Application of Linear Programming for Profit Maximization of the Bank and the Investor

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Abstract: The main objective of this paper is to optimize (maximize) the net return of Central Bank of India in the area of interest from loans such as Personal loan, Car loan, Home loan, Agricultural loan, Commercial loan, Education loan and also maximize the net return of the investor by investing some amount in the investment policy of Central Bank of India such as Fixed Deposit, Saving Account, Public Provident fund and other investment Policies. The linear programming technique is applied to maximize profit of the Bank and the investor.

Index Terms- Linear Programming Model, Objective function, Constraints, Decision variables, Simplex method, Maximization, Marginal cost of funds based Lending Rate (MCLR).

I. INTRODUCTION

Linear Programming is a mathematical technique which is used to determine the optimal allocation of the limited resources, among the competitive activities provided all the relationships among the variables are linear. It is mainly concerned with a method of finding the optimum value (Maximum or Minimum) of a function of n variables. It is used extensively in business, economics and engineering. An example of an engineering application would be maximising profit in a factory that manufactures a number of different products from same raw material using same resources. The constraints would be decided by amount of raw materials available. The problems related to product mix and distribution of goods is solved by the technique of linear programming for optimization. In a business setting, profit maximization is always emphasized which inevitably means the minimization of some cost function. For Linear Programming Problems, the Simplex algorithm provides a powerful computational tool, able to provide fast solution to very large-scale application. Many Researchers worked for maximum profit using Linear Programming in different fields as:


The intention of this paper is to find the maximum profit of Central Bank of India in the area of interest from loans and maximum profit of investor by investing some amount in different policies of the Bank under some conditions. The general form of linear programming problem is Optimize

\[ Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n \] (objective function) .......(2.1)

subject to the constraints

\[
\begin{align*}
 a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &\leq b_1 \\
 a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &\leq b_2 \\
 &\ldots \ldots \ldots \ldots \ldots \ldots \\
 a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &\leq b_m
\end{align*}
\]

and non-negative restrictions

\[ x_j \geq 0, \quad j = 1, 2 \ldots n \]

Where \(a_{ij}'s, b_i's\) and \(c_j's\) are constants and \(x_j's\) are variables. In the conditions given by (2.2) there may be any of the three signs \(\leq, =, \geq\).

Standard form of a Linear programming problem for solving by simplex method is as:

(a) Using slack and surplus variables to express all constraints as equation.
(b) For each constraint all \(b_i \geq 0\), if any \(b_i\) is negative then multiply the corresponding constraint by \(-1\).
(c) Always, problem must be of maximization type if not convert it in maximization type by multiplying objective function by \(-1\).

Using slack and surplus variables the linear programming problem of \(n\) variables and \(m\) constraints can be written as follows: Optimize

\[ Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n + 0.s_1 + 0.s_2 + \ldots + 0.s_m \] (objective function) .......(2.3)

subject to the constraints
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\[
\begin{align*}
& a_1x_1 + a_2x_2 + \ldots + a_nx_n + s_1 = b_1 \\
& a_2x_2 + a_2x_2 + \ldots + a_mx_m + s_2 = b_2 \\
& \vdots \\
& a_mx_m + a_mx_m + \ldots + a_mx_m + s_m = b_m \\
\end{align*}
\]

and non-negative restrictions

\[x_1 \geq 0, \ s_1 \geq 0, \ j = 1, 2, \ldots, n, \ i = 1, 2, \ldots, m\]

Where \(a_i\)'s, \(b_i\)'s and \(c_i\)'s are constants and \(x_i\)'s and \(s_i\)'s are variables.

II. ASSUMPTION FOR PROBLEM

(a) It is assumed that the total loan amount of Bank for maximum profit is fixed.
(b) It is assumed that the total amount for investing for maximum profit is fixed.
(c) It is assumed that there is a linear relationship among the variables used in the problem.

III. DATA PRESENTATION AND ANALYSIS FOR BANK PROFIT

The data for this paper is collected by Central Bank of India, Kota, Rajasthan.

The total fund is Rs. 150 crore for different loan products. The bank is faced with following constraints:

(a) Amounts allocated to the different loan products not more than the total fund.
(b) The total amount for Home loan, Agricultural loan and Educational loan not more than 60% of the total fund.
(c) The total amount for Personal loan and Car loan not more than 50% of the total fund.
(d) The total amount for Commercial loan (Secured) not more than 40% of the total fund.
(e) The total amount for Commercial loan (Unsecured) not more than 20% of the total fund.
(f) The total amount allocated to Home Loan is at least 5% of the total fund.
(g) The total amount allocated to Agricultural loan is at least 5% of the total fund.
(h) The total amount allocated to Car loan is at least 5% of the total fund.

The interest rate of different loans are given in following table: (MCLR = 8.60%)

<table>
<thead>
<tr>
<th>Types of Loan</th>
<th>Rate of Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home</td>
<td>8.60%</td>
</tr>
<tr>
<td>Agricultural</td>
<td>8.60%</td>
</tr>
<tr>
<td>Car</td>
<td>MCLR + 0.4% = 9.00%</td>
</tr>
<tr>
<td>Personal</td>
<td>MCLR + 3% = 11.60%</td>
</tr>
<tr>
<td>Educational</td>
<td>MCLR + 2% = 10.60%</td>
</tr>
<tr>
<td>Commercial (Secured / Unsecured)</td>
<td>MCLR + 1.5% = 10.10%</td>
</tr>
</tbody>
</table>

IV. MODEL FORMULATION

Let amount for Home Loan = \(x_1\)
Let amount for Agricultural Loan = \(x_2\)
Let amount for Car Loan = \(x_3\)
Let amount for Personal Loan = \(x_4\)

Let amount for Educational Loan = \(x_5\)
Let amount for Commercial Loan (Secured) = \(x_6\)
Let amount for Commercial Loan (Unsecured) = \(x_7\)

Let the objective function which is to be maximize is denoted by \(Z\).

The mathematical form of the above problem (data) is

\[\text{Max} \ Z = 0.086x_1 + 0.086x_2 + 0.09x_3 + 0.116x_4 + 0.106x_5 + 0.101x_6 + 0.101x_7\]

Subject to

\[\begin{align*}
& x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \leq 150 \\
& x_1 + x_2 + x_5 + s_1 = 150 \\
& x_3 + x_4 + s_3 = 90 \\
& x_6 + s_4 = 60 \\
& x_7 + s_5 = 30 \\
& x_1 - s_6 = 7.5 \\
& x_2 - s_7 = 7.5 \\
& x_3 - s_8 = 7.5 \\
& x_1, x_2, x_3, x_4, x_5, x_6, x_7, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8 \geq 0
\end{align*}\]

Using simplex method the optimal solution of the above problem is

\[\begin{align*}
& x_1 = 7.5 \quad , x_2 = 7.5 \\
& x_3 = 7.5 \quad , x_4 = 67.5 \\
& x_5 = 60 \quad , x_6 = 0 \\
& x_7 = 0
\end{align*}\]

and Max \(Z = 16.155\) Crore

V. INTERPRETATION OF RESULT

The optimum result acquire from the model, based on the data collected specified that the maximum profit of Bank is 16.155 crore and allocated amount for Home Loans is 7.5 crore, for Agricultural Loans is 7.5 crore, for Car Loans is 7.5 crore, for Personal Loans is 67.5 crore, for Educational Loans...
Loans is 60 crore, for Commercial Loans Secured is 0 and for Commercial Loans Unsecured is 0.

VI. DATA PRESENTATION AND ANALYSIS FOR INVESTOR PROFIT

The total fund is Rs. 2 Lakh for different investment policies by Senior Citizen (S.C.) Investor. The Investor has following constraints.

(a) Total invested amount not more than the total fund.
(b) Total invested amount in Saving Account is atleast 20% of the total fund.
(c) Total invested amount in Fixed Deposit is atleast 25% of the total fund.
(d) Total invested amount in Public Provident Fund (PPF) and Senior Citizen Savings Scheme (SCSS) is atleast 10% of the total fund
(e) Total invested amount in Public Provident Fund (PPF) and Senior Citizen Savings Scheme (SCSS) is not more than 20% the total fund.

The interest rate of different investment policies are given in the following table:

<table>
<thead>
<tr>
<th>Investment Policies</th>
<th>Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saving Account</td>
<td>3.5%</td>
</tr>
<tr>
<td>Fixed Deposit (S.C.)</td>
<td>6.65% + 0.5%</td>
</tr>
<tr>
<td>PPF</td>
<td>8%</td>
</tr>
<tr>
<td>SCSS</td>
<td>8.6%</td>
</tr>
</tbody>
</table>

The mathematical form of the above problem (data) is

Let amount for Saving Account = $x_1$
Let amount for Fixed deposit (S.C.) = $x_2$
Let amount for PPF = $x_3$
Let amount for SCSS = $x_4$
Let the objective function which is to be maximize is denoted by $Z$.

The mathematical form of the above problem (data) is

Max $Z = 0.035x_1 + 0.0715x_2 + 0.08x_3 + 0.086x_4$

Subject to

\[
\begin{align*}
&x_1 + x_2 + x_3 + x_4 \leq 2,00,000 \\
&x_1 \geq 40,000 \\
&x_2 \geq 50,000 \\
&x_3 + x_4 \geq 20,000 \\
&x_2 + x_3 \leq 40,000
\end{align*}
\]

and $x_1, x_2, x_3, x_4 \geq 0$

Using slack and surplus variables the problem Converted to

Max $Z = 0.035x_1 + 0.0715x_2 + 0.08x_3 + 0.086x_4 + s_1 + s_2 + s_3 + s_4 + s_5$

Subject to

\[
\begin{align*}
&x_1 + x_2 + x_3 + x_4 + s_1 = 2,00,000 \\
&x_1 - s_2 = 40,000 \\
&x_2 - s_3 = 50,000 \\
&x_3 + x_4 - s_4 = 20,000 \\
&x_4 + s_5 = 40,000
\end{align*}
\]

and $x_1, x_2, x_3, x_4, s_1, s_2, s_3, s_4, s_5 \geq 0$

Using simplex method the optimal solution of the above problem is

$x_1 = 40,000$, $x_2 = 1,20,000$, $x_3 = 0$, $x_4 = 40,000$

and $Max Z = 13,420$

VIII. INTERPRETATION OF RESULT

The optimum result acquire from the model, based on the data collected specified that the maximum profit of the Investor is 13,420 and invested amount for saving account is 40,000, for Fixed deposit is 1,20,000, for PPF is 0, and for SCSS is 40,000.

IX. CONCLUSION

From the above discussion it is concluded that the maximum net return of the Bank is Rs. 16.155 Crore by allocate the amount of Rs. 7.5 Crore to Home loan, Rs. 7.5 Crore to Agricultural loan, Rs. 7.5 Crore to Car loan, Rs. 67.5 Crore to Personal Loan, Rs. 60 Crore to Education loan and none of the amount is for Commercial loan (Secured and Unsecured) and the maximum profit of investor is Rs. 13,420 by investing the amount of Rs. 40,000 to Saving account, Rs. 1,20,000 to fixed deposit, Rs. 40,000 to SCSS and none of the amount is for PPF.

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REFERENCES