\ldots\ldots,1) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

And

$$\left(\mu^1{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n) - \mu^2{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right) = (\alpha_{x_1}-\beta_{x_1})\times(\alpha_{x_2}-\beta_{x_2})\times\ldots\ldots\times(\alpha_{x_n}-\beta_{x_n}) \quad \ldots\ldots\ldots(6)$$

is called membership value of the indicator function $\mu_{N_{X_1X_2\ldots\ldots X_n}}(x_1,x_2,\ldots\ldots,x_n)$.

Where

$\mu^1{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n) = (\alpha_{x_1},\alpha_{x_2}\ldots\ldots,\alpha_{x_n})$ and

$\mu^2{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n) = (\beta_{x_1},\beta_{x_2}\ldots\ldots,\beta_{x_n})$

C. Definition: For the n[th]-dimensional normal imprecise number,

$$\mu_{N_{X_1X_2\ldots\ldots X_n}} = \left\{\left(\mu_{N_{X_1X_2\ldots\ldots X_n}}(x_1,x_2,\ldots\ldots,x_n)\right),(0,0,\ldots\ldots,0): (x_1,x_2,\ldots\ldots,x_n)\in X_1\times X_2\times\ldots\times X_n\right\}$$

as defined above, the complement $\mu_{N^C{}_{X_1X_2\ldots\ldots X_n}}$

$$= \left\{(1,1,\ldots\ldots,1),\left(\mu_{N_{X_1X_2\ldots\ldots X_n}}(x_1,x_2,\ldots\ldots,x_n)\right): (x_1,x_2,\ldots\ldots,x_n)\in X_1\times X_2\times\ldots\times X_n\right\},$$

will have membership function $(1,1,1,\ldots\ldots,1)$and the reference function $\mu_{N_{X_1X_2\ldots\ldots X_n}}(x_1,x_2,\ldots\ldots,x_n) < 1$

for $-\infty < (x_1,x_2,\ldots\ldots,x_n) < \infty$

Thus the n[th]-dimensional imprecise numbers is characterized by

$$\left\{\left(\mu^1{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right),\left(\mu^2{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)\right): (x_1,x_2,\ldots\ldots,x_n)\in X_1\times X_2\times\ldots\times X_n\right\}$$

Where, $\mu^1{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)$

and $\mu^2{}_{X_1X_2\ldots\ldots X_n}(x_1,x_2,\ldots\ldots,x_n)$ are called membership function and the reference function of the indicator function $\mu_{N_{X_1X_2\ldots\ldots X_n}}(x_1,x_2,\ldots\ldots,x_n)$.

In particular, $\{(\mu^1{}_X(x)),(\mu^2{}_X(x)): x\in X\}$ is a characterized form of the one-dimensional imprecise number.

$$\{(\mu^1{}_{X_1X_2}(x_1,x_2)),(\mu^2{}_{X_1X_2}(x_1,x_2)): (x_1,x_2)\in X_1\times X_2\}$$

If the membership value is equal to 1, then the imprecise number is called the normal imprecise number otherwise subnormal.

D. Definition: Intersection and union of n[th] dimensional imprecise numbers is defined as follows.

If $A\left(\mu_{X_i}(x_i)\right) = \left\{\left(\mu^1{}_{X_i}(x_i)\right),\left(\mu^2{}_{X_i}(x_i)\right): (x_i)\in X_i; 1\le i\le n\in N\right\}$

And $B\left(\mu_{X_i}(x_i)\right) = \left\{(\mu^3{}_{X_i}(x_i)),(\mu^4{}_{X_i}(x_i)): (x_i)\in X_i; 1\le i\le n\in N\right\}$ be two imprecise numbers.

Then, intersection and the union of imprecise numbers is defined by

$$A\left(\mu_{X_i}(x_i)\right)\cap B\left(\mu_{X_i}(x_i)\right) = \left\{min\left(\left(\mu^1{}_{X_i}(x_i)\right),\left(\mu^3{}_{X_i}(x_i)\right)\right), max\left(\left(\mu^2{}_{X_i}(x_i)\right),\left(\mu^4{}_{X_i}(x_i)\right)\right)\right\}; (x_i)\in X_i, 1\le i\le n\in N \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

$$A\left(\mu_{X_i}(x_i)\right)\cup B\left(\mu_{X_i}(x_i)\right) = \left\{max\left(\left(\mu^1{}_{X_i}(x_i)\right),\left(\mu^3{}_{X_i}(x_i)\right)\right), min\left(\left(\mu^2{}_{X_i}(x_i)\right),\left(\mu^4{}_{X_i}(x_i)\right)\right)\right\}; (x_i)\in X_i, 1\le i\le n\in N \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

III. PROPERTIES OF IMPRECISE NUMBERS & RESULTS

Based on classical set theory properties under the operations of intersection and union we can obtain the n[th] dimensional imprecise numbers.

Property (Universal Laws)

(i) $A\left(\mu_{X_i}(x_i)\right)\cap A^C\left(\mu_{X_i}(x_i)\right) = \emptyset\left(\mu_{X_i}(x_i)\right)$

and (ii) $A\left(\mu_{X_i}(x_i)\right)\cup A^C\left(\mu_{X_i}(x_i)\right) = \Omega\left(\mu_{X_i}(x_i)\right)$

Where, $A^C\left(\mu_{X_i}(x_i)\right),\emptyset\left(\mu_{X_i}(x_i)\right)$ and $\Omega\left(\mu_{X_i}(x_i)\right)$ complement, null and the universal imprecise numbers respectively.

Proof:

Let $A\left(\mu_{X_i}(x_i)\right) = \left\{\left(\mu^1{}_{X_i}(x_i)\right),(0): x_i\in X_i; 1\le i\le n\in N\right.$

$$A^C\left(\mu_{X_i}(x_i)\right) = \left\{(1),\left(\mu^1{}_{X_i}(x_i)\right): x_i\in X_i; 1\le i\le n\in N\right\},$$

Where, $0<\mu^1{}_{X_i}(x_i)<1;\ i=1,2,3\ldots\ldots\ldots.n$ are individually imprecise numbers for the respective dimension.

Now, $A\left(\mu_{X_i}(x_i)\right)\cap A^C\left(\mu_{X_i}(x_i)\right)$

$$= \left\{\left(\min\left(\mu^1{}_{X_i}(x_i),1\right)\right),\left(\max\left(0,\mu^1{}_{X_i}(x_i)\right)\right): x_i\in X_i; 1\le i\le n\in N\right\}$$

$$= \left\{\left(\mu^1{}_{X_i}(x_i)\right),\left(\mu^1{}_{X_i}(x_i)\right)\right\}$$

Its membership value is, $\left(\mu^1{}_{X_i}(x_i)-\mu^1{}_{X_i}(x_i)\right)=0.$

So the intersection of imprecise number and its complement is a null set.

And $A\left(\mu_{X_i}(x_i)\right)\cup A^C\left(\mu_{X_i}(x_i)\right)$

$$= \left\{\left(\max\left(\mu^1{}_{X_i}(x_i),1\right)\right),\left(\min\left(0,\mu^1{}_{X_i}(x_i)\right)\right): (x_i)\in X_i; 1\le i\le n\in N\right\}$$

$$= \{(1)(0): x_i\in X_i; 1\le i\le n\in N\}$$

Its membership value is, $(1-0)=1$.

So, union of imprecise number and its complement is the universal set.

Remaining properties are discussed at the below.

Universal laws in term of fuzzy numbers:

(ii) $A\left(\mu_{X_i}(x_i)\right) \cap A^C \neq \emptyset\left(\mu_{X_i}(x_i)\right)$

and (ii) $A\left(\mu_{X_i}(x_i)\right) \cup A^C\left(\mu_{X_i}(x_i)\right) \neq \Omega\left(\mu_{X_i}(x_i)\right)$

Where, $A^C\left(\mu_{X_i}(x_i)\right), \emptyset\left(\mu_{X_i}(x_i)\right)$ and $\Omega\left(\mu_{X_i}(x_i)\right)$ complement, null and the universal imprecise numbers respectively.

Property (Distributive Laws)

(i) $A\left(\mu_{X_i}(x_i)\right) \cap \left(B\left(\mu_{X_i}(x_i)\right) \cup C\left(\mu_{X_i}(x_i)\right)\right)$

$= A\left(\mu_{X_i}(x_i)\right) \cap B\left(\mu_{X_i}(x_i)\right) \cup A\left(\mu_{X_i}(x_i)\right) \cap C\left(\mu_{X_i}(x_i)\right)$

$: (x_i) \in X_i; 1 \leq i \leq n \in N$

(ii) $A\left(\mu_{X_i}(x_i)\right) \cup \left(B\left(\mu_{X_i}(x_i)\right) \cap C\left(\mu_{X_i}(x_i)\right)\right)$

$= A\left(\mu_{X_i}(x_i)\right) \cup B\left(\mu_{X_i}(x_i)\right) \cap A\left(\mu_{X_i}(x_i)\right) \cup C\left(\mu_{X_i}(x_i)\right)$

Proof:

Let, $A\left(\mu_{X_i}(x_i)\right) = \left\{\left(\mu^1{}_{X_i}(x_i)\right), (0): x_i \in X_i; 1 \leq i \leq\right.$ *n∈N, BμXixi=μ2Xixi,0: xi∈Xi;1≤i≤n∈N*

$C\left(\mu_{X_i}(x_i)\right) = \left\{\left(\mu^3{}_{X_i}(x_i)\right), (0): x_i \in X_i; 1 \leq i \leq n \in N\right\}$

Where, $0 < \mu_{X_i}(x_i)^1{}_{X_i}(x_i) < \mu^2{}_{X_i}(x_i) < \mu^3{}_{X_i}(x_i) < 1$; $i = 1,2,3 \ldots \ldots \ldots . n$ are individually imprecise numbers for the respective dimension. Now,

(i) $A\left(\mu_{X_i}(x_i)\right) \cap \left(B\left(\mu_{X_i}(x_i)\right) \cup C\left(\mu_{X_i}(x_i)\right)\right)$

$$= \left((\mu^1{}_{X_i}), (0)\right) \cap \left(\left(max\left(\mu^2{}_{X_i}(x_i), \mu^3{}_{X_i}(x_i)\right)\right), (min(0,0))\right)$$

$$= \left((\mu^1{}_{X_i}), (0)\right) \cap \left((\mu^3{}_{X_i}), (0)\right); i = 1,2, \ldots, n$$

$$= \left(\left(min\left(\mu^1{}_{X_i}(x_i), \mu^3{}_{X_i}(x_i)\right)\right), (max(0,0))\right)$$

$$= \left((\mu^1{}_{X_i}), (0)\right); = 1,2, \ldots, n$$

$\left(A\left(\mu_{X_i}(x_i)\right) \cap B\left(\mu_{X_i}(x_i)\right)\right) \cup \left(A\left(\mu_{X_i}(x_i)\right) \cap C\left(\mu_{X_i}(x_i)\right)\right)$

$$= \left(\left(min\left(\mu^1{}_{X_i}(x_i), \mu^2{}_{X_i}(x_i)\right)\right), (max(0,0))\right) \cup \left(\left(min\left(\mu^1{}_{X_i}(x_i), \mu^3{}_{X_i}(x_i)\right)\right), (max(0,0))\right)$$

$$= \left((\mu^1{}_{X_i}), (0)\right) \cup \left((\mu^1{}_{X_i}), (0)\right); = 1,2, \ldots, n$$

$$= \left(\left(max\left(\mu^1{}_{X_i}(x_i), \mu^1{}_{X_i}(x_i)\right)\right), (min(0,0))\right)$$

$$= \left((\mu^1{}_{X_i}), (0)\right); = 1,2, \ldots, n$$

Hence Proved

(ii) $A\left(\mu_{X_i}(x_i)\right) \cup \left(B\left(\mu_{X_i}(x_i)\right) \cap C\left(\mu_{X_i}(x_i)\right)\right)$

$$= \left((\mu^1{}_{X_i}), (0)\right) \cup \left(\left(min\left(\mu^2{}_{X_i}(x_i), \mu^3{}_{X_i}(x_i)\right)\right), (max(0,0))\right)$$

$$= \left((\mu^1{}_{X_i}), (0)\right) \cup \left((\mu^2{}_{X_i}), (0)\right); i = 1,2, \ldots, n$$

$$= \left(\left(max\left(\mu^1{}_{X_i}(x_i), \mu^2{}_{X_i}(x_i)\right)\right), (min(0,0))\right)$$

$$= \left((\mu^2{}_{X_i}), (0)\right); = 1,2, \ldots, n$$

$\left(A\left(\mu_{X_i}(x_i)\right) \cup B\left(\mu_{X_i}(x_i)\right)\right) \cap \left(A\left(\mu_{X_i}(x_i)\right) \cup C\left(\mu_{X_i}(x_i)\right)\right)$

$$= \left(\left(max\left(\mu^1{}_{X_i}(x_i), \mu^2{}_{X_i}(x_i)\right)\right), (min(0,0))\right) \cup \left(\left(max\left(\mu^1{}_{X_i}(x_i), \mu^3{}_{X_i}(x_i)\right)\right), (min(0,0))\right)$$

$$= \left((\mu^2{}_{X_i}), (0)\right) \cap \left((\mu^3{}_{X_i}), (0)\right); = 1,2, \ldots, n$$

$$= \left(\left(min\left(\mu^2{}_{X_i}(x_i), \mu^3{}_{X_i}(x_i)\right)\right), (max(0,0))\right)$$

$$= \left((\mu^2{}_{X_i}), (0)\right); = 1,2, \ldots, n$$

Hence Proved

Property (Associatively Laws)

(i) $A\left(\mu_{X_i}(x_i)\right) \cup \left(B\left(\mu_{X_i}(x_i)\right) \cup C\left(\mu_{X_i}(x_i)\right)\right) = \left(A\left(\mu_{X_i}(x_i)\right) \cup B\left(\mu_{X_i}(x_i)\right)\right) \cup C\left(\mu_{X_i}(x_i)\right)$

(ii) $A\left(\mu_{X_i}(x_i)\right) \cap \left(B\left(\mu_{X_i}(x_i)\right) \cap C\left(\mu_{X_i}(x_i)\right)\right) = \left(A\left(\mu_{X_i}(x_i)\right) \cap B\left(\mu_{X_i}(x_i)\right)\right) \cap C\left(\mu_{X_i}(x_i)\right)$

Proof:

Let, $A\left(\mu_{X_i}(x_i)\right) = \left\{\left(\mu^1{}_{X_i}(x_i)\right), (0): x_i \in X_i, 1 \leq i \leq\right.$ *n∈N*

$B\left(\mu_{X_i}(x_i)\right) = \left\{\left(\mu^2{}_{X_i}(x_i)\right), (0): x_i \in X_i, 1 \leq i \leq n \in N\right\}$

$C\left(\mu_{X_i}(x_i)\right) = \left\{\left(\mu^3{}_{X_i}(x_i)\right), (0): x_i \in X_i, 1 \leq i \leq n \in N\right\}$

Where, $0 < \mu^1{}_{X_i}(x_i) < \mu^2{}_{X_i}(x_i) < \mu^3{}_{X_i}(x_i) < 1$; $i = 1,2,3 \ldots \ldots \ldots . n$ are individually imprecise numbers for the respective dimension. Now,

(i) $A\left(\mu_{X_i}(x_i)\right) \cup \left(B\left(\mu_{X_i}(x_i)\right) \cup C\left(\mu_{X_i}(x_i)\right)\right)$

$$= \left((\mu^1{}_{X_i}), (0)\right) \cup \left(\left(max\left(\mu^2{}_{X_i}(x_i), \mu^3{}_{X_i}(x_i)\right)\right), (min(0,0))\right)$$

$$= \left((\mu^1{}_{X_i}), (0)\right) \cup \left((\mu^3{}_{X_i}), (0)\right); i = 1,2, \ldots, n$$

$$= \left(\left(max\left(\mu^1{}_{X_i}(x_i), \mu^3{}_{X_i}(x_i)\right)\right), (min(0,0))\right)$$

$$= \left((\mu^3{}_{X_i}), (0)\right); = 1,2, \ldots, n$$

$$\left(A\left(\mu_{X_i}(x_i)\right) \cup B\left(\left(\mu_{X_i}(x_i)\right)\right)\right) \cup C\left(\left(\mu_{X_i}(x_i)\right)\right)$$
$$= \left(\left(max\left(\mu^1{}_{X_i}(x_i), \mu^2{}_{X_i}(x_i)\right)\right), (min(0,0))\right)$$
$$\cup \left((\mu^3{}_{X_i}), (0)\right); i = 1,2,\dots,n$$
$$= \left((\mu^2{}_{X_i}), (0)\right) \cup \left((\mu^1{}_{X_i}), (0)\right); i = 1,2,\dots,n$$
$$= \left(\left(max\left(\mu^2{}_{X_i}(x_i), \mu^3{}_{X_i}(x_i)\right)\right), (min(0,0))\right)$$
$$= \left((\mu^3{}_{X_i}), (0)\right); = 1,2,\dots,n$$

Hence Proved

Similarly, proof of the property III (C). (ii) can be done.

A. Property (De-Morgan's Laws)

(i) $\left(A\left(\mu_{X_i}(x_i)\right) \cup B\left(\mu_{X_i}(x_i)\right)\right)^C = A^C\left(\mu_{X_i}(x_i)\right) \cap B^C\left(\mu_{X_i}(x_i)\right)$

(ii) $\left(A\left(\mu_{X_i}(x_i)\right) \cap B\left(\mu_{X_i}(x_i)\right)\right)^C = A^C\left(\mu_{X_i}(x_i)\right) \cup B^C\left(\mu_{X_i}(x_i)\right)$

Proof:

Let, $A\left(\mu_{X_i}(x_i)\right) = \left\{\left(\mu^1{}_{X_i}(x_i)\right), (0): x_i \in X_i; 1 \le i \le n\right\}$ and $B\left(\mu_{X_i}(x_i)\right) = \left\{\left(\mu^2{}_{X_i}(x_i)\right), (0): x_i \in X_i; 1 \le i \le n\right\}$.

Where, $0 < \mu^1{}_{X_i}(x_i) < \mu^2{}_{X_i}(x_i) < 1;\ i = 1,2,3 \dots\dots\dots n$ are individually imprecise numbers for the respective dimension. Then,

Then, $A^C\left(\mu_{X_i}(x_i)\right) = \left\{(1), \left(\mu^1{}_{X_i}(x_i)\right): x_i \in X_i; 1 \le i \le n\right\}$ and *BCμXixi=1,μ2Xixi: xi∈Xi;1≤i≤n*. Now,

(i) $\left(A\left(\mu_{X_i}(x_i)\right) \cup B\left(\mu_{X_i}(x_i)\right)\right)^C =$
$$\left(\left(max\left(\mu^1{}_{X_i}(x_i), \mu^2{}_{X_i}(x_i)\right)\right), (min(0,0))\right)^C$$
$$= \left(\left((\mu^2{}_{X_i}), (0)\right)\right)^C$$
$$= \left((1), (\mu^2{}_{X_i})\right); i = 1,2,\dots,n$$

$A^C\left(\mu_{X_i}(x_i)\right) \cap B^C\left(\mu_{X_i}(x_i)\right) =$
$$\left((min(1,1)), \left(max\left(\mu^1{}_{X_i}(x_i), \mu^2{}_{X_i}(x_i)\right)\right)\right)$$
$$= \left((1), (\mu^2{}_{X_i})\right); i = 1,2,\dots,n$$

Hence proved

(ii) $\left(A\left(\mu_{X_i}(x_i)\right) \cap B\left(\mu_{X_i}(x_i)\right)\right)^C =$
$$\left(\left(min\left(\mu^1{}_{X_i}(x_i), \mu^2{}_{X_i}(x_i)\right)\right), (max(0,0))\right)^C$$
$$= \left(\left((\mu^1{}_{X_i}), (0)\right)\right)^C$$
$$= \left((1), (\mu^1{}_{X_i})\right); i = 1,2,\dots,n$$

$A^C\left(\mu_{X_i}(x_i)\right) \cup B^C\left(\mu_{X_i}(x_i)\right) =$
$$\left((max(1,1)), \left(min\left(\mu^1{}_{X_i}(x_i), \mu^2{}_{X_i}(x_i)\right)\right)\right)$$
$$= \left((1), (\mu^1{}_{X_i})\right); i = 1,2,\dots,n$$

Hence proved

IV. CONCLUSIONS

For the dynamic and elasticity objects, fuzzy problem is depending on the study of the whole dimension of the body. So the effect on the different faces of those type objects is suggested to study along with all the axes. Common effect and the whole effect of the fuzzy type object can be obtained in the form of intersection and the union of set. So, intersection and union of the finite dimensional imprecise numbers are expressed in terms of maximum and minimum operators. With these definitions, properties of finite dimensional imprecise numbers have been discussed in the various section of this article.

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