Active Suspension with Model Predictive Control

Rashmi Ranjan Das, Vinodh Kumar E

Abstract— This paper examines the performance of Model Predictive Control (MPC) scheme for an Active suspension. A vehicle suspension is designed to provide superior ride comfort and road handling characteristics. Unlike passive suspensions, the Active suspension can change the dynamic of suspension in real-time by injecting force into the system. MPC allows the active suspension to provide better and consistent passenger comfort and road handling capabilities for different road profile. Even though long back, the idea of active suspension conceived, the prohibitive cost and complexity restricted its usage. In recent years active suspension is receiving more and more attention with users preferring a high-end car. In an active suspension for the real-time adjustment of the control force, need a design of a controller. In literature, many controllers used such as Proportional Integral Derivative (PID), Linear Quadratic regulators (LQR), Fuzzy logic controller, Artificial Neural Networks (ANN). In this paper, revealed a model predictive control arrangement for Active suspension model. MPC is an optimal control scheme which uses a model of plant for predicting the future output. The control inputs are optimized such that these predicted outputs meet the desired level of performance. Tested the MPC control scheme is using a bench-scale replica of Quarter active suspension model from QUANSER. To better appreciate the capabilities of the MPC Control Scheme, compared the performance of the active suspension with that of an LQR control scheme, and passive suspension.

Keywords—Model Predictive Control, Active suspension, Qunaser, Linear Quadratic Regulator.

I. INTRODUCTION

Since the invention of the first car back in 1886 by Karl Benz, the quest for the car with better ride comfort and handling has begun, which led to the invention of suspension. Up until the late '70s, the passive suspension had the monopoly of the suspension system. The passive suspension has evolved time to time into different kinds with variation in structure, operating principle of springs and dampers. Even with all these advancements, the passive suspension could not provide consistent performance with varying road terrain. Engineers solved this problem by developing an active suspension.An Active suspension primarily consists of an actuator in addition to the spring and damper found in a passive suspension. The actuator injects an independent force to manipulate the suspension deflection in real-time [8] [4].

Rashmi Ranjan Das*, School of Electrical Engineering, Vellore Institute, of Technology, Vellore, India. Email:rashmiranjandas@vit.ac.in
Vinodh Kumar E, School of Electrical Engineering, Vellore Institute of Technology, Vellore, India. Email: Vinodhkumar.e@vit.ac.in

Actuator makes a continuous adjustment to the suspension in real-time based on the profile of road ahead.

Over the years, different control schemes such as PID, Linear Quadratic regulator, LQR, Artificial Neural Network (ANN), Fuzzy logic, etc. are used to calculate the actuation signal [9].

Model Predictive Control (MPC) designed an advanced control strategy which is in the time domain. Its ability to handle constraints on system explicitly sets it apart from other controllers. Initially use of MPC restricted to slow process in industries such as petrochemical, paper and pulp mill due to computational requirement complexity. Recently with the advent of high-speed processor and memory, fast optimization algorithm, MPC is applied to quick process with the bandwidth of 50 kHz. An effort is made to apply linear MPC algorithm to the active suspension on quarter car model experimental setup compared the experimental results for MPC and LQR control scheme for and active suspension in this work.
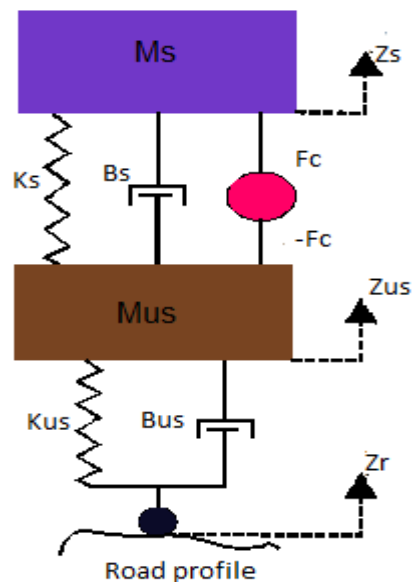

Fig. 1.Schematic of Quarter Car Model

II. MATHEMATICAL MODELLING

A. Quarter Car model

A Quarter Car model with two degrees of freedom is used to study the performance of the Model Predictive control and LQR Scheme for active suspension. Even though it's a simplified model of the suspension system, it is useful in studying the vertical motion of the suspension. It is represented using double spring mass damper, as shown in Fig.1. The top mass M_s is the sprung mass, which represents the weight of the passenger and the vehicle body.

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Sprung mass mounted on the suspension. Active suspension modeled as a combination of spring of stiffness K_s, a damper with damping coefficient B_s and an actuator capable of exerting an independent force F on the vehicle body. The bottom mass in Fig.1 represents the mass of the unsprung mass (weight of tire and linkages) connecting the wheels to the suspension. The tire modeled as spring constant and damper. The lower spring and damper represent the tire stiffness coefficient K_{us} and damping factor B_{us}, respectively.

The mathematical modeling of the quarter car model is obtained by using the first principle of (Newton's Laws of Motion). The equation of motion for quarter car model, free body diagram of two mass considered and the dynamics equation of the quarter car model, captured in the differential equation, (1) and (2) given below.

$$M_s \ddot{Z}_s = F_c + B_s \dot{Z}_{us} - B_s \dot{Z}_s - (Z_s - Z_{us})K_s \tag{1}$$

$$M_{us} \ddot{Z}_{us} = -F_c - B_s \dot{Z}_{us} - B_{us} \dot{Z}_{us} + B_s \dot{Z}_s + B_{us} \dot{Z}_r \\ -(Z_{us} - Z_r)K_{us} \tag{2}$$

B. State-space Modelling

MPC controller often uses the state-space representation of the plant due to the ease of handling Multi-Input Multi-Output (MIMO) systems and availability of control theorems for state-space formulation. A Quarter car model has four energy storage element viz. the sprung mass, unsprung mass, springs representing the stiffness of the tire and the suspension. The energy storage element represents four state variables are required to capture the dynamic of Quarter car active suspension.

The combination of states representing system is not unique, but with suitable transformation can be converted from one combination to another. Often state variables are chosen such that they reflect the parameters to be optimized, which makes the formulation of optimization problem much more accessible. Here, the vector representing the states of quarter car active suspension is given by

$$x = \begin{bmatrix} Z_s - Z_{us} \\ \dot{Z}_s \\ Z_{us} - Z_r \\ \dot{Z}_{us} \end{bmatrix} \tag{3}$$

Where the four elements of state vector x represent the suspension deflection, sprung mass velocity, tire deflection, unsprung mass velocity in order.

The Quarter car active suspension model in state-space representation can be described by a mathematical equation in terms of the state-space form, given by

$$\dot{x} = Ax + Bu \\ y = Cx + Du \tag{4}$$

Where $x \in \mathbb{R}^{n \times 1}, y \in \mathbb{R}^{p \times 1}, u \in \mathbb{R}^{m \times 1}$ Are the state vector, output vector, and input vector respectively. The output of the state variable of the system measured, which means y equals x. the input vector denoted by equation

$$u = \begin{bmatrix} \dot{z}_r \\ F_C \end{bmatrix} \tag{5}$$

Where Fc is the force exerted by the actuator. $\dot{z}_r$ is the vertical velocity of the tire The system matrix (A), input matrix(B), output matrix(C) and feed through the matrix (D) for active suspension system are described by (6) to (9)

$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ \frac{-K_s}{M_s} & \frac{-B_s}{M_s} & 0 & \frac{B_s}{M_s} \\ 0 & 0 & 0 & 1 \\ \frac{K_s}{M_{us}} & \frac{B_s}{M_{us}} & \frac{-K_{us}}{M_{us}} & \frac{-(B_s + B_{us})}{M_{us}} \end{bmatrix} \tag{6}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{M_s} \\ -1 & 0 \\ \frac{B_{us}}{M_{us}} & \frac{-1}{M_{us}} \end{bmatrix} \tag{7}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{9}$$

The C and D matrix varies with the combination of outputs used for control.

III. METHODOLOGY

A. Linear Quadratic Regulator

LQR is an optimal control scheme which aims to maintain the states of the system at zero. The cost function has penalized if any deviation of states from zero references by the square of two norms of states. The cost function minimized by an LQR control scheme is given by (10).

$$J = \int_0^\infty (x^T Q x + u^T R u) \ dx \tag{10}$$

The weighting matrix Q determine the relative importance of each state. The scaling factor is taken into consideration while selecting the Q matrix. The control inputs also penalized for two reasons. Firstly, it is logical to impose a penalty on control input since it cannot have an arbitrarily high value as there are always limits on the control input. For the higher cost of the R, the matrix reduced the magnitude. Secondly, to ensure that the solution to optimization exists, the hessian must be positive definite. Weighting matrix R must be positive definite. That means t penalized all the control inputs.

The solution of the LQR control problem is in the form (10)

$$u = -kx$$
(11)

For a finite time LQR control problem, the solution obtained by solving the differential Riccati equation turns out to be a time-varying feedback control law. When the problem extended to infinite time, the feedback law converges to time-invariant one. The feedback control law obtained by solving the algebraic Riccati equation for infinite time LQR problem described by (10) and the LQR feedback gain is given by (13)

$$A^T P + PA - PBR^{-1}B^T P + Q = 0$$
(12)

$$K = R^{-1}B^T P$$
(13)

B. Model Predictive Control

Model Predictive Control is an advanced optimal control strategy. Initially use of MPC was limited to slow process in industries like petrochemical, paper, and pulp, etc. due computation burden of solving the optimization problem within one sampling instant. Recently with availability high-speed processor and memory, fast optimization algorithms MPC is applied to processes with fast dynamics. Unlike classical controllers like PID, Model Predictive controllers have the handle MIMO system and manage the constraints on outputs and inputs in a systematic manner[1].

MPC is not a single control algorithm; instead, it can be treated as a group control algorithms with a specific
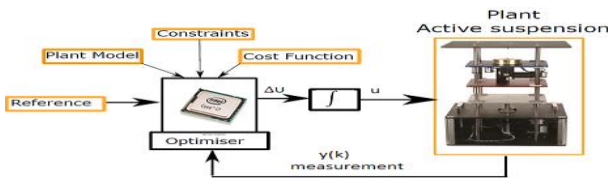

Fig.2. MPC block diagram

standard feature as listed below Fig. 2 Block diagram of MPC Control Use a model of the plant for predicting outputs Receding horizon strategy Optimization of cost function supporting constraints also Fig.2. Shows that the block diagram of MPC control. It consists of a model of plant for predicting how the system evolves in time when subjected to the control input sequence. The cost function is scalar quantity reflecting performance requirement, which is minimized by proper selection of control input. In the active suspension experiment, a quadratic cost function used with linear constraints on outputs and inputs. The current state of the system is measured or estimated are used as initial states for prediction in the future, this action introduces the feedback mechanism in an indirect way the general mathematical formulation linear MPC problem is given by (14)

$$J = \sum_{k=0}^{N_p} f(x_k, \Delta u_k)$$
(14)

$$subject\ to$$

$x_{k=0} = x(t)$ (measured or estimated value)
$x(k + 1) = Ax(k) + Bu(k)$
$y(k) \ \epsilon \ Y$ (constraints on outputs)
$\Delta u(k) \ \epsilon \ \Delta \mathbb{u}$ (constraints on rate of control input)
$u(k) \ \epsilon \ \mathbb{u}$ (constraints on outputs)

MPC uses the plant model to predict the outputs of the system for N_p sampling instant in the future known as prediction horizon, and the input calculated for instantaneous and the number of sampling instant in future N_C. The period for which the control input calculated is known as a control horizon (N_C). At each instant prediction horizon and control, the horizon remains the same. MPC creates an impression that the prediction and control horizon is receding in time [2][10]. Fig.3 depicts the receding horizon strategy. And optimal input is calculated such that the evolution of outputs in future comply with performance requirements and does not violate the constraints. Even though optimal control computed the sequence for the entire control horizon at each instant, the only the first component in the sequence is applied to the plant and discarded the rest. At the next sampling instant again, the optimization problem is solved with new initial states and the cycle. The flow chart in Fig. 4 enumerated the algorithm for a linear MPC.
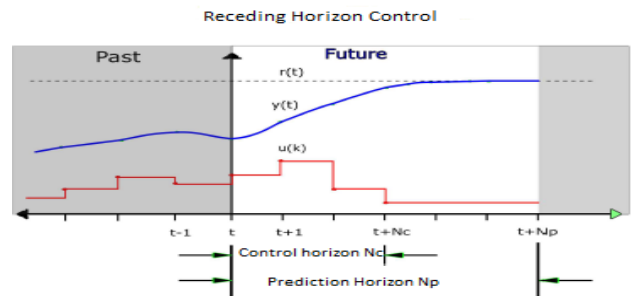

Fig. 3. Receding horizon strategy

Here the MPC algorithm is designed such that it calculates increment of the control signal, which ensures offset free tracking output and makes it easy to impose constraints rate of control. Firstly, control input and previous input expressed in terms of increment of the control signal. The state-space model is reformulated to include embedded an integrator so that the new plant model accepts the increase of control signal as input [2]. The new plant model is given by (13)
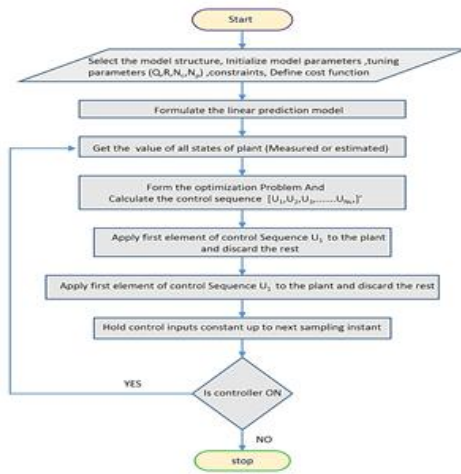
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$$\begin{bmatrix} u_0 \\ u_1 \\ \vdots \\ u_{Nc} \end{bmatrix} = \begin{bmatrix} I \\ I \\ \vdots \\ I \end{bmatrix} u(k-1) +$$
$$\underbrace{\begin{bmatrix} u_0 \\ u_1 \\ \vdots \\ u_{Nc} \end{bmatrix}}_{U} = \underbrace{\begin{bmatrix} I \\ I \\ \vdots \\ I \end{bmatrix}}_{S_1}$$

$$\underbrace{\begin{bmatrix} I_{m\times m} & 0 & 0 & \dots & 0 \\ I & I & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ I & I & I & I & I \end{bmatrix}}_{S_2} \underbrace{\begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+N_c-1) \end{bmatrix}}_{\Delta U} \tag{19}$$

Thus finally, all the constraints are translated to linear constraints on the decision variable ΔU and is represented in a compact form given by (20).

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \Delta U = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} \tag{20}$$

where

$$\left. \begin{array}{l} Z_1 = \begin{bmatrix} -I \\ I \end{bmatrix} \\ g_1 = \begin{bmatrix} -\Delta U_{min} \\ \Delta U_{max} \end{bmatrix} \end{array} \right\} \text{Constraints on the rate of input}$$

$$\left. \begin{array}{l} Z_2 = \begin{bmatrix} -S_2 \\ S_2 \end{bmatrix} \\ g_2 = \begin{bmatrix} S_1 u(k-1) - U_{min} \\ -S_1 u(k-1) + U_{max} \end{bmatrix} \end{array} \right\} \text{Constraint on input}$$

$$\left. \begin{array}{l} Z_3 = \begin{bmatrix} -\Omega \\ \Omega \end{bmatrix} \\ g_3 = \begin{bmatrix} \Theta x(k) - Y_{min} \\ -\Theta x(k) + Y_{max} \end{bmatrix} \end{array} \right\} \text{Constraint on outputs}$$

Fig. 4.Flow Chart of Linear MPC

$$\begin{bmatrix} x(k+1) \\ u(k+1) \end{bmatrix} = \begin{bmatrix} A & B \\ 0_{p\times n} & I_{p\times p} \end{bmatrix} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} + \begin{bmatrix} B \\ I_{p\times p} \end{bmatrix} \Delta u \tag{15}$$

In model predictive control scheme, the future input sequence generated as such that predicted output meets the desired performance requirement and constraints, are not violated the model predictive control scheme for active suspension minimizes a quadratic objective function defined by (16)

$$J = y_0^T Q_0 y_0 + (\text{Ref} - Y_P)^T Q(\text{Ref} - Y_P) + \Delta U^T R \Delta U \tag{16}$$

Where Y_P is a vector representing of all outputs in prediction horizon and U is the vector of control inputs to be applied in control horizon. Ref represents the output reference at each sampling instant within the prediction horizon, and for the active suspension system, all outputs (states) are required to maintained at zero.

The predictions of outputs also expressed in terms as a function of the increment of control inputs as described by (17), which then substituted to the objective function and constraints.

$$Y_p = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^{N_p} \end{bmatrix} x_0 + \begin{bmatrix} CB & 0 & 0 & \dots & 0 \\ CAB & CB & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & 0 \\ CA^{N_P-1}B & CA^{N_P-2}B & \dots & \dots & CA^{N_P-N_C}B \end{bmatrix} \Delta U$$

$$Y_p = \Theta x(k) + \Omega \Delta U(k) \tag{17}$$

In the newly formed optimization problem, the decision variable is a vector of the control signal to be applied in the control horizon. As written, the objective function in (18).

$$\begin{aligned} J(\Delta U, x_0) = & (REF - \Theta x(k)^T \bar{Q}(REF - \Theta x(k) \\ & - 2\Delta U^T(k)\Omega^T(REF - \Theta x(k) \\ & + 2\Delta U^T(k)\Omega^T(REF \\ & - \Theta x(k)\Omega\Delta U) \end{aligned} \tag{18}$$

Imposed the constraints on outputs and states throughout the prediction horizon, and limitations on inputs and control increment rate are imposed throughout the control horizon Similar to the objective function the restriction also parameterized in terms of the decision variable ΔU [11]. parameterized the control sequence in the control horizon as described by (19)

IV. HARDWARE

The effectiveness of LQR and MPC controls scheme for Active suspension tested on a bench-scale model of Quarter Active suspension. Fig.5 shows the block diagram of the hardware setup of the Active suspension experiment

Three high-resolution Quadrature optical encoder is used to measure suspension deflection, the road profile (motion of silver plate) and the movement of the sprung mass. The Data Acquisition (DAQ) unit communicate the encoder measurements and control signals to actuator via amplifier from target and host running in PC.The top blue plate and golden mass above it represents the vehicle body and passenger. The middle red plate represents the unsprung mass, which includes the weight of the tire. The two springs and two dampers placed diagonally represent the suspension of the car. A capstan drive using a dc motor is employed as an actuator to inject independent force between the unsprung mass and sprung mass — the bottom silver plate connected to a fast dc motor response to emulate the different road conditions.

V. EXPERIMENTAL RESULTS

The Performance of MPC and LQR control scheme for quarter car active suspension bench-scale carefully examined for various road profile. The quality of ride depends on acceleration and frequency of vibration to which the passenger is exposed.

Fig.5 . Block diagram of hardware setup

Practical control of the active suspension model of the quarter car, the active suspension is subjected to bump road profile of magnitude .02 and period one cycle from Fig.6. The open-loop mode shows, the plate has more vibration and overshot when subjected to the bump also it takes too much time for the waves to settle. Means that passenger subjected to more vibration, which is not desirable. Fig.7. Shows that the plate position of the active suspension with LQR controller. For the active suspension LQR controller, for both the sprung mass and unsprung mass the vibration and overshoot reduced considerably. With MPC Control scheme, the oscillation and overshoot reduce drastically in sprung mass and un\sprung caused by road profile. Fig.8.Shows that the plate displacement of the active suspension model with MPC control scheme.
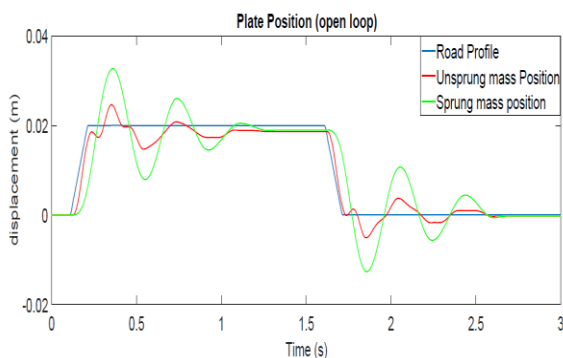

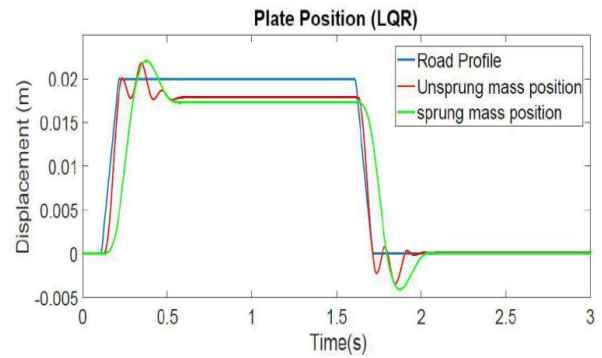**Fig. 6. .The open-loop unsprung and sprung mass movement**

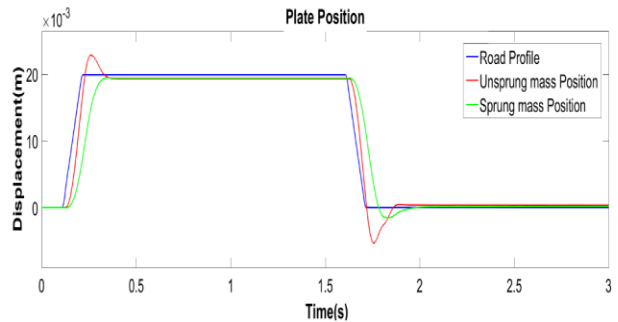**Fig. 7. Movement of unsprung and sprung mass (LQR)**

Fig.8. Movement of Unsprung and sprung mass

To demonstrate the constraint handling capabilities of the MPC control Scheme. The maximum absolute value of the control force is explicitly limited to 8 Newton even though the hardware is capable of developing ±38 newton. Fig.9. Show actuation force designed with LQR and MPC control scheme. For LQR control the actuation force goes slightly below -8 Newton and slightly below -8 Newton. With MPC control, we can limit the actuation force is ± 8 Newton.Then that the energy of the actuation signal with MPC compared with the LQR control, which attributed to better control of plate positions.
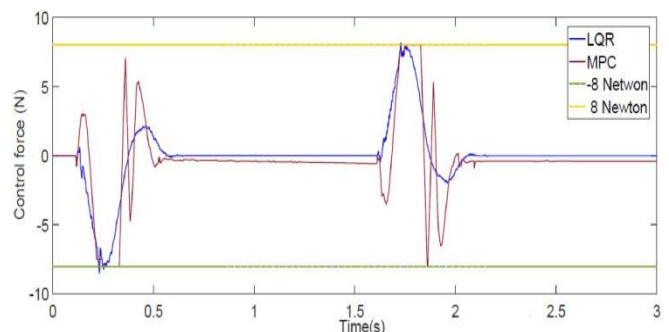

Fig.9.Actuation Force for MPC and LQR Control Scheme

Next plate position for random road profile examined.Fig.10. Shows movement of the sprung mass and unsprung mass position for random road profile. For random road profile, MPC control scheme can eliminate overshoot and oscillation in sprung mass, and unsprung is firmly following road profile leading to improved road handling.

VI. CONCLUSION

Fig.10.Unsprung and sprung mass motion for random road profile (MPC)

This work mainly focuses on developing a linear model predictive control and LQR control scheme for the active suspension. A Quarter model of Active suspension designed for simulation purpose and to use as an internal model of MPC control. The effectiveness of LQR and MPC controls scheme for Active suspension studied by analyzing the movement of sprung and unsprung masses, actuator force and vertical acceleration of sprung mass for various road profile. Active suspension with MPC is more resilient to irregularities of the road, the oscillation, and overshoot of sprung mass and unsprung masses associated with vibration drastically reduced. The performance improvement MPC compared to that of an LQR when there is an active constraint — the steady-state offset in position of sprung attributed to the nonlinearities present in the hardware setup. It can summarize from above discussion predictive control scheme for active suspension Model predictive control scheme is far superior to LQR in all aspects.

REFERENCES

1. Manfred Morari, Jay H. Lee "Model predictive control: past, present, and future," Computers and Chemical Engineering 23 (1999) 667–682
2. Kenneth R. and James B. Rawlings "Model predictive control with linear models."AIChE Journal, February 1993 Vol. 39, No. 2
3. Mayne, D.Q., Michalska, H.," Robust Receding Horizon Control of Constrained Nonlinear Systems," IEEE Transactions on Automatic Control, 38, pp 1623-1633, (1993).
4. Bart L.J. Gysen, Johannes J. H. Paulides, Jeroen L. G. Janssen, and Elena A.Lomonova, "Active Electromagnetic Suspension System for Improved Vehicle Dynamics," IEEE transactions on vehicular technology, vol. 59, no. 3 March 2010.
5. The ride comfort vs. handling compromise for off-road vehicles P.S. Els, N.J. Theron, P.E. Uys, M.J. Thoresson Department of Mechanical and Aeronautical Engineering, University of Pretoria, Pretoria 0002, South Africa. Received 10 July 2006; received in revised form 29 March 2007; accepted 2 May 2007, Available online 27 June 2007.
6. Garcia, C.E. Prett, D.M., Morari, M., Model Predictive Control Theory and Practice ñ A Survey, *Automatica*, 25, 3, pp. 335-348 (1989).
7. Alberto Bemporad, Panagiotis Patrinos 'Simple and Certifiable Quadratic Programming Algorithms for Embedded Linear Model Predictive Control' 4th IFAC Nonlinear Model Predictive Control Conference International Federation of Automatic Control Noordwijkerhout, NL. August 23-27, 2012
8. Dishant, Er.Parminder Singh, Er.Mohit Sharma 'Suspension Systems: A Review' International Research Journal of Engineering and Technology (IRJET), Page 1377, Volume: 04 Issue: 04 | Apr -2017
9. Wissam H. Al-Mutar Turki Y. Abdalla 'Quarter Car Active Suspension System Control Using PID Controller tuned by PSO'Iraq J. Electronic and Electrical engineering vol.11 No.2,2015
10. A. Bemporad and M. Morari, "Robust model predictive control: A survey," Robustness in identification and control, pp. 207–226, 1999.
11. L. Wang, Model Predictive Control System Design and Implementation Using MATLAB, ser. Advances in Industrial Control. Springer London, 2009.

AUTHORS PROFILE

Rashmi Ranjan Das, Assistant Professor(SG), School Electrical Engineering, Vellore Institute of Technology, Vellore-14.

Dr.Vinodh Kumar E, Associate Professor, School of Electrical Engineering, Vellore Institute of Technology, Vellore-14.