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Mathematical Analysis of Queueing-Inventory Model with Compliment and Multiple Working Vacations

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Abstract: ***In this paper, we provide independent continuous review of stock for two commodities, namely commodity 1(C1) and 2 (C2). The type 1(T1) customer demands C1 and Type 2(T2) customer demands C2 but C2 is also given to T1 customer as a compliment (i.e. C2 is also given as a compliment to C1). The arrival pattern of customers is from a finite source of population. The ordering policies of both commodities are independent and also, each customer demands service with positive time. A single finite retrial orbit is allowed for any customer, when the demanding item is stocked out or the server is busy. The pre-emptive priority service policy is assigned to T1 customer over T2 customer. The server may go for the working vacation as C1 becomes zero, in which T2 customer who is taking his service is allowed to orbit whenever T1 customer demands the service with both commodities and when the orbit is not full. The joint limiting distributions of four random variables are studied under steady state. Long run expected total cost rate and measures of system performance are derived.***

Keywords: Complement items, Finite population, Retrial orbit, Working vacations.

I. INTRODUCTION

A company / organization can show a positive turnover by changing their marketing philosophy. In order to attract new customers and to enhance the sales of new products, new marketing techniques are practiced. One of the new techniques is providing basic service with additional free service to customers. In order to sell other related substitutable products, at the time of customer purchasing the product, some auxiliary services are provided. An organization can gain the trust of its customers by providing the demanded features to the customer's expectations and by persuading them to by the product by showcasing its additional features. Some organization in place of providing additional service to the customers, they provide them with offers [compliments] to deflect the attraction of the buyer, thereby saving time and creating a demand for the product. So far only few papers published in the literature of complement inventory model. For detail see [1,2].

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II. MODEL DESCRIPTION

The system consists of two types of customers, two types of commodities and a single server. The customers are classified as Type 1(T1) customer and Type 2(T2) customer. The commodities are termed as commodity 1(C1) and commodity 2(C2). The T1 customer demands C1 and the T2 customer demands C2. On demand of an unit of item, the T1 customer is given C2 as a compliment whereas T2 customers not given any compliment. The service time of each customer is positive, where both the demands come from a single finite population of size N, which follows independent quasi-random distribution. The ordering policy of each commodity is considered independently. Let S_1 and S_2 be maximum inventory level of C1 and C2 respectively. At that point of time, the inventory level of C1 drops to s_1 and to place the order of quantity Q_1, $Q_1=S_1-s_1$. The replenishment time of an order for C1 is exponential at rate, but the replenishment of C2 is instantaneous ((0, S_2) ordering policy). Let λ_1 and λ_2 be the arrival rates of T1 customer and T2 customer respectively. If the inventory level of C1 is zero, the server changes to working vacation idle state(WVIS) from regular period(RP) with probability q, otherwise he stays in idle state in regular period with probability p, p + q =1. In a regular period, the service time of T1 customer and T2 customer are independent exponential with rates μ_1 and μ_2 respectively. Similarly, in a working vacation, the service times of T1 customer and T2 customer are independent exponential with rates μ_3 and μ_4 respectively. The completion time of working vacation is exponential at rate η. Whenever the server is busy, the arriving of any type of customer goes to finite retrial orbit of size N. The demand retrial rates of T1 customer and T2 customer in the orbit are given by $p_1\theta$ and $p_2\theta$ respectively, where $p_{1+}\ p_{2=1}$. The pre-emptive priority service policy is considered in this system at which T2 customer taking his service is sent to orbit whenever T1 customer demands the service with both commodities available. After completion of working vacation, the server may take another vacation if there is no positive inventory level of C1.

III. ANALYSIS OF THE MODEL

Let $u_t \in \{0,1,2,\ldots,S_1\}$, $v_t \in \{0,1,2,\ldots,S_2\}$ respectively, be the level of first and second commodity, $w_t \in \{0,1,2,3,4,5\}$ be the current situation of the server, w_t =0 denotes server is idle (SI) in the regular period (RP), w_t =1 denotes server is busy(SB) with T1 customer in the RP, w_t =2 denotes SB with T2 customer in the RP, w_t =3 denotes SI in working vacation period(WVP), w_t =4 denotes SB with T1 customer in the WVP, w_t =5 denotes SB with T1 customer in the WVP and $x_t \in \{0,1,2,\ldots,N\}$ is the level of orbit at time t. Then L(t) is a continuous time Markov chain (CTMC) with state space(SS)

given by $E=b_1 \cup b_2 \cup b_3$, where

$b_1=\{0,1,2,\ldots,S_1,\ 1,2,\ldots,S_2,\ 0,3,\ 1,2,\ldots,N\}$

$b_2=\{1,2,\ldots,S_1,\ 1,2,\ldots,S_2,\ 1,4,\ 0,1,2,\ldots,N-1\}$

$b_3=\{0,1,2,\ldots,S_1,\ 1,2,\ldots,S_2,\ 2,5,\ 0,1,2,\ldots,N-1\}$

The generator matrix of CMCT $\{L(t), t\geq 0\}$ is

$$[\Phi]_{l_1 m_1} = \begin{cases} C_0, & a' = Q, \quad a = 0 \\ C_1, & a' = Q + a, \quad a = 1,2,..,S \\ F_a, & a' = a - 1, \quad a = 1,2,..,S \\ E_a, & a' = a \quad a = 0,1,2,..,S \\ 0, \textit{otherwise}. \end{cases}$$

Dimensions of C_0 and F_0 are $2S_2(2N + 1) \times 2S_2(3N + 1)$ and $2S_2(3N + 1) \times 2S_2(2N + 1)$ respectively. E_0 has dimension $2S_2(2N + 1)$. Other matrices are have size $2S_2(3N + 1)$. Elements of the above matrices can obtained easily, for details see [3]. From the construction of Φ, it is trivially say that L(t) on the finite SS E is Regular.

Then the steady state distribution (SSD) Π satisfies

$\Pi\ \Phi=0$ and Π e=1. (1)

Also (1) yields the following equations:

$\Pi^{(a)}E_0+\Pi^{(a+1)}F_0=0,\ a=0,$

$\Pi^{(a)}E_1+\Pi^{(a+1)}F_1=0,\ a=1,2,\ldots,s,$

$\Pi^{(a)}E_2+\Pi^{(a+1)}F_1=0,\ a=s+1,\ldots,Q-1,$

$\Pi^{(0)}C_0+\Pi^{(a)}E_2+\Pi^{(m_1+1)}F_1=0,\ a=Q,$ (*)

$\Pi^{(a-Q)}C_1+\Pi^{(a)}E_2+\Pi^{(a+1)}F_1=0,\ a=Q+1,\ldots,S-1,$

$\Pi^{(a-Q)}C_1+\Pi^{(a)}E_2=0,\ a=S,$

After solving these equations recursively, (except (*)), we get $\Pi^{(i_1)}$. For more details see [3].

IV. SYSTEM PERFORMANCE MEASURES

The following performance measures are useful for calculate the total cost (TC)

(i) Average inventory level for commodity-I η_{I1} and commodity-II η_{I2}:

$$\eta_{I1} = \sum_{a=1}^{S_1}\sum_{b=1}^{S_2}\sum_{d=0}^{N} a\ [\phi^{(a,b,0,d)}+\ \phi^{(a,b,3,d)}]+\sum_{a=1}^{S_1}\sum_{b=1}^{S_2}\sum_{c=1}^{2}\sum_{d=0}^{N-1} a\ \phi^{(a,b,c,d)}+\sum_{a=1}^{S_1}\sum_{b=1}^{S_2}\sum_{c=4}^{5}\sum_{d=0}^{N-1} a\ \phi^{(a,b,c,d)}$$

Similarly we can obtain η_{I2} .

(ii) Average reorder rate for commodity-I η_{R1} and commodity-II η_{R2}:

$$\eta_{R2} = \sum_{a=1}^{S_1}\sum_{d=0}^{N-1} \mu_1\ \phi^{(a,1,1,d)}+\sum_{a=1}^{S_1}\sum_{d=0}^{N-1} \mu_3\ \phi^{(a,1,4,d)}+\sum_{a=0}^{S_1}\sum_{d=0}^{N-1} \mu_2\ \phi^{(a,1,2,d)}+\sum_{a=0}^{S_1}\sum_{d=0}^{N-1} \mu_4\ \phi^{(a,1,5,d)}$$

Similarly we can obtain η_{R1}.

(iii) Average number of customers in the orbit:

$$\eta_{O1} = \sum_{a=0}^{S_1}\sum_{b=1}^{S_2}\sum_{d=1}^{N} d\ [\phi^{(a,b,0,d)}+\ \phi^{(a,b,3,d)}]+\sum_{a=1}^{S_1}\sum_{b=1}^{S_2}\sum_{d=1}^{N-1} d\ [\phi^{(a,b,1,d)}+\ \phi^{(a,b,2,d)}]+\sum_{a=0}^{S_1}\sum_{b=1}^{S_2}\sum_{d=1}^{N-1} d\ [\phi^{(a,b,2,d)}+\phi^{(a,b,5,d)}]\ .$$

(iv) Total Cost:

$$TC\ (S_1, S2) = c_{a1}\eta I_1+ c_{a2}\eta I_2+ c_{r1}\eta R_1+ c_{r2}\eta R_2+ c_o\eta o\ .$$

Where

- c_{a1} = Holding cost for I-commodity.
- c_{a2} = Holding cost for II-commodity.
- c_{r1} = Setup cost for I-commodity.
- c_{r2} = Setup cost for II-commodity.
- c_o = Waiting cost of an orbiting demand.

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