

# Buckling analysis of Functionally Graded Natural Fiber-Flyash-Epoxy (FGNFFE) Cylinder

Sarada Prasad Parida, Saritprava Sahoo, Basna Bidisha Bal, Pankaj Charan Jena



**Abstract:** A hollow cylinder or a pipe is a common element used in structural applications. Now days in the era of new material development, replacement of conventional materials by modern material are of primary choice for the researchers and developers as well. This paper presents the buckling analysis of functionally graded natural-fiber-flyash-epoxy (FGNFFE) hollow cylinders using FEA. In the first part, a mathematical model for buckling analysis is developed to get the dynamic behavior of hollow cylinder under free vibration. Initial five modes of buckling analysis are performed by theoretical, finite element analysis and experimentation. Accordingly Mechanical properties are obtained and used for buckling study in FEA environment as being a cylindrical structure to the design, it is subjected to compression and buckling due to self weight and due to external load is very common. The critical buckling load is determined by FEA study and compared with the experimental value. Further the study extended by optimizing the critical buckling load and stress with respect to the ingredients and other designed parameters and discussed.

**Keywords:** Buckling, functionally graded, cylinder, filler.

## Nomenclature

$L$  length of beam

$U_i$  mechanical property of inner layered material

$U_o$  mechanical property of outer layered material

$H$  thickness of cylinder

$Z_k$  thickness of  $k$ th layer

$\beta_R$  stiffness of cylinder

$P_{cr}$  critical load

$EI$  flexural rigidity

## I. INTRODUCTION

Selection of suitable material is an important aspect in product design for which product to be sustainable under

different loads. The mechanical property of newly designed material plays an important role. However, both physical and mechanical properties of products play additional attributes for design aspect. The selections of material also have a great effect on any design to bring off low cost. By looking into the present industry demand and keeping knowledge of environmental effect, the suitable product's material along with its reliability is currently very important factor for an engineering product design. A number of works had been conducted in literatures mentioned with an emphasis to develop new kind of material from the industrial wastes and natural fibres as well. Cherradi et al. 1994 [1] have presented a study on FGM in structural application by designing from its microstructure level. Kieback et al. [2] have outlined the different preparing technologies of functionally graded materials (FGM). They have considered processing of powder and approaches concerning the processing of metal melts and graded polymers and their dependency gradient extension and the geometry of the components. They have also discussed microstructure gradients. Udupa et al. [3] have overviewed the study on functionally graded composite material (FGCM) as fundamental approach, their classifications, properties and their preparation procedure. They have considered a case study as CNT-reinforced Al functionally graded materials. They have used powder metallurgy methodology for the fabrication of FGCM. Through case studies, they have observed the effects of layered CNT enhancement, which changed physical properties due to uneven microstructure and spatial changes. Chumanov et al. 2011 [4] have described a process for introducing a diffuse powder into a metal melt to manufacture FGM. Jamian et al. [5] have fabricated functional graded composite using slurry in centrifugal casting technique. Zhang and Wang [6] have proposed thermal bonding lamination method for producing a polymer FGMs having a closed cell and an open cell under the enforced compressive load, controlled heating and suitable holding time. Galy et al. [7] have investigated the horizontal centrifugal casting using SiCp+Al concept of FGM to produce multifunctional components of metal matrix composites. They have considered dissimilar mass fractions (0%, 2.5%, 5%, 7.5%, 10% and 15%) of SiCp having element size  $16\mu\text{m}$ ,  $23\mu\text{m}$  and  $500\mu\text{m}$  and three rotational speed 800 rpm, 900 rpm and 1000 rpm. They have found that the SiCp is deposited more at outer surface and gradually decreases to interior surface. Elghazouli et al.

Revised Manuscript Received on October 30, 2019.

\* Correspondence Author

**Sarada Prasad Parida**, Department of Production Engineering, Veer Surendra Sai University of Technology, Burla, Odisha, India.

**Saritprava Sahoo**, Department of Production Engineering, Veer Surendra Sai University of Technology, Burla, Odisha, India.

**Basnabidisha Bal** Department of Mechanical Engineering, Gandhi Institute of Technology and Management, Gangapada, Bhubaneswar, Odisha, India.

**Pankaj Charan Jena\***, Department of Production Engineering, Veer Surendra Sai University of Technology, Burla, Odisha, India-768018,

© The Authors. Published by Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP). This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>)

## Buckling analysis of Functionally Graded Natural Fiber-Flyash-Epoxy (FGNFFE) Cylinder

[8] present the results of buckling tests on laminated composite cylinders made of glass fiber reinforced plastic (GFRP). The results of this experimental study demonstrate the influence of laminate orientation on the buckling strength of composite cylinders under axial compression. Dong and Mistry [9] carried out an experimental investigation into the buckling

failure of filament wound glass reinforced plastic cylinders subjected to a combination of external pressure and axial compression. Chryssanthopoulos et al. [10] give a detailed research on the numerical finite element validation for laminated GFRP cylinders subjected to concentric and eccentric compression. Both linear eigen value analysis and geometrically nonlinear simulations are undertaken using a general purpose finite element program. An analytical and experimental study of the buckling behavior of thin-walled carbon fiber reinforced polymer (CFRP) laminated shells under combined axial and torsion loading is done by Meyer-Piening et al. [11]. Spagnoli et al. [12] studied a numerical simulation for the buckling behaviour of laminated glass-reinforced plastic cylinders under axial compression. Hilburger et al. [13] presented buckling results from a numerical study of the response of thin-walled compression-loaded quasi isotropic laminated composite cylindrical shells with unreinforced and reinforced square cutouts. Topal [14] studied multi objective optimization of laminated cylindrical shells to maximize a weighted sum of the frequency and buckling load under external load. He used the first order shear deformation theory for the finite element formulation of the laminated shells. Wu et al. [15] have developed unified formula for the virtual displacement (PVD) principle and the finite cylindrical layer method (FCLMs). They have performed buckling analysis of the FGM hollow cylinder under axial compression and external pressure. Moon et al. [16] have presented a FEA study on buckling of filament wound composite cylinder subjected to hydrostatic pressure generally used for underwater applications for different winding angles and optimized the failure modes. Paor et al. [17] have predicted the buckling pressure for thin walled cylinders with geometrical imperfections induced to external pressure. White et al. [18] analyzed the stiffness of composite cylinder under axial loading using Abacus. Yu et al. [19] have stated the functionally graded material manufacturing via heat treatment or material consolidation. Jena et al. [20] have studied the free vibration analysis of AMMC using finite element method.

The buckling load of a composite cylindrical shell depends on a variety of parameters, such as the geometry of the cylinder, properties of reinforcement and boundary conditions.

### II. MATHEMATICAL MODEL

The effective material properties such as elastic modulus, shear moduli, density, poisson's ratio etc. of FGM depends upon the volume fraction distribution of constituent materials. There are various idealization schemes to evaluate the material properties. The exponential law, power law and Mori Tanka scheme are widely accepted. These formulations are used for the mathematical model. According to exponential law the effective properties of FGM can be calculated as;

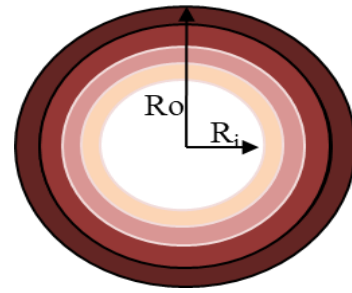


Fig.1. Schematic view of FG cylinder.

$$U(Z) = (U_t - U_b) \left( \frac{Z}{H} + \frac{1}{2} \right)^k + U_b \quad (1)$$

The governing equation of deformation due buckling of a column subjected to axial load is given by

$$\frac{d^2 y}{dx^2} + \frac{Py}{EI} = 0 \quad (2)$$

The general solution to this equation is given by

$$y = C_1 \sin ax + C_2 \cos ax \quad (3)$$

Where,  $a = \sqrt{P/EI}$

The critical load of a column subjected to axial compression or buckling is given by

The buckling load and stress for different mode of the column is given by

$$P = \frac{n^2 \pi^2 EI}{L^2} \quad (4)$$

$$\sigma_c = \pi^2 E / L^2 A \quad (5)$$

The lateral buckling deflection of an elastic beam is given by

$$\delta_1(0) = \delta_0 [\sigma' / (\sigma_c - \sigma')] \quad (6)$$

Where  $\delta_0$  is initial deflection,  $\sigma'$  is normal stress developed due to normal loading and  $\sigma_c$  is critical buckling stress and is given by

$$\sigma_c = \pi^2 E / 4 \left( \frac{L}{b} \right)^2 \quad (7)$$

The compressive stress in the left and right of the column is given by

$$\sigma_L(0) = \sigma' \left[ 1 + 2 \frac{\delta_0}{b} \left( \frac{\sigma_c}{\sigma_c - \sigma'} \right) \right] \quad (8)$$

$$\sigma_R(0) = \sigma' \left[ 1 - 2 \frac{\delta_0}{b} \left( \frac{\sigma_c}{\sigma_c - \sigma'} \right) \right] \quad (9)$$

### III. CYLINDER AND PROPERTY

The test samples were manufactured by horizontal centrifugal settling method. For the purpose a hollow steel cylindrical mould was designed and assembled. The properties are presented in Table I.



Fig.2: Prepared FGNNFE cylinder specimens.

IV. FINITE ELEMENT ANALYSIS

Initially the hollow cylinder is assumed of as a hollow circular beam constrained at bottom face and loaded at the top faces. Beam 188 element is chosen for analysis. A uniform load is applied Eigen buckling is carried on the hollow cylinder after successively conducting the static analysis. Here the hole is assumed to be five layers with equal thickness an each layer assumed to be isotropic. Hence the cylinder is assumed to be layered isotropic with functionally graded to their densities. It is assumed the heavier material at outer layer and the lighter settled to interior surface of FG cylinder due to centrifugal action. It is assumed this settling of particles is assumed to the exponential function of density. Likewise other mechanical properties of the FG pipe are varied and can be calculated accordingly. The material a property of the hollow pipe is calculated from the exponential formula as described and the properties are as depicted in Table-II.

Table –I. Material properties.

Material properties	Youngs modulus	Density	Poison's Ratio
values	13.14MPa	1226Kg/m <sup>3</sup>	0.33

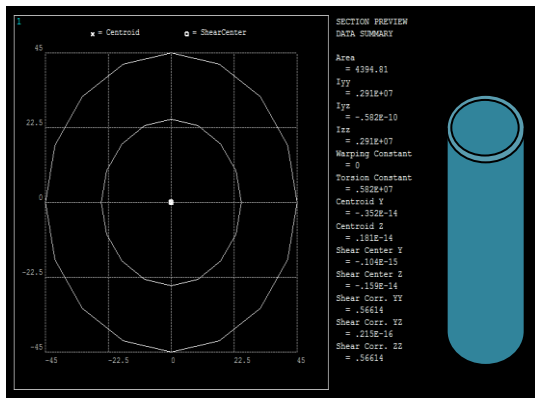


Fig.3 Sectional view of hollow cylindrical beam modelled in FEA.

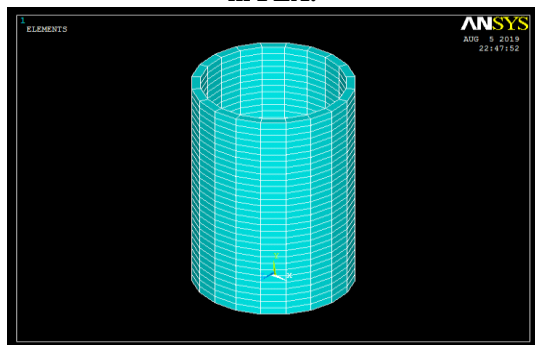
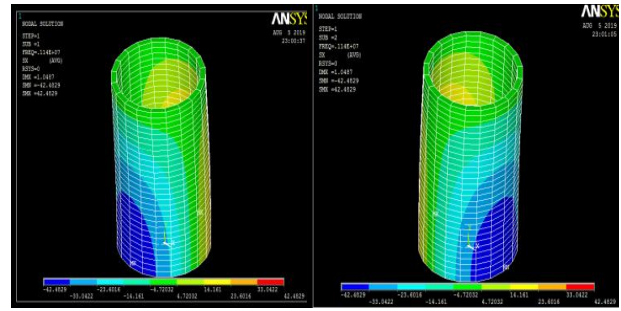
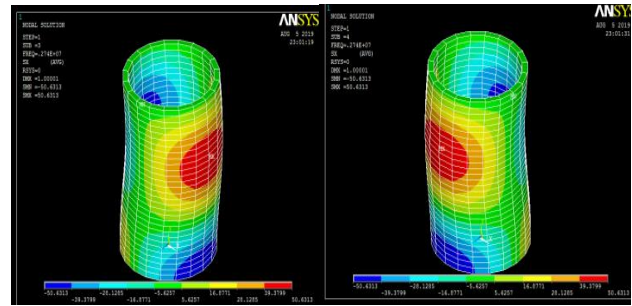


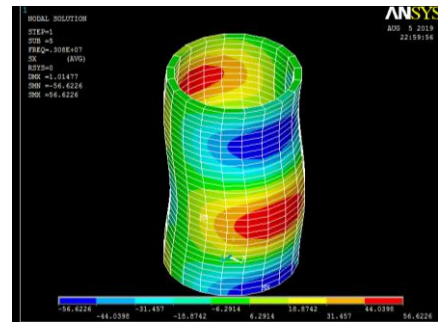
Fig.4 Beam 188 element used in FEA for buckling analysis.



(a) Buckling mode-I (b) Buckling mode-II



(c) Buckling mode-III (d) Buckling mode-IV



(e) Buckling mode-V

Fig.5 First five modes of FGNNFE cylinder in buckling analysis.

V. RESULT AND DISCUSSION

The preparation of the specimens are carried out by centrifugal casting method, it is obvious that the heavier material settles at outer periphery an the lighter material settles at inner region. It is as assumed in the analysis and presented in the Figure 1. Fabricated cylinders are shown in Figure 2. The FEA model and meshing of cylinder are shown in Figure 3 and Figure 4. Buckling analysis of hollow FGNNFE cylinder is carried for first five modes of buckling (shown in Figure 5). The maximum buckling stress obtained are enlisted in Table-II for different height to length ratios of the cylinders and are compared with the theoretical values of critical stresses. It has been found out that by increasing the thickness to length ratio of a FGNNFE cylinder the buckling stress intensity decreases as depicted in the Figure 6. It is as the increase in cross sectional area as a general convenience to take higher load at the lower concentration of stress.



## Buckling analysis of Functionally Graded Natural Fiber-Flyash-Epoxy (FGNFFE) Cylinder

This decrease in buckling stress FGNFFE cylinders with increase in thickness to length ratio obtained from FEA study shows an good agreement with the theoretical values of buckling stresses. Figure 7 (a-e) represent the variation of buckling stress with thickness to length ratio for buckling modes of I, II, III, IV and V respectively. From the figures it can be observed and concluded that the buckling stresses shows good resemblance with the theoretical values calculated with minimum variations. By increasing the thickness of the cylinder it becomes more stable as a result the stress intensity and concentration decreases which is a general convention. Both FEA and theoretical study shows the same agreement. The density of the prepared sample depends upon the speed of the cylindrical mould as presented in Figure 8. It is observed that with the increase in speed the density increases. However the optimum density of the material is obtained for the mould speed of 1485 rpm and the sample of lowest density is obtained for mould speed of 1475 rpm. The rotation speed in return decides the distribution of constituent materials. By increasing the speed the cylinder wall becomes more denser and make it capable to take higher load, however increasing the rotation speed further flyash being the heaviest material deposited at the outer periphery and at the core only fibers deposited causing decrease in compressive strength at the core, so increasing the speed after certain level, there is decrease in compressive load carrying capacity. The buckling load can be related with the content of constituent ingredients. Figure 9 represents the variation of buckling load with the weight percentage of content of banana fiber. From the figure it is observed with increase in banana fiber weight percentage in the constituent material, the buckling strength increases abruptly and then decreases gradually when the total fiber content reaches 6%. This is due to increase in fiber content decreases compressive strength in contrast to initial increase in intermolecular bonding. It is ascertained that increase in fiber content increase damping coefficient of the material as a result, it can carry the load however at certain limit when the fiber content exceeds and segregates more at inner region due to centrifugal action of the mould, lesser bonding in inner part of the cylinder is resulted. Coherently it results decrease in buckling strength of the cylinders. However in FEA study it is found out that with increase in banana fiber content in FGNFFE, there is decrease in Buckling load as presented in Figure 10 with increase in flyash content. Increase in flyash in the specimen decreases bonding strength and made it more fragile. Figure 11 represents the variation of buckling load of FGNFFE specimens with variation of particle size of flyash as obtained from experimental investigation and FEA study. It is observed that with increase in particle size of flyash initially buckling strength increases and later further increase in particle size lead to decrease in buckling strength. Figure 12 presents the variation of deflection with respect to buckling load.

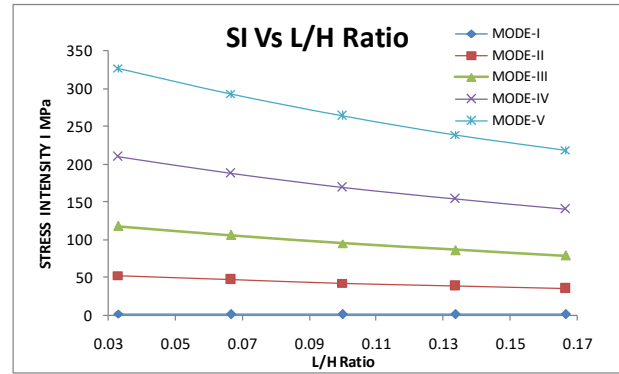


Fig.6 Variation of Buckling stress Vs L/H .

Table-II Critical buckling stress for different thickness to length ratios.

H/L	Buckling mode	$\sigma_{cr}$ theoretical in MPa	$\sigma_{cr}$ FEA in MPa
0.1667	1	8.7	8.2
	2	34.93	32.13
	3	78.59	73.25
	4	139.7	133.1
	5	218.3	208.9
0.1337	1	9.5	9.2
	2	38.17	36.78
	3	85.88	83.57
	4	152.6	147.68
	5	238.5	232.3
0.1	1	10.5	10.6
	2	42.13	45.62
	3	94.80	91.2
	4	168.5	160.5
	5	263.3	256.8
0.0667	1	11.7	10.48
	2	46.81	42.48
	3	105.3	101.2
	4	187.2	168.3
	5	292.5	278.5
0.0333	1	13.1	10.6
	2	52.22	48.03
	3	117.4	115.431
	4	208.8	201.3
	5	326.3	317.8

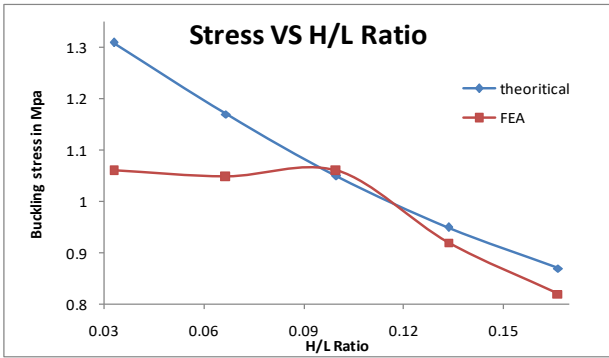


Fig.7(a) H/L Ratio Vs Buckling Stress in Mode-I.

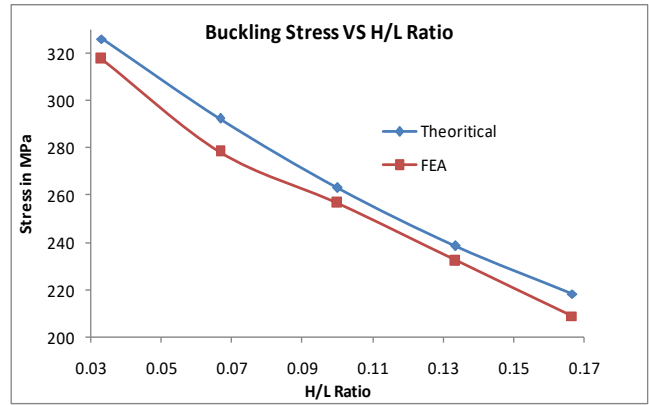


Fig.7(e) H/L Ratio Vs Buckling Stress in Mode-V.

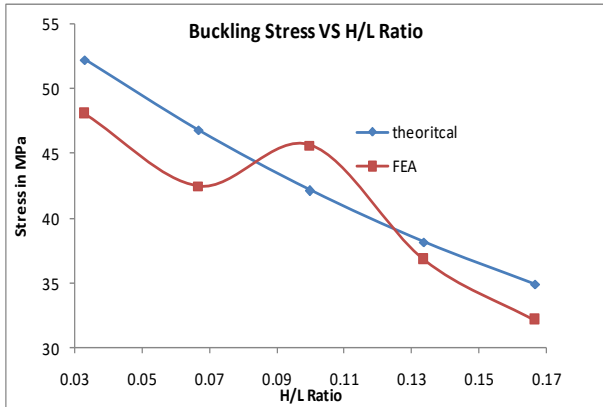


Fig.7(b) H/L Ratio Vs Buckling Stress in Mode-II.

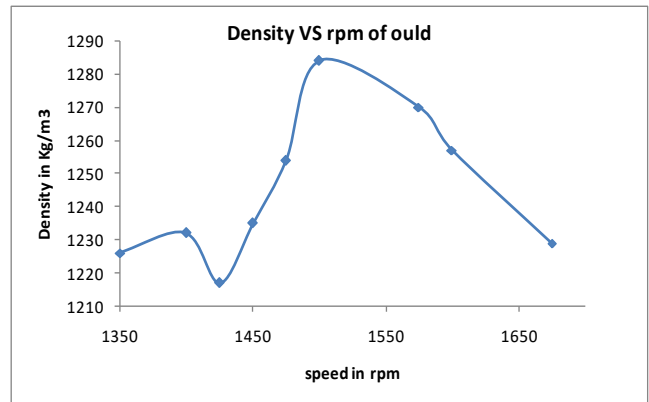


Fig.8 variation of density with speed of RPM.

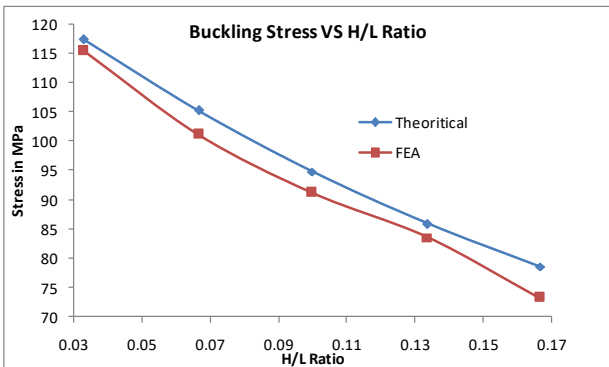


Fig.7(c) H/L Ratio Vs Buckling Stress in Mode-III.

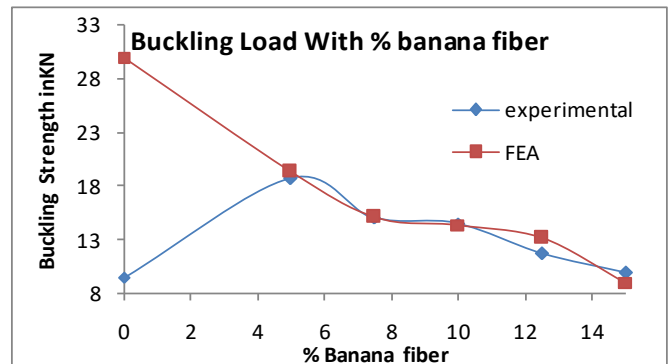


Fig.9 Compressive strength Vs banana fiber content.

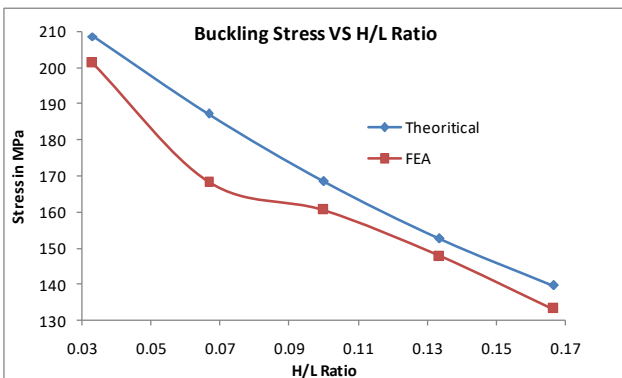


Fig.7(d) H/L Ratio Vs Buckling Stress in Mode-IV.

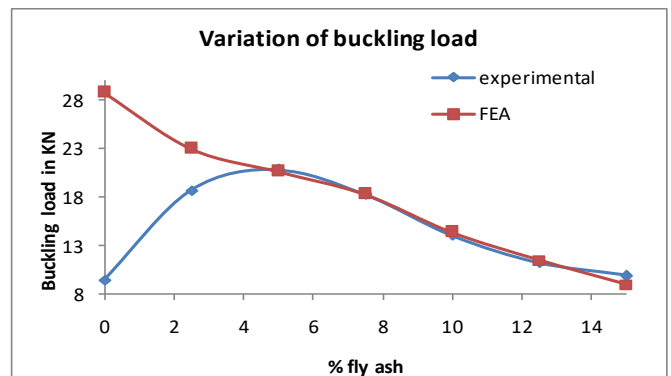
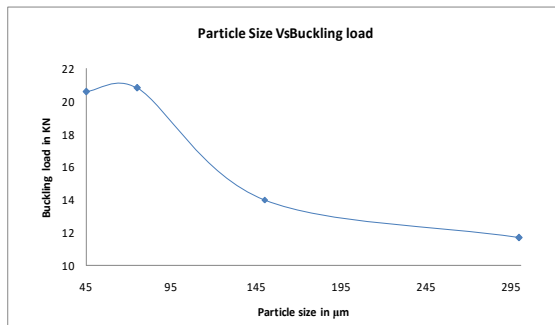
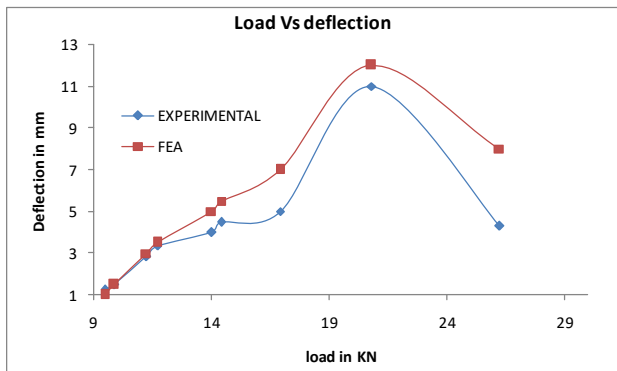


Fig.10 Variation of Buckling strength with flyash content.

# Buckling analysis of Functionally Graded Natural Fiber-Flyash-Epoxy (FGNFFE) Cylinder



**Fig.11 Variation of Buckling strength with particle size of flyash.**



**Fig.12 Buckling deflection cylinder with load.**

## VI. CONCLUSION

The buckling analysis of fabricated Functional Graded hollow cylinder has been performed using analytical method, finite element analysis and experimentation by varying constituent (flyash and banana fiber) percentage in epoxy under free vibration. It has been found out that by increasing the thickness to length ratio of a FGNFFE cylinder the buckling stress intensity decreases. The buckling strength increases abruptly and then decreases gradually when the total fiber content reaches 6 %. The increase in flyash causes decrease in buckling strengths.

## REFERENCES

1. N. Cherradi, Akira Kawasaki and M. Gasik, worldwide trends in functional gradient materials research and development, *Composites Engineering*, vol. 4, no. 8, pp. 883–894, 1994
2. B. Kieback, A. Neubrand, and H. Riedel, "Processing techniques for functionally graded materials," *Mater. Sci. Eng. A*, vol. 362, no. 1–2, pp. 81–106, 2003.
3. G. Udupa, S. S. Rao, and K. V. Gangadharan, "Functionally Graded Composite Materials: An Overview," *Procedia Mater. Sci.*, vol. 5, pp. 1291–1299, 2014.
4. I.V. Chumanov, A. N. Anikeev, and V. I. Chumanov, "Fabrication of functionally graded materials by introducing wolframium carbide dispersed particles during centrifugal casting and examination of FGM's structure," *Procedia Eng.*, vol. 129, pp. 816–820, 2015.
5. S. Jamian, S. N. F. Razali, and M. R. Z. Abidin, "Fgnf/epoxy composite fabricated using centrifugal slurry-pouring method," *ARNP J. Eng. Appl. Sci.*, vol. 11, no. 12, pp. 7759–7764, 2016.
6. Y. Zhang and J. Wang, "Fabrication of Functionally Graded Porous Polymer Structures Using Thermal Bonding Lamination Techniques," *Procedia Manuf.*, vol. 10, pp. 866–875, 2017.
7. I.M. El-Galy, M. H. Ahmed, and B. I. Bassiouny, "Characterization of functionally graded Al-SiC p metal matrix composites manufactured by centrifugal casting," *Alexandria Eng. J.*, vol. 56, no. 4, pp. 371–381, 2017.
8. S. Janardhanan and G. Engineering, "Numerical Simulation of Centrifugal Metal-Matrix Composites," vol. 8, no. 4, pp. 66–74, 2017.

9. Elghazouli A.Y., Chryssanthopoulos M.K., Spagnoli A.: Experimental response of glass reinforced plastic cylinders under axial compression. *Marine Structures*, 1998, Vol. 11, pp. 347-371.
10. Dong L., Mistry J.: An experimental study of the failure of composite cylinders subjected to combined external pressure and axial compression. *Composire Strucfures*, 1998, Vol. 40, No. 1. pp. 81-94.
11. Chryssanthopoulos M.K., Elghazouli A.Y., Esong I.E.: Validation of FE models for buckling analysis of woven GFRP shells. *Composite Structures*, 2000, Vol. 49, pp. 355-367.
12. Meyer-Piening H.-R., Farshad M., Geier B., Zimmermann R.: Buckling loads of CFRP composite cylinders under combined axial and torsion loading- experiments and computations. *Composite Structures*, 2001, Vol. 53, pp. 427-435.
13. Spagnoli A., Elghazouli A.Y., Chryssanthopoulos M.K.: Numerical simulation of glass reinforced plastic cylinders under axial compression. *Marine Structures*, 2001, Vol. 14, pp.353-347.
14. Hilburger M.W., Starnes J.H.J.: Buckling behavior of compression-loaded composite cylindrical shells with reinforced cutouts. *International Journal of Non-Linear Mechanics*, 2005, Vol. 40, Issue 7, pp. 1005-1021.
15. Topal U.: Multiobjective optimization of laminated composite cylindrical shells for maximum frequency and buckling load. *Materials & Design*, 2009, Vol. 30, Issue 7, pp. 2584-2594.
16. C.J Moon, I.H Kim, B.H Choi, J.H Kweon, J.H Choi, Buckling of filament-wound composite cylinders subjected to hydrostatic pressure for underwater vehicle applications, [Composite Structures, Volume 92, Issue 9](#), August 2010, Pages 2241-2251.
17. C.D. Paor, D.Kelliher, K.Cronin, W.M.D.Wright, S.G.McSweeney, Prediction of vacuum-induced buckling pressures of thin-walled cylinders, [Thin-Walled Structures, Volume 55](#), June 2012, Pages 1-10.
18. S. C White, P.M. Weaver, K.C Wu, Post-buckling analyses of variable-stiffness composite cylinders in axial compression, [Composite Structures, Volume 123](#), May 2015, Pages 190-203.
19. I.C. Wu, Y. Chen, and S. Peng, "Thin-Walled Structures Buckling analysis of functionally graded material circular hollow cylinders under combined axial compression and external pressure," *Thin Walled Struct.*, Vol. 69, pp. 54–66, 2013.
20. Jena, Pankaj C., Parhi, Dayal R., Pohit, G. and Samal, B.P., Crack Assessment by FEM of AMMC Beam Produced by Modified Stir Casting Method, *Journal of Materials Today: Proceedings*, 2267 - 2276, 2015.

## AUTHORS PROFILE



**Mr. Sarada Prasad Parida** is presently pursuing PhD in the Department of Production Engineering at Veer Surendra Sai University of Technology, Burla, Odisha, India. His research interest includes Mechanical Vibration, FEM and Composite Material.



**Ms. Saritprava Sahoo** is presently pursuing PhD in the Department of Production Engineering at Veer Surendra Sai University of Technology, Burla, Odisha, India. Her research interest includes FEM and Composite Material and machining.



**Basnabidisha Bal** is working as assistant professor, Department of Mechanical Engineering, Gandhi Institute of Technology and Management, Gangapada, Bhubaneswar. Her research includes composite materials and vibration analysis.



**Dr. Pankaj Charan Jena\*** (Ph.D in Mechanical Engineering, Jadavpur University, Kolkata, India) is working as Associate Professor, Department of Production Engineering, Veer Surendra Sai University of Technology, Burla, Odisha, India. His current research interests include Composite material design

and modelling and casting, control, dynamics and fault diagnosis using Artificial Intelligence.

