International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249-8958 (Online), Volume-8 Issue-6, August, 2019

Modelling of Shear Strength for Reinforced Concrete Beams Provided with Side-Face Reinforcement in Dependence of Crack Inclination Angle

A. A. Manar, T. M. Mansour

Abstract: ***Shear behavior of reinforced concrete beams (RC-beams) is proved to be influenced by different parameters such as web reinforcement, beam size, shear span-to-depth ratio, concrete strength, and longitudinal reinforcement. In addition to these parameters, researches acknowledge the significant contribution of side-face reinforcement (SFR) in shear strength of RC-beams. This paper aims at proposing a new model for predicting shear strength of RC-beams that accounts for the contribution of SFR in shear strength along with the other above-mentioned parameters. An explicit formula is derived based on a mechanical conceptual model that considers the variation of the inclination angle of diagonal shear cracking. The derived formula is verified on the basis of numerical analysis results in addition to the available results from relevant experimental researches in literature. Reliability of the proposed formula is investigated compared to design provisions in different codes. Results demonstrates that the proposed formula is more capable of predicting shear strength of RC-beams provided with SFR rather than shear design codes. Consistency of the proposed formula in predicting shear strength implies that the mechanical concept, on which the proposed formula is derived, is in consistent with the actual mechanical behavior.***

Keywords: ***Shear Strength Model, Reinforced Concrete Beam, Side-Face Reinforcement, Web Reinforcement, Variable Inclination Angle, Shear Span-to-depth Ratio.***

I. INTRODUCTION

Shear strength of reinforced concrete beams (RC-beams) depends on various parameters, underestimating the contribution of any component in shear resistance results in conservative design. During the past few decades, huge number of researches have been conducted to investigate the different parameters that affect shear behavior of RC-beams, such as web reinforcement [1-5], longitudinal reinforcement [5-8], beam size [5,9,10], and concrete strength [2,3]. Many researches are associated with modelling of shear strength of RC-beams with stirrups as web reinforcement [1,11,12].

So far, shear design provisions differ from each other even from theoretical basis [13-16]. ACI 318 [13] adopts the truss model concept to evaluate shear force V as the sum of concrete and stirrups contributions (V_c and V_s^v, respectively) based on the original 45-degree truss model [17], as

$$V = 0.17\sqrt{f_c'}bd + (A_s^v d/a^v)f_y^v \qquad (1)$$

where, $V_c = 0.17\sqrt{f_c'}bd$, and $V_s^v = A_s^v f_y^v d/a^v$. Eurocode 2 [14] acknowledges only concrete or stirrups contribution in evaluating shear strength of RC-beams, but accounts for the variation of the inclination angle of diagonal shear cracking based on variable angle truss model, the range of variation of the inclination angle θ is ($21.8^o \le \theta \le 45.0^o$), and the shear force is given as,

$$V = min\begin{cases} (A_s^v/a^v)\, z f_y^v \cot\theta \\ \alpha_c b z \omega f_c'/(\cot\theta + \tan\theta) \end{cases} \qquad (2)$$

where α_c is a coefficient takes into account the effect of normal stresses on shear strength, it is taken to be 1 for non-prestressed members, and ω is a coefficient that takes into account the reduction in shear force transferred by aggregate interlock with the increase of concrete compressive strength.

Canadian CSA-A23.3 code [15] adopts an alternative method for shear design, named the general method, which considers the compatibility and equilibrium together. The method takes into account the effect of deformation due to flexure in evaluating the contribution of concrete and stirrups to shear strength;

$$V = \beta\sqrt{f_c'}bz + (A_s^v z/a^v)\, f_y^v \cot\theta, \quad \theta \ge 29.0^o \qquad (3)$$

where, β is function of the longitudinal strain at mid-depth, this general method is a simplification of the modified compression field theory [18-20] based on the variable truss angle model. British Standard BS8110 [16] employs the truss mechanism, the ultimate shear strength of RC-member is calculated by summing the contributions of transverse reinforcement and concrete, where the concrete contribution V_c is taken as,

Revised Manuscript Received on August 30, 2019.
* Correspondence Author
A. A. Manar*, Assistant professor at Civil Engineering Department, Faculty of Engineering, Suez Canal University, Egypt.
Email: manar_abdelshakour@eng.suez.edu.eg
T. M. Mansour, Assistant professor at Mechanical Engineering Department, Faculty of Engineering, Suez Canal University, Egypt.
Email: tmansour@eng.suez.edu.eg

$$V_c = \frac{0.79bd}{\gamma_c}\left(\frac{100A_{st}}{bd}\right)^{\frac{1}{3}}\left(\frac{400}{d}\right)^{\frac{1}{4}}\left(\frac{f_c'}{25}\right)^{\frac{1}{3}} \tag{4}$$

where, A_{st} is the area of the longitudinal tension reinforcement and γ_c is a partial safety factor of concrete.

Contribution of main longitudinal steel in shear and torsion resistance of beams beside bending and axial loads is acknowledged by researchers [1,2] and some design provision codes [15,21].

Installing secondary longitudinal reinforcement along the side-faces of large concrete beams is required by standard codes to control cracks that are expected under service loads due to low tensile strength of concrete. There are considerable differences in standard codes about how much side-face reinforcement (SFR) is appropriate [13,15,22,23]. According to the arrangement of this SFR, some may be placed in compression or in tension zones.

Experimental and numerical studies were conducted to investigate the structural behavior of RC-beams provided with SFR. Ten RC-beams were tested [24], results showed that, providing SFR to RC-beams has significant effect on shear response of beams. RC-beams provided with SFR were tested in [25], it was concluded that SFR has considerable contribution to strength of RC-beams. Results of an experimental study conducted by [26] showed that, shear capacity of RC-beams reinforced with SFR is 41.1% greater than shear capacity of similar beams without SFR. It was concluded from the experimental study conducted by [27] that, SFR plays an important role in improving ultimate load capacity of RC-beams. Numerical parametric study was carried out by the author [28] to investigate the effect of SFR on the strength of RC-beams, it was found that, the presence of SFR significantly contributes to the beam strength in general, although this contribution is affected by other parameters.

Researches [24-28] emphasize on that SFR not only controls cracking of RC-beams, but also has an important contribution to the strength of such beams, and hence, it is recommended to be taken into account in design provisions.

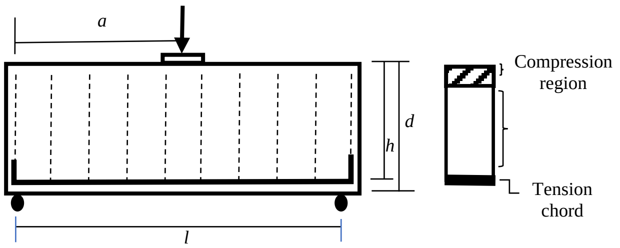
This work aims to introduce a general formula for predicting shear strength of RC-beams. The proposed formula takes into account the contribution of SFR along with concrete, transverse reinforcement, and main reinforcement contributions. The formula is derived for varied inclination angle of cracking. The formula is verified on the basis of the numerical analysis results that was conducted by the author in [28], in addition to the existing experimental results [24,25,26,27]. The parametric study [28] was carried out for considerable wide range of parameters that affect shear behavior of RC-beams, which guarantee wide-ranging verification of the proposed formula.

The following parameters were considered:

- Arrangement of SFR along the beam depth,
- Amount of SFR,
- Ratio of transverse shear reinforcement,
- Dimensions of beam cross-section,
- Shear span-to-depth ratio,
- Amount of tension and compression reinforcement.

II. MODEL BASES

Truss model for RC-beam with web reinforcement provides an excellent theoretical representation to express the forces that exist in cracked beam under shear-flexure interaction. A combination of truss model concept and sectional analysis is used in this work. Typical RC-beam with rectangular cross-section is represented on Fig (1-a), the beam is loaded at a distance a from the mid-point of the nearest support of the beam, the main longitudinal reinforcement is located at a distance $(h-d)$ from the beam bottom surface, where, h is the beam depth and d is the effective beam depth.



(a) (b)

Fig (1): (a) Outline of an RC-beam.(b) Idealization of RC-beam into top compression region, web shear region, and bottom tension chord.

A. *Idealization of the beam*

The shown RC-beam, on Fig (1-a), is subject to flexural-shear stresses, it is idealized herein into three parts; top compression region, web shear region, and bottom tension chord. The expected cracking pattern is shown on Fig (2-a), it consists of vertical flexural cracks at mid-span, inclined web shear cracks near the supports and inclined flexure shear cracks in-between. The crack pattern shown on Fig (2-a) suggests that the beam acts like a truss with the top compression region forming the compression chord members of the truss, the main longitudinal reinforcement forming the tension chord members, the transverse reinforcement in the web shear region acting as the vertical tension members, and the concrete web between diagonal cracks providing the compression diagonal struts. In the traditional truss-model, the compression stresses in the diagonal struts are assumed to remain at 45 degrees [17], as in Fig (2-b), while in the variable-angle truss model, the cracks inclination angle is not equal to 45 degrees, and it might vary along the beam span, see Fig (2-c). Choosing a flatter inclination of the truss diagonals leads to less transverse reinforcement while more longitudinal reinforcement will be required.

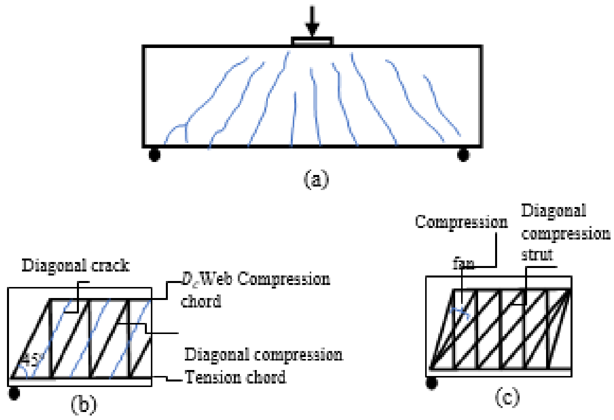


Fig (2): (a) Crack pattern due to flexure and shear stresses. (b) Original truss model with constant diagonal inclination angle of 45º. (c) Variable angle truss model; concrete diagonal strut inclination θ varies dependent on rebars arrangement.

In the present idealization, it is assumed that:

- concrete and web reinforcement in web shear region resist only shear stresses developed in the beam, assisted with the contribution of concrete in compression region and the main longitudinal reinforcement,
- both concrete and web reinforcement in the web shear region resist shear stresses independently,
- flexural stresses are fully resisted by the top compression region along with the main longitudinal reinforcement.

B. Direction of inclined cracking

Referring to Fig (2), concrete in web shear region between two successive diagonal cracks is idealized as a diagonal compression strut, the direction angle of the force in the inclined strut with the longitudinal direction is assumed to be the same as of the crack inclination angle, θ. Structural analysis using truss model with variable inclination angle of compressive struts based on plasticity theory implies that, the element has sufficient ductility which ensures internal forces redistribution. That is, a permissible stress field is satisfied, in which the yield strength of reinforcement and the compressive strength of concrete are not exceeded anywhere. Hence, the ultimate shear resistance may be associated by an arbitrary inclination angle θ of the concrete compressive struts. Yet, since concrete exhibits limited plastic deformations, the compressive strut inclination angle should be limited in range. Moreover, for low inclination angle θ a large strain of web reinforcement is expected, thus, widens the inclined cracks and reduces the resistance of the inclined compressive struts. Therefore, minimum value of the compressive strut inclination angle should be defined. Various recommended values of compressive strut inclination angle, θ proposed by different codes for shear design of reinforced concrete beams are given in table (1).

Table 1: Various compressive strut inclination angle proposed by codes.

Code	Angle θ
Eurocode 2 EN 1992-1-1 [14]	$21.8^o \le \theta \le 45.0^o$
Canadian Code CSA-A23.3 [15]	$29^o \le \theta$
German Code DIN 1045-1 [23]	$18.4^o \le \theta \le 59.9^o$

For RC-beam subject to shear and bending only, the detailed condition for the compressive strut inclination angle adopted by German Code [23], in table (1), can be expressed as

$$0.58 \le \cot\theta \le \min\left\{\frac{1.2}{1-\frac{V_c}{V}}\ ,\ 3.0\right\} \qquad (5)$$

where, V_c is the concrete contribution in resisting shear force, and V is the overall shear force resisted by the cross-section. Compressive strut inclination angle θ can be calculated as presented in [29] as follows,

$$\theta = \tan^{-1}\left(\left(\frac{E_c}{v_n}\,\varepsilon_x + 2\right)/\left(\frac{E_c\tan\theta}{E_s\rho_v} + 2\right)\right)^2 \qquad (6)$$

where, ε_x is the longitudinal strain, v_n is the nominal shear stress, ρ_v is the volumetric ratio of transverse shear reinforcement, and E_c and E_s are the material elasticity moduli. The longitudinal strain used in (6), if not measured in experiment, is evaluated by conducting general strain compatibility analysis, where the tension stiffening and the contribution from longitudinal reinforcement can be accounted for. Another equation to calculate the diagonal crack angle was developed in [30], the equation was derived by considering energy minimization on the work done over the components of truss model, the derived expression of θ is,

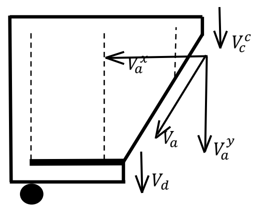
$$\theta = \tan^{-1}\left(\left(\rho_v\frac{E_s}{E_c} + \lambda\frac{\rho_v A_v}{\gamma_s A_g}\right)/\left(1 + \rho_v\frac{E_s}{E_c}\right)\right)^{\frac{1}{4}} \qquad (7)$$

where, γ_s is the volumetric ratio of longitudinal reinforcement, A_g is the gross sectional area of concrete element, A_v is the shear area of concrete section, and λ is the member end-fixity factor.

C. Shear resisted by concrete

Shear force resisted by concrete at an inclined crack is transferred across the crack as shown on Fig (3), where V_a is the force transferred by interlock of aggregate, V_a^x and V_a^y are its components, V_c^c is the shear force in the top compression region, and V_d is the dowel action of longitudinal reinforcement. So, the shear force at an inclined crack without web reinforcement is resisted by the sum of V_c^c, V_a^y, and V_d, as shown on Fig (3).

Fig (3): Components of shear force carried by concrete through an inclined crack.



It is difficult to separately quantify the contribution of each of these components; thus, these are lumped as the concrete contribution in resisting shear force in RC-beam, V_c, and is taken equal to the failure capacity of the beam without web reinforcement. Shear stress distribution along non-cracked compression region of cracked rectangular cross section of RC-beam has parabolic distribution [31], while it exhibits a uniform distribution through the cracked web shear region [19], as shown on Fig (4).

The contribution of the top compression region in resisting shear force is calculated to $0.07V_c$ by considering the geometrical shape of shear stress distribution, while $0.93V_c$ is resisted by the concrete in web shear region [12].

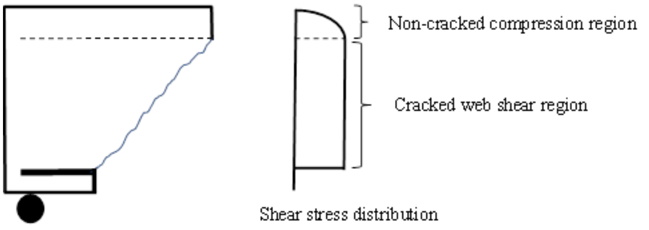


Fig (4): Shear stress distribution along the depth of cracked rectangular cross section of RC-beam.

Denoting the compression force in the inclined concrete strut that transfers shear resisted by concrete in the web region as D_c, and equilibrate forces acting on the beam portion shown on Fig (5-a), thus, the shear force resisted by concrete in the web shear region, V_c^w, equals the vertical component of the compression force D_c,

$$V_c^w = D_c\, sin\theta \quad (8)$$

then, concrete contribution in resisting the shear force at a diagonal crack in RC-beam, V_c is equal to,

$$V_c = \frac{D_c \sin\theta}{0.93} \quad (9)$$

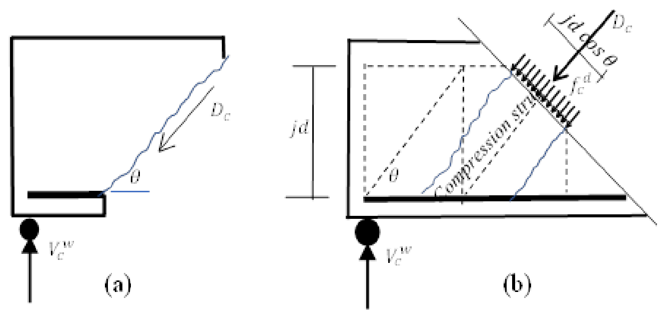


Fig (5): (a) Diagonal compression force transfers the shear carried by concrete in the web shear region. (b) Diagonal concrete strut carries the shear force resisted by concrete in the web region.

Referring to Fig. (5-b), The cross-sectional area of the diagonal concrete strut equals to $bjd\cos\theta$, where b is the beam width, hence, the diagonal force, D_c, in the concrete compression strut can be expressed as,

$$D_c = f_c^d bjd\cos\theta \quad (10)$$

From (9) and (10), the concrete contribution to shear force, V_c, is expressed as,

$$V_c = \frac{f_c^d bjd \sin\theta \cos\theta}{0.93} \quad (11)$$

where, f_c^d is the average compression stress in the diagonal concrete strut, and j is the lever arm coefficient, given as,

$$j = 1 - \frac{k}{3} \quad (12)$$

The resulting expression for k which was derived from the classical bending theory for a single reinforced concrete section [32] is used here,

$$k = \sqrt{2\gamma_s m + (\gamma_s m)^2} - \gamma_s m \quad (13)$$

where, m is the modular ratio between the moduli of elasticity of steel and concrete respectively, $(m = E_s/E_c)$, and γ_s is the volumetric ratio of the main longitudinal reinforcement, $(\gamma_s = A_{st}/bd)$.

D. Concrete compressive strength in diagonal strut

Concrete subject to diagonal compressive stress and cracked parallel to the direction of compression is weaker than concrete in uniaxial compression, as in a cylinder or cube test, this is known as the compression softening effect due to the presence of transverse tensile strain, see Fig (6). Due to compression softening effect, the average compression stress in the diagonal concrete strut, f_c^d, in the presence of the transverse tensile strain is given by,

$$f_c^d = \beta f_c \quad (14)$$

where, f_c is the uniaxial compressive stress in concrete and β is a softening coefficient accounts for the compression softening effect, analogously,

$$f_c^e = \beta f_c' \quad (15)$$

where $f_c^e$is the effective concrete compressive strength in the diagonal strut direction, and $f_c'$is the characteristic compressive strength of concrete.

In early softening model proposed by [33], based on test data carried out on concrete members with characteristic strength less than 40Mpa, modifications were made to both the pick stress and the corresponding strain of Hognestad parabola.

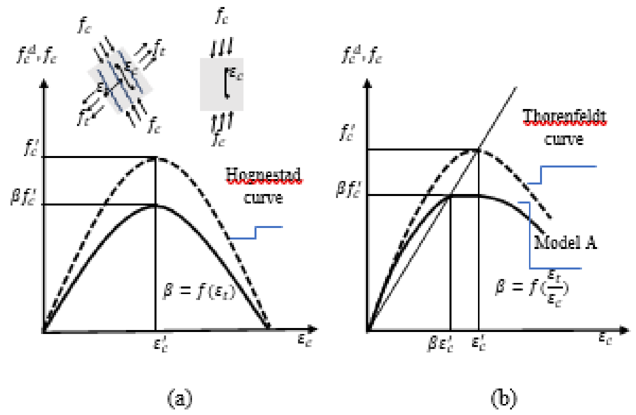


Fig (6): Compressive stress–strain response of concrete subject to transverse tensile stress. (a) Softening model proposed in [18]. (b) Softening model proposed in [34].

To facilitate the use of softening model in design procedure, a simplified model was presented [18], based on Hognestad parabola as the base curve, in which the coefficient β depends only on the transverse tensile strain ε_t, see Fig (6-a), the softening coefficient was presented as,

$$\beta = \frac{1}{0.8-0.34\frac{\varepsilon_t}{\varepsilon_c'}} \qquad \leq 1.0 \qquad (16)$$

where, ε_c' is the compressive strain corresponding to the characteristic compressive strength of concrete, f_c' taken equals to 0.002. If the value of the transverse tensile strain corresponding to the transverse concrete tensile stress acting on the diagonal strut at cracking is ε_r, hence, the effective concrete strength in the diagonal strut direction f_c^e can be expressed as,

$$f_c^e = \frac{f_c'}{0.8+170\varepsilon_r} \qquad \leq f_c' \qquad (17)$$

The value of f_c^e, in (17) is a function of the unknown strain ε_r, the stress-strain curve for concrete in tension is assumed straight up to the tensile strength [33], see Fig (7), within this range, the modulus of elasticity in tension can be taken the same as in compression [34], thus, ε_r can be expressed as $\varepsilon_r = f_r/E_c$,where f_r is the transverse concrete tensile stress acting on the concrete strut at cracking. The concrete strut element in the web shear region is subject to biaxial tension-compression stress state, the tensile strength in a biaxial state is lower than that of the concrete loaded in uniaxial regime, f_{ct}. Therefore, the maximum value of f_r which can be attained is less than the limiting value f_{ct}.

By adopting the tension stiffening model [33], presented on Fig (7), the principal tensile stress of concrete in the web shear element can be calculated as,

$$f_t = E_c\varepsilon_t \qquad for \quad \varepsilon_t \leq \varepsilon_r \qquad (18)$$

$$f_t = f_{ct}/(1+\sqrt{200\varepsilon_t}) \leq f_r \quad for \quad \varepsilon_t > \varepsilon_r \qquad (19)$$

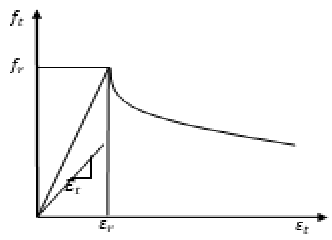


Fig (7): Tensile stress–strain response of concrete in biaxial stress state [33].

By imposing $f_r = f_{ct}$ as the transverse concrete tensile stress in the web shear region at failure, the effective concrete strength in the diagonal strut direction f_c^e in (17) becomes,

$$f_c^e = \frac{f_c'}{0.8+170\frac{f_{ct}}{E_c}} \qquad \leq f_c' \qquad (20)$$

Another softening model, referred to as Model A, was proposed by [34] based on Thorenfeldt model [35] as base curve, see Fig (6-b), the softening coefficient was presented as,

$$\beta = \frac{1}{1+k_c k_f} \qquad (21)$$

where,

$$k_c = 0.35\left(\frac{-\varepsilon_t}{\varepsilon_c} - 0.28\right)^{0.8} \qquad \geq 1.0\,, \qquad (22)$$

$$k_f = 0.1825\sqrt{f_c'} \qquad \geq 1.0 \qquad (23)$$

where k_c represents the effect of transverse cracking and straining, and k_f represents the effect of concrete strength.

German Code [23] defines the effective design value of concrete compressive strength of normal weight concrete as,

$$f_{c,design}^e = 0.75\left(min\left\{1.1-\frac{f_c'}{500}\,,1.0\right\}\right)f_c'(\alpha_{cc}/\gamma_c) \qquad (24)$$

where, f_c' is in Mpa, γ_c is a partial safety factor, and α_{cc} is a coefficient accounts for the way the load is applied, from (24), the effective compression strength in the concrete strut may be expressed as,

$$f_c^e = 0.75\left(min\left\{1.1-\frac{f_c'}{500}\,,1.0\right\}\right)f_c' \qquad (25)$$

According to Eurocode 2 EN 1992-1-1, the effective design value of concrete compressive strength for non-prestressed members is obtained by reducing the design compressive strength as

$$f_{c,design}^e = 0.6\left(1-\frac{f_c'}{250}\right)(\alpha_{cc}f_c'/\gamma_c) \qquad (26)$$

with f_c' is in Mpa, therefore, the effective concrete compressive strength may be expressed as,

$$f_c^e = 0.6\left(1-\frac{f_c'}{250}\right)f_c' \qquad (27)$$

The present softening coefficients are summarized in table (2).

Table 2: Softening coefficients adopted in different approaches

Source	Softening coefficient	Comments
Ref. [18]	$\beta = \dfrac{1}{0.8 + 170\dfrac{f_{ct}}{E_c}} \leq 1.0$	$f_c' < 40$ MPa
Ref. [34]	$\beta = \dfrac{1}{1 + k_c k_f}$ $k_c = 0.35\left(\dfrac{-\varepsilon_t}{\varepsilon_c} - 0.28\right)^{0.8} \geq 1.0$ $k_f = 0.1825\sqrt{f_c'} \quad \geq 1.0$	
Eurocode 2 EN 1992-1-1	$\beta = 0.6\left(1 - \dfrac{f_c'}{250}\right)$	non-prestressed member
German codes DIN 1045-1	$\beta = 0.75\left(min\left\{1.1 - \dfrac{f_c'}{500}, 1.0\right\}\right)$	normal weight concrete

The softening model which is proposed by [34] and referred to as “Model A”, shown on Fig (6-b), is selected from among the other softening models and adopted in this work to calculate the effective concrete compressive strength f_c^e. This softening model is chosen since the effect of transverse straining along with the concrete strength is included in the model. In addition, this model gives greater reduction for concrete with greater compressive strength, what is in agreement with the reduction models of Eurocode 2 and German codes listed in table (2). Moreover, it gives good correlation for wide range of concrete strengths as stated in [34], on the contrary, the softening model shown on Fig (6-a), presented by [18], does not give good correlation for high strength concrete neither for concrete with $f_c' < 20$ Mpa as mention by [34].

By substituting $f_c^d = f_c^e$ in (11), the limiting concrete contribution, V_c^{max} , to shear resistance is expressed as,

$$V_c^{max} = \frac{f_c^e bjd \sin\theta \cos\theta}{0.93} \tag{28}$$

by using (15),

$$V_c^{max} = \frac{\beta f_c'(bjd \sin\theta \cos\theta)}{0.93} \tag{29}$$

where, β is the softening coefficient, define γ as,

$$\gamma = \frac{j \sin\theta \cos\theta}{0.93}\beta \tag{30}$$

then we get,

$$V_c^{max} = \gamma f_c' bd \tag{31}$$

here, V_c^{max} is the maximum contribution of concrete in resisting shear force, no matter how much web reinforcement is provided. Concrete contribution in resisting shear force at diagonal crack in RC-beam, V_c , is obtained as following,

$$V_c = \mu V_c^{max} \tag{32}$$

where, μ is a reduction factor that is applied to the limiting value V_c^{max} to abtain V_c, therefore,

$$V_c = \mu\gamma f_c' bd \tag{33}$$

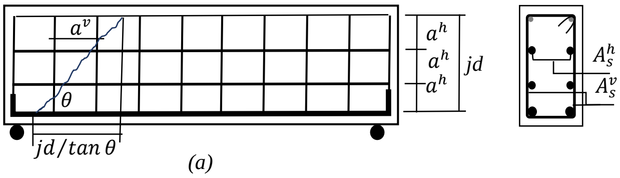
E. Shear resisted by web reinforcement

Concrete in web shear region is stiffened by web reinforcement to resist shear stresses. It is widely accepted in literature and codes of practices that shear capacity of RC-beam equals the sum of the contribution from concrete V_c and that from web reinforcement represented by the provided stirrups V_s^v. This paper considers the SFR contributes as web reinforcement along with the stirrups in shear resistance of RC-beams. Thus, the stirrups act as transverse web reinforcement and the provided side-face bars work as longitudinal web reinforcement, Fig (8-a).

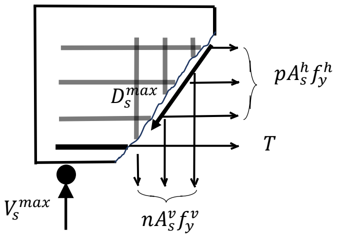
Consequently, the shear force V_s resisted by the web shear reinforcement is given by the sum of two components as,

$$V_s = V_s^v + V_s^h \tag{34}$$

where, V_s^h represents the shear force resisted by SFR, and V_s^v is the shear force resisted by vertical stirrups. The ultimate force carried by n-number of identical stirrups crossing the diagonal crack, Fig (8-b), equals $nA_s^v f_y^v$ where A_s^v is the total cross-sectional area of one stirrup branches and f_y^v is the yield strength of stirrups steel. Alike, the ultimate force carried by p-number of horizontal side-face bars equals $pA_s^h f_y^h$, where A_s^h is the total cross-sectional area of one row of side-face bars, Fig (8-b), and f_y^h is the yield strength of SFR.



(a)



(b)

Fig (8): (a) Arrangement of transverse stirrups and longitudinal side-face bars in reinforced concrete beam. (b) Schematic of shear resistance by means of reinforcement at an inclined crack of RC-beam.

The number of stirrups, n crossing the diagonal crack, Fig (8-a), is defined as,

$$n = \frac{jd/\tan\theta}{a^v} \tag{35}$$

and a^v is the equal distance between stirrups. Similarly, the number of side-face bars rows, p, is expressed as,

$$p = \frac{jd}{a^h} - 1 \tag{36}$$

where, a^h is the equal distance between side-face bars along the beam depth.
Sectional analysis of forces acting on the beam portion shown on Fig (8-b) leads to the following equilibrium equations,

$$D_s^{max} \sin\theta + nA_s^v f_y^v = V_s^{max} \tag{37}$$

$$D_s^{max} \cos\theta = pA_s^h f_y^h + T \tag{38}$$

where, V_s^{max} and D_s^{max} are the limiting shear resistance and the limiting compression force in the diagonal strut, respectively, which can be carried by horizontal and vertical web reinforcement all yielding, and T is the main longitudinal reinforcement contribution in shear.
Assuming that $(\mu_1 f_y^v)$ and $(\mu_2 f_y^h)$ are the average tensile stresses in the vertical stirrups and the horizontal side-face bars crossing the diagonal crack, respectively. The reduction factors $\mu_1$and μ_2 are included in accounting for the possibility that some vertical and horizontal web reinforcement bars may not reach yielding, depending on their location across the crack, and still functional even after extensive cracking of the beam web prior to failure [10]. Analogously to (37) and (38),

$$D_s \sin\theta + nA_s^v(\mu_1 f_y^v) = V_s \qquad \mu_1 \leq 1.0 \tag{39}$$

$$D_s \cos\theta = pA_s^h(\mu_2 f_y^h) + T \qquad \mu_2 \leq 1.0 \tag{40}$$

The contribution of the main longitudinal reinforcement in shear resistance, T, can be expressed as

$$T = (\mu_3 A_{st}) f_y \qquad \mu_3 \leq 1.0 \tag{41}$$

where, A_{st} and f_y are the main reinforcement cross-sectional area and the yield strength of the main reinforcement, respectively, and $(\mu_3 A_{st})$ is the main reinforcement ratio, unavoidably, contributes in shear resistance. Therefore, from (39) to (41), the shear force resisted by web reinforcement both vertical stirrups and horizontal side-face bars assisted by main longitudinal reinforcement is concluded as,

$$V_s = \mu_1 nA_s^v f_y^v + \left(\mu_2 pA_s^h f_y^h + \mu_3 A_{st} f_y\right) \tan\theta \tag{42}$$

Equation (42), by means of (35) and (36) becomes,

$$V_s = \mu_1 \left(\frac{jd}{a^v \tan\theta}\right) A_s^v f_y^v + \left(\mu_2 \left(\frac{jd}{a^h} - 1\right) A_s^h f_y^h + \mu_3 A_{st} f_y\right) \tan\theta \tag{43}$$

The nominal shear stress, v_n carried, independently, by concrete and reinforcement in RC-beam is given as,

$$v_n = \frac{1}{bd}(V_c + V_s) \tag{44}$$

by means of (33) and (43), (44) becomes,

$$v_n = \mu\gamma f_c' + \mu_1 \left(\frac{j}{a^v b \tan\theta}\right) A_s^v f_y^v + \left(\mu_2 \left(\frac{j}{a^h b} - \frac{1}{bd}\right) A_s^h f_y^h + \frac{1}{bd}\mu_3 A_{st} f_y\right) \tan\theta \tag{45}$$

thus, the proposed parametric formula for computing the nominal shear strength of reinforced concrete beam with stirrups, and provided with SFR is obtained as,

$$v_n = \mu\gamma f_c' + \mu_1 \gamma_v f_y^v \cot\theta + \left(\mu_2 \gamma_h f_y^h + \mu_3 \gamma_s f_y\right) \tan\theta \tag{46}$$

where, γ is calculated from (31), $(\gamma_v = jA_s^v/ba^v)$, $(\gamma_h = j\,A_s^h/ba^h - A_s^h/bd)$, and $(\gamma_s = A_{st}/bd)$.
Equation (46) is a function of the unknown coefficients μ, μ_1, μ_2, and μ_3, which will be determined on the basis of experimental and numerical results. The proposed formula in (46) is function of the varied diagonal crack inclination angle θ. The inclination angle θ can be calculated using (7) for data set with constant value of shear span-to-depth ratio, a/d , as (7) seems to be insensitive to the variation of a/d.
To validate the present formula for varied shear span-to-depth ratios, (6) is rather used to calculate angle θ, as it is implicitly sensitive to a/d variation.

III. COEFFICIENTS OF THE PROPOSED FORMULA

Numerical analysis results [28] along with experimental results [24,25,26,27] are utilized here as the verification data, v_{ver}. This verification data is used to verify the present proposed formula. The unknown coefficients μ , μ_1, μ_2, and μ_3 in (46) are calculated based on minimizing the coefficient of variation (CV), which is calculated as the ratio between the standard deviation and the average (SD/AVG) of the ratios between the shear strength verification data, v_{ver}, and the nominal shear strength predicted by the proposed formula, v_n.

The values of μ, μ_1, μ_2, and μ_3 have been found to be 0.18, 0.76, 0.43, and 0.12 respectively, consequently (46) becomes,

$$v_n = 0.18\gamma f_c' + 0.76\gamma_v f_y^v \cot\theta + \left(0.43\gamma_h f_y^h + 0.12\gamma_s f_y\right)\tan\theta \qquad (47)$$

An average (AVG) value of 1.01 and a coefficient of variation (CV) of 0.113 are found on the comparison between the verification results v_{ver} and the values v_n predicted by (47).

IV. COMPARISON TO SHEAR DESIGN PROVISIONS IN DIFFERENT CODES

Reliability of (47) in predicting shear strengths of RC-beams is investigated through comparing its results with the predictions of ACI 318, CSA A23.3, and BS8110 codes. Fig (9-a) shows the nominal shear strengths, v_n , calculated using the ACI shear design formula (1) versus the shear strengths verification results, v_{ver} , while Fig (9-b) shows the nominal shear strengths, v_n, calculated with the proposed formula (47) versus v_{ver} . The (AVG) and (CV) values of the ratios between the nominal shear strengths v_n and the verification shear strengths v_{ver} have been stated on figures (9-a) and (9-b), the ACI 318 prediction has mean value of 1.17 and (CV) of 0.19. The (CV) obtained for the results predicted with the proposed formula is 40% less, which indicates that the proposed formula predicts shear strength more satisfactorily than ACI code, that is because the proposed formula takes into account the contribution of SFR to shear strength.

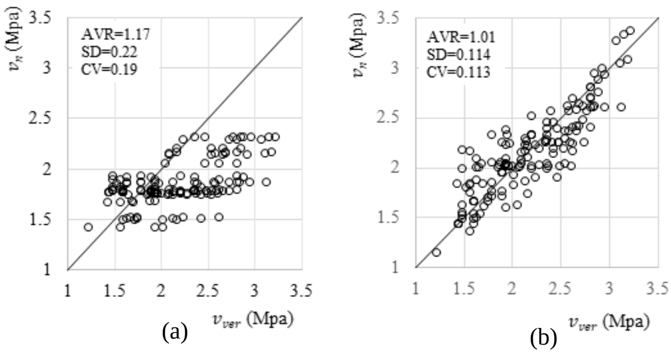


Fig (9): Verification shear strengths versus nominal shear strengths predicted by: (a) ACI 318 code, (b) Proposed formula.

The shear strength results obtained with the proposed formula (47) have been compared also with the results obtained by Equation (3) of the Canadian code and Equation (4) of the British code, the results are plotted on Fig (10) and Fig (11).

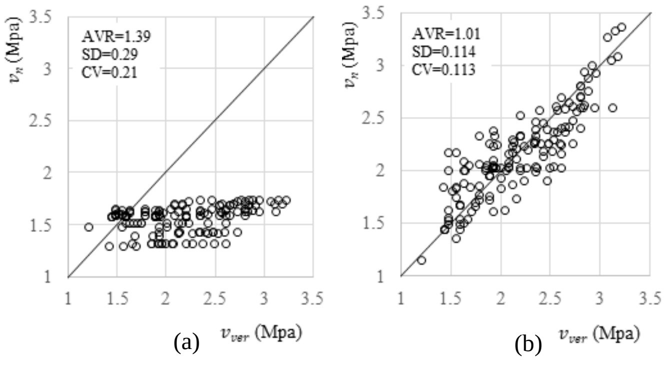


Fig (10): Verification shear strengths versus nominal shear strengths predicted by: (a) CSA A23.3 code, (b) Proposed formula

While (47) provides an (AVG) of 1.01 and a (CV) of 0.11, the predicted results by CSA A23.3 has an (AVG) and a (CV) of 1.39 and 0.21, respectively, and the predicted results by BS8110 has an (AVG) and a (CV) of 1.21 and 0.2, respectively. Comparison of results emphasizes on the superiority of (47) to predict the shear strength of RC-beams provided with SFR.

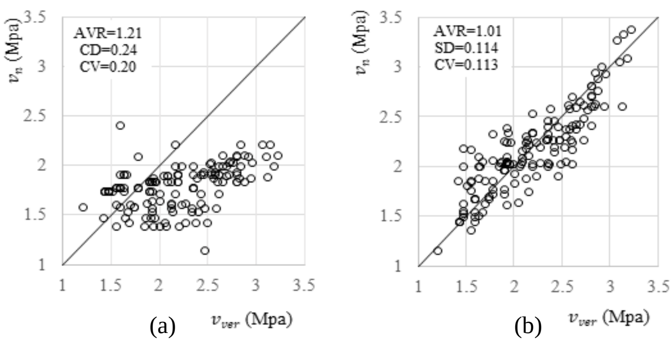


Fig (11): Verification shear strengths versus nominal shear strengths predicted by: (a) BS8110 code, (b) Proposed formula.

The standard deviation (SD) of the ratios (v_{ver}/v_n) obtained for Equation (47) and the provisions codes are plotted versus (a/d) on Fig (12). It can be observed that the results predicted by the proposed formula have the least (SD) among the predictions of all codes in comparison, which means that the proposed shear strength formula gives more consistent prediction for considerable range of shear span-to-depth ratios of RC-beams.

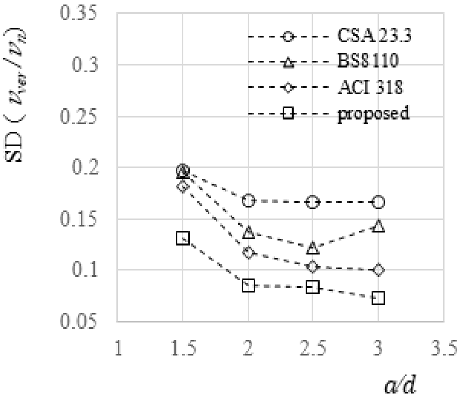


Fig (12): Effect of shear span-to-depth ratio on shear strength prediction with the proposed formula and the code of provisions.

The reliable prediction of the proposed formula is owed to that it takes into account the contribution of SFR to shear resistance. Besides that, the proposed formula is derived based on mechanical concept that considers the variation of the inclination angle of cracking.

V. CONCLUSION

In this paper, an analytical formula for predicting the shear strength of RC-beams provided with SFR is developed. The experimental and numerical studies available in the literature, and relevant to shear resistance of RC-beams provided with SFR, is used for the verification of the proposed model. The following conclusions can be drawn:

- A single explicit formula for predicting shear strength of RC-beams provided with SFR is obtained based on varied inclination angle of cracking.
- The developed formula considers the effect of main parameters such as shear span–depth ratio, stirrup ratio, main reinforcement ratio, and beam depth and width.
- The proposed formula takes into account the amount and distribution of SFR.
- The coefficient of variation (CV) between shear strength predicted with the proposed formula and the verification shear strength data is 0.113.
- Coefficients of variation (CV) for shear strengths predicted with the proposed formula are 40%, 46%, and 44% less, compared to predictions of ACI 318, CSA A23.3, and BS8110 design provisions, respectively, which highlights its superiority to predict shear strength of RC-beams provided with SFR.
- The formula gives reliable prediction of shear strength for RC-beams with shear-span-to-depth ratios ranges from 1.5 to 3.0.
- Consistency of the proposed formula in predicting shear strength of such beams emphasizes on the important contribution of SFR on shear response of RC-beams, and hence, it is recommended to be taken into account in designing of such beams.
- Since only force equilibrium is considered in the present analysis, it is recommended to extend the study to conduct compatibility analysis to take into consideration the effect of deformation due to flexure.

REFERENCES

1. G. Russo, D. Mitri, M. Pauletta, Shear Strength Design Formula for RC Beams with Stirrups, Engineering Structures, 51, (2013), 226–235.
2. Y. S. Yoon, W. D. Cook, D. Mitchell, Minimum shear reinforcement in normal, medium, and high-strength concrete beams, ACI Structural Journal, 93(5), (1996), 576–584.
3. D. Angelakos, E. C. Bentz, M. P. Collins, Effect of Concrete Strength and Minimum Stirrups on Shear Strength of Large Members, ACI Structural Journal, 98(3), (2001), 290–300.
4. G. Arslan, Shear strength of reinforced concrete beams with stirrups, Materials and Structures, 41(1), (2008), 113–122.
5. E. J. Tompos, R. J. Frosch, Influence of Beam Size, Longitudinal Reinforcement, and Stirrup Effectiveness on Concrete Shear Strength, ACI Structural Journal, 99(5), (2002), 559–567.
6. G Russo, G Somma, D. Mitri, Shear strength analysis and prediction for RC beams without stirrups., Journal of Structural Engineering ASCE, 131(1), (2005), 66–74.
7. N. Khaldoun, Longitudinal Steel Stresses in Beams Due to Shear and Torsion in AASHTO-LRFD Specifications, ACI Structural Journal, 102(5), (2005), 689-698.
8. N. Khaldoun, Minimum Transverse Reinforcement In 65 Mpa Concrete Beams, ACI Structural Journal, 101(6), (2004), 872-878.
9. C. Chiu, T. Ueda, K. Chi, S. Chen, Shear Crack Control for High Strength Reinforced Concrete Beams Considering the Effect of Shear-Span to Depth Ratio, International Journal of Concrete Structures and Materials, 10(4), (2016), 407–424.
10. B. Hu, Y. Wu, Effect of Shear Span-To-Depth Ratio on Shear Strength Components of RC Beams, Engineering Structures, 168, (2018), 770–783.
11. G. Davor, S. Paulo, G. Ivana, Shear Resistance of Reinforced Concrete Beams in Dependence on Concrete Strength in Compressive Struts, Tehnički Vjesnik 22(4), (2015), 925-934.
12. E. Jude, CH. Oh, SC. Lee, Single Web Shear Element Model for Shear Strength of RC Beams with Stirrups, International Journal of Concrete Structures and Materials, 12(1), (2018), Article 27.
13. ACI Committee 318-14, Building Code Requirements for Structural Concrete and Commentary, ACI 318-14/ACIR-14, Farmington Hills, MI: American Concrete Institute, (2014).
14. Eurocode 2, European Committee for Standardization. Pren 1992-1-1:2003. Design of Concrete Structures, Part 1: General Rules and Rules for Buildings. Revised Final Draft, Brussels, Belgium, December (2003).
15. CAN/CSA-A23.3, Design of Concrete Structures, Rexdale, Canadian Standards Association, (2014).
16. BS8110, Structural Use of Concrete. Part 1: Code of Practice for Design and Construction, British Standards Institution (BSI), Milton Keynes, (1997).
17. E. Morsch, Der Eisenbetonbau - Seine Theorie und Anwendung (Reinforced Concrete Construction - Theory and Application), 5th Edition, 1(2), K. Wittwer, Stuttgart, (1922).
18. F.J. Vecchio, M.P. Collins, The Modified Compression Field Theory for Reinforced Concrete Elements Subjected to Shear, ACI Journal Proceedings, 83(2), (1986), 219-231.
19. F. J. Vecchio, M. P. Collins, Predicting the Response of Reinforced Concrete Beams Subjected to Shear Using the Modified Compression Field Theory, ACI Structural Journal, 85(3), (1988), 258-268.
20. M. P. Collins, D. Mitchell, Prestressed Concrete Structures, Prentice Hall, Englewood Cliffs, N.J., (1991), 766.
21. American Association of State Highway and Transportation Officials., AASHTO LRFD Bridge Design Specifications, Sixth Edition, Washington, (2012).
22. CP114, Code of Practice for The Structural Use of Concrete, British Standard Institute, (1972).
23. DIN 1045-1, Tragwerke Aus Beton, Stahlbeton Und Spannbeton; Teil 2: Bemessung Und Konstruktion, Betonkalender, Ernst & Sohn, Berlin, (2009), 478-584.

24. A. Yehia, Shear Response of Large R.C. Beams Provided with Side Bars Under Repeated Loading, Journal of Engineering Sciences, Assiut University, 34(4), (2006), 1113-1131.
25. A. Luiz, V. Alexandre, Analise Experimental Da Influência Da Armadura De Pele No Combate Ao Cisalhamento Em Vigas De Concreto Armado, UNESC- Universidade Do Extremo Sul Catarinense, (2017).
26. P. Janilson, V. Alexandre, Analise Experimental Da Influência Da Armadura De Pele No Combate Ao Cisalhamento Em Vigas De Concreto Armado, 16 F. Trabalho De Conclusão De Curso (Graduação Em Engenharia Civil), Universidade Do Extremo Sul Catarinense, Criciúma, (2016).
27. E. Zainab, A. Yehia, M. Mohamed, A. Ahmed, Contribution of Shrinkage Bars in Behaviour of Reinforced Concrete Beams under Static Loading, M. Sc. Thesis, Department of Civil Engineering, Faculty of Engineering, Assiut University, (2002).
28. A. A. Manar, E. A. Zainab, Strength of Reinforced Concrete Beams Provided with Side-Face reinforcement, International Journal of Latest Trends in Engineering and Technology, 14(2), (2019), 10-16.
29. P. Adebar, J. Leeuwen, Side-Face Reinforcement for Flexural and Diagonal Cracking in Large Beams, ACI Structural Journal, (1999), 698-704.
30. J. Kim, J. Mander, Theoretical Crack Angle in Reinforced Concrete Elements Subjected to Strong Earthquakes, 12th World. Conf. On Earthquake Eng., Auckland, New Zeeland, 944, (2000).
31. J. Gere, B. Goodno, Mechanics of Materials (8th Ed.), Boston, MA: Cengage Learning, (2013).
32. S. U. Pillai, M. Devdas, Reinforced Concrete Design, 3rd Edition, Tata Mcgraw Hill, New Delhi.
33. F. J. Vecchio, M. P. Collins, The Response of Reinforced Concrete to In-Plane Shear and Normal Stresses, Publication No. 8243. Toronto, ON: Department of Civil Engineering, University of Toronto, (1982).
34. F. J. Vecchio, M. P. Collins, Compression Response of Cracked Reinforced Concrete, Journal of Structural Engineering, 119(12), (1993), 4184.
35. E. Thorenfeldt, A. Tomaszewicz, J. Jensen, Mechanical Properties of High-Strength Concrete and Application in Design, Proc. Symposium Utilization of High-Strength Concrete. Stavanger, Norway, Tapir Trandheim, (1987).

AUTHORS PROFILE

A. A. Manar, Assistant professor, Civil Engineering Department, Faculty of Engineering, Suez Canal University, Egypt (2013- until now).
Assistant professor, UTM, Malaysia (2011-2013).
Ph.D., Graduate School of Engineering, Tohoku University, Japan.

Email: manar_abdelshakour@eng.suez.edu.eg

T. M. Mansour, Assistant professor, Mechanical Engineering Department, Faculty of Engineering, Suez Canal University, Egypt (2013- until now).
Visiting researcher, Graduate School of Engineering, Tohoku University, Japan (2008 - 2011).
Ph.D., Graduate School of Engineering, Tohoku University, Japan.
Email: tmansour@eng.suez.edu.eg