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Wavelet Solution for Biosensor Response at Mixed Enzyme Kinetics

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Abstract*: *This paper presents the approximate solution of the reaction diffusion equation based on the hybridization of classical polynomials and Legendre wavelets. The systems of equations are generated first for the differential equations by the properties of Legendre wavelets. Theoretical analysis for the proposed scheme is discussed and computed solutions are also compared with other numerical solutions available in the literature.

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Index Terms*:*Legendre wavelets; Convergence analysis; Reaction-diffusion system.

I. INTRODUCTION

In recent days, differential equations are used to analyze the behaviour of Semiconductors gas sensors and biosensors. One such equation is the steady-state solution of the substrate reaction-diffusion process given by

$$\frac{\partial^2 u}{\partial x^2} - \frac{Ku}{1+\alpha u+\beta u^2} = 0 \qquad (1)$$

Subject to the boundary conditions u(1)=1 and $\frac{\partial u(0)}{\partial x} = 0$.

The gas sensors [1-7] and biosensors [8-16] models have been studied by differential equations in the literature.

To improve the sustainability of our society, we use the gas sensors to detect, monitor and control the presence of dangerous gases in the atmosphere. Many metal oxide semiconductor-based gas sensors are used to detect the different kinds of gas molecules namely H_2, CO,NH_3, H_2S, SO_2, odours,other applications are reported in the field such as the detection of explosive gases, fire detection, and air quality[1-7]. This semiconductor-based gas sensor consists of sintered block, thick film and thin film. The manufacturing and production of this gas sensor considered towards those higher sensitivities, quicker selectivity and response. They also concentrate on easier fabrication and portability.

The analytical devices made up of biological entity. Usually enzyme are termed as biosensors[8-16]. These sensors recognize a particular analyte and translates the changes in the bio-molecules into an electric signal. Due to high sensitivity, simplicity and low cast these sensors are widely used in many industries. The mathematical models have discussed the analytical behavior of bio sensors. In particular, authors [16] studied the enzymatic reaction in the enzyme membrane and mass transport from both sides, which is used to study the biochemical behavior in dimensionlessform. This model is categorized as a nonlinear reaction-diffusion equations in which the nonlinear term related to non-Michaelis-Menton kinetics of the enzymatic reaction, its steady-state version is given in (1), and the authors have studied the analytical and numerical solution for (1) by using the variational iteration method(VIM) and Homotopy perturbation method (HPM) rigorously.

They have also compared the simulation results with average percentage deviation of 0.516 and 0.4676 for α =10 and K=0.1 for VIM and HAM. The error percentage (0.5656,1.8451) and (7.1357,1.5644) subject to(α=1,K=1) and (α =0.1,K=5) for VIM and HAM respectively. To reduce this error percentage and accuracy of concentration, we try Legendre wavelet-based technique to solve (1) in this paper.

Generally solving a nonlinear equation is a tedious task. To solve these kinds of equations, we have approximate, numerical and semi-analytical techniques. At the same time, each method has its own merits and demerits based on their applicability. Not all nonlinear problems have been solved by a single approach. These methods have deviated on their nature of the nonlinearity, initial and boundary conditions. Symmetries and transformations methods are also used to find the exact solutions for the same without considering their initial conditions. So the researchers keep on trying the new methods to overcome the limitations stated in the nonlinear differential equations Many researchers have investigated the substrate reaction-diffusion process through differential equations in nonlinear form, and they have also discussed the various methods to solve, and it can be found in [1-16]. The dynamic behaviour of such a process is encountered in various physical and chemical phenomena and very helpful in modeling certain parameters like reaction-diffusion equations. Chemical kinetics, Populationdynamics, neurophysiology are the areas where this equation plays a pivotal role.In this paper, we solve this partial differential equation through Legendre wavelet-based hybrid method. Wavelet-based methods are usually applied in image processing, restoration, compression etc. The orthogonal properties of the various wavelets are effectively used to solve the differential equations in nonlinear nature.The remaining sections organized as In section 2,

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we describe the Legendre wavelets and function approximation. Section 3 is devoted to the proposed approach for solving Eq.
(1). In Section 4, theconvergence and error analysis of the same is given. Numerical examples and concluding remarks are presented in section 5 and 6.

II. WAVELETS AND FUNCTION APPROXIMATION

Recently, Wavelets, their various types and applications of wavelets are studied by different researches for solving both ordinary differential equations and partial differential equations in the non-linear sense[17-21].
Wavelets are basically derived from a class of polynomials with the shifting and scaling parameters. The polynomials are continuously differentiable in certain domain of the real line. Any function can be expressed as the linear combination of orthogonal polynomials where the coefficients easily identified by orthogonal property.
For achieving the solution of the equation numerically, we discretize the domain first and convert the given polynomials in terms of wavelets. The researchers currently are developing a different type of wavelets, derived from classical and orthogonal polynomials. The classical polynomial based wavelets are applicable for solving different type of linear problems where the properties of classical polynomials fulfil the properties of wavelets also. The orthogonal polynomial based wavelets are effectively applied to create sparse matrices in solving both linear and nonlinear equations.
The main advantage of this wavelet-based methods is the non-linear differential equations solved by the respective system of algebraic equations. The process of such transformation has done by the wavelet-based procedure. Here we consider Legendre wavelet to study and analyze the solution of (1). The Legendre wavelets [17, 18] defined on the interval [0,1) as

$$\psi_{nm}(t)=\begin{cases}\sqrt{m+\frac{1}{2}}\,2^{\frac{k}{2}}P_m\left(2^k t-\hat{n}\right), & for\ \frac{\hat{n}-1}{2^k}\le t\le\frac{\hat{n}+1}{2^k},\\ 0 & ,\ otherwise\end{cases} \quad (2)$$

where m = 0,1,2, … , M-1, n = 1,2,3, … , 2^{k-1}.

Infinite series expansion of function u(x) defined over [0,1) is expanded as

$$u(x)=\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}c_{nm}\psi_{nm}(x) \quad (3)$$

where $c_{nm}=(2m-1)\int_0^1\int_0^1 u(x)\psi_{nm}(x)\,dx$.

In practice, we consider a finite number of terms to approximate the same and then it can be assumed as

$$u_{nm}(x)\cong\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1}c_{nm}\psi_{nm}(x)\ =C^T\Psi(x) \quad (4)$$

where C and $\psi(x)$ are 2^{k-1}M X 1matrices of unknown and known.
In order to prove the accuracy of this approximate solution, we need to study the theoretical analysis. The accuracy of the solution depends on the number of terms in the truncated series. If we increase the number of the term, then the error should tend to zero. For this purpose,we discuss the mathematical analysis and also study the applicability of this method in the following sections.

III. THE SOLUTION OF THE BOUNDARY VALUE PROBLEM USING LEGENDRE WAVELETS METHOD

Consider the Eq. (1)

$$\frac{\partial^2 u}{\partial x^2}-\frac{Ku}{1+\alpha u+\beta u^2}=0$$

$$\int_0^x u_{xx}\,dx+\alpha\int_0^x uu_{xx}dx+\beta\int_0^x u^2u_{xx}dx-\int_0^x Ku\,dx=0.$$

$$\int_0^x\int_0^x u_{xx}\,dxdx+\alpha\int_0^x\int_0^x uu_{xx}dxdx$$
$$+\beta\int_0^x\int_0^x u^2u_{xx}dxdx-\int_0^x\int_0^x Ku\,dxdx=0$$

By using (4), we have

$$\int_0^x\int_0^x C^T\Psi''(x)dxdx+\alpha\int_0^x\int_0^x C^T\Psi(x)C^T\Psi''(x)dxdx$$
$$+\beta\int_0^x\int_0^x (C^T\Psi(x)^2)C^T\Psi''(x)dxdx-\int_0^x\int_0^x KC^T\Psi(x)dxdx=0$$

We now collocate the above equation at 2^{k-1}M-2 points at x_i as

$$\int_0^x\int_0^x C^T\Psi''(x_i)dxdx+\alpha\int_0^x\int_0^x C^T\Psi(x)C^T\Psi''(x_i)dxdx$$
$$+\beta\int_0^x\int_0^x (C^T\Psi(x_i)^2)C^T\Psi''(x_i)dxdx-\int_0^x\int_0^x KC^T\Psi(x_i)dxdx=0$$

The system of equations in 2^{k-1}M unknowns are constructed by assigning the values to k and M.

IV. CONVERGENCE ANALYSIS

In this section, we discuss the theoretical background for the function approximation defined in the previous section through the process of convergence and error estimation. We prove the idea of convergence by restricting the given connection coefficients by some constants, and thereby, it converges to the original solution uniformly. For uniform convergence, the δ value depends only on ε but not on *x*. For error estimate, we go with the difference of the original approximation with the truncated series in the approximation. Hence the estimate or the bound is attained for those terms that remain after truncation. The bound value that is obtained is validated through some examples, and the error value sticks to the upper bound. This error bound expression falls to zero uniformly when the value of M gets increased.

Theorem 1 (Convergence Theorem)

The function $u_{nm}(x)$ converges towards the exact solution $u(x)$ where $u(x)$ is continuous and bounded in the second derivative.

Proof:

$$\left|\sum_{n=1}^{\infty}\sum_{m=0}^{\infty} c_{nm}\psi_{nm}(x_i)\right| \leq \sum_{n=1}^{\infty}\sum_{m=0}^{\infty}|c_{nm}|\,|\psi_{nm}(x_i)|$$

$$\leq \sum_{n=1}^{\infty}\sum_{m=0}^{\infty}|c_{nm}|$$

We can compute the coefficients c_{nm} through $2^{k-1}M$ system of equations.

Theorem 2 (Error Analysis)

The error between exact and approximate solution is given by

$$\varepsilon_{mn} \leq 2^{\frac{-(k+1)}{2}}\lambda\sqrt{6}\,2^{k}\sum_{i=2^k}^{\infty}\sum_{j=M}^{\infty}\frac{1}{\sqrt{2j+1}}\frac{1}{\sqrt{2^k(j-2)+2i-3}}$$ where $\left|\frac{\partial^2 u}{\partial x^2}\right| \leq \lambda$ on [0,1)) and $\varepsilon_{mn} = \left\|\left[u(x)-u_{nm}(x)\right]\right\|_{L^2}$

.Numerical Application

We apply the Legendre wavelets method to the steady-state solution of Eq.(1) for k=2 and M=7 is given by $\frac{1}{cosh\sqrt{K}}\left(1+\frac{Kx^2}{2!}+\frac{(Kx^2)^2}{4!}+\frac{(Kx^2)^3}{6!}\right)$and for larger values of K, the solution converges to $u(x) = \frac{cosh(\sqrt{K}x)}{cosh(\sqrt{K})}$ coincides with the solution given in [16].

The values of u(x) are reported in Tables 1, and 2 for various values α, $\beta$and K. Table 1 exhibits the values of u(x) for K= 0.1;1;5 at α= β=0 and it also confirms that the LWM solution converges to exact solution when x approaches 1. The case is shown in Table 2 as the values of u(x), 0<x<1 at different values α, β and K. Comparison of LWM solution with other methods VIM and HPM has been reported in the same Table. From Both Tables 1 and 2, we conclude that this LWM solution coincides with exact and accurate in less computational effort than the other methods reported in the literature. Figures 1(a) to 1(d) are also confirmed that this LWM converges to the exact solution as x tends to 1. The analytical conclusion about this model is the concentration decreases when K increases for all values of $\alpha$and β. The values of u(x) are almost equal to 1 for K≤ 1. The simulation results [16] of this biosensor study coincide with LWM solution with zero percentage error so that the simulation results have not been shown explicitly in the table.

Table 1: Steady-state concentration comparison when α=0 and β=0.

	$K=0.1$			$K=1$			$K=5$		
x	VIM	HPM	LWM	VIM	HPM	LWM	VIM	HPM	LWM
0	0.9520	0.9520	0.9522	0.6471	0.6481	0.6580	0.1892	0.2113	0.2114
0.25	0.9550	0.9550	0.9559	0.9550	0.9550	0.9559	0.2201	0.2452	0.2450
0.50	0.9639	0.9639	0.9634	0.7298	0.7308	0.7306	0.3286	0.3578	0.3589
0.75	0.9789	0.9789	0.9786	0.8384	0.8390	0.8386	0.5622	0.5851	0.5849
1	1	1	1	1.0001	1	0.9999	1	1	1

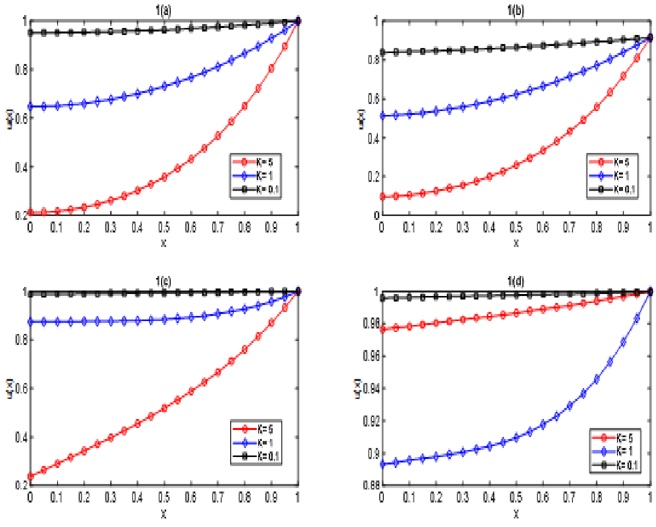

Figure. 1. du(x) vs dx for different values of K when α=0 and β=0.

Table 2: Comparison of steady-state concentration for various values of α , β and K with VIM and HPM

	$\alpha=10, K=0.1$			$\alpha=1, \beta=0.1, K=1$			$\alpha=0.1, \beta=1, K=5$		
x	VIM	HPM	LWM	VIM	HPM	LWM	VIM	HPM	LWM
0	0.9958	0.9962	0.9960	0.7829	0.7836	0.7830	0.2366	0.2497	0.2499
0.25	0.9960	0.9964	0.9962	0.7959	0.7966	0.8362	0.2708	0.2860	0.2855
0.50	0.9968	0.9971	0.9970	0.8355	0.8361	0.8355	0.3852	0.4034	0.4034
0.75	0.9979	0.9983	0.9980	0.9028	0.9032	0.9030	0.6128	0.6274	0.6273
1	0.9996	1	0.9998	1	1	0.9999	1	1	1

Accuracy wise the wavelet-based solution supports exact and experimental solutions. Both VIM and HPM also provide the closest solutions for this problem, but the amount of computational work involved in these methods are very high, time-consuming, and tediousness to evaluate non-linear integration. HPM requires the calculation of coefficients of the infinite series wherein VIM needs more function integrals which are a complicated task to get the good solution. LWM requires only coefficients which can be identified through by a solving system of algebraic equations, can be identified by using collocation methods and solved by traditional numerical techniques. With regard to computational effort and solution accuracy, we conclude that LWM is superior to the other methods for this study.

V. CONCLUSION

In this paper, the solution of the partial differential equation in the action of a biosensor at mixed enzyme kinetics has been discussed using wavelets. The theoretical analysis, such as the convergence analysis and the error estimation for the proposed technique has been discussed. The solution obtained through the proposed solution has been compared with Homotopy perturbation method and Variational Iteration methods. The quality of the solution has also been investigated through tables and figures. The numerical results are in good agreement with the solution obtained through other traditional methods.

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