

Range Estimation of Battery Electric Vehicle by Mathematical Modelling of Battery's Depth-of-Discharge

M. Yerri Veeresh, V. Naga Bhaskar Reddy, R. Kiranmayi



Abstract: This paper presents the mathematical modelling of Battery Electric Vehicle (BEV) based on the depth-of-discharge (DOD) for range estimation of the vehicle using MATLAB/Simulink software. In this scheme, the lead acid battery and lithium ion battery are considered for depth of discharge computation and the range is estimated for the Simplified Federal Urban Drive Cycle (SFUDC) and European urban drive cycle. The analysis comprises with the tractive effort, machine and accessories losses are accounted. The potential values of the BEV are assessed with design, type and parameters of the vehicle. The complete mathematical model is simulated and the comprehensive results are tabulated.

Keywords : Depth- of- discharge; Electric vehicle; Range; Tractive effort.

I. INTRODUCTION

In the growing population of all over the world, Electric vehicle (EV) is the perfect alternative to the internal combustion engine (ICE) vehicles for the transportation through road ways and rail ways. The major factor for this change is that the billions of population with ICE vehicles on roads are polluting the environment more and more by H. Wakefield in [7]. On the other hand, the fossil fuels of petrol/diesel used in the ICE vehicle will extinct as they are non-renewable. Then, what is the alternative fuel? The answers are like Ethane, Methane etc., but still these are not available in abundant. So, again need other alternative, now the answer is Electricity in [1] - C. C. Chan *et.al* in [2]. In the developing countries like India, the cost of fuel is very significant. The cost of electricity to charge the battery, which is the fuel of EVs, is very low compared to the cost of petrol/Diesel.

Electric vehicles are broadly includes battery electric vehicles (BEVs), hybrid electric vehicles (HEVs) and fuel cell electric vehicles (FCEVs). Generally, EVs are considered as a multidisciplinary subject that covers broad portion. Even

though the different stages of development on the challenging problems are taking place in BEVs, HEVs

and FCEVs. Today most of BEVs are used for short - range transportation with low speed application, since limited with battery size and ratings. HEVs can meet the long-range high-speed transportation but the cost is higher. FCEVs are still in the developing stage to meet the consumer's demand.

The EVs having major issues like battery, battery management, charging facilities in BEVs, management of multiple energy sources, battery management in HEVs and fuel processor and fuel cell cost in FCEVs by C. C. Chan in [4], [9] and K. T. Chau *et.al* in [11]. In this paper, the battery electric vehicle BEVs is mainly considered for discussion and their mathematical modelling equations are carried at different stages of the overall system.

II. BATTERIES

The energy source of an electric vehicle as battery is termed as Battery Electric vehicle (BEV). There are various rechargeable batteries available in the market, but still the battery electric vehicles are not competing with IC engine vehicle. The main drawbacks of batteries are their specific energy (KWh/Kg), energy density (KWh/L) and power density (KW/L) values are not up to level of petrol/diesel values. The low values of specific energy take the penalty of large number of battery packs to drive the vehicle over the long distance. Therefore, the battery consumes more volume and weight of the vehicle and also leads to more cost. The BEVs are also suffering with slow charging converters, which take long time to charge the battery by James Larminie *et.al* in [3].

On the other hand, the benefits with BEVs are many over the long run. The BEVs are absolutely best alternatives of IC/SI Engine vehicles. The fuel cost (i.e., cost of electricity) of the BEVs is as low as 1/3 times to 1/5 times of the cost of petrol or diesel vehicle. Even though the BEVs initial cost is high with the cost of batteries, the batteries charging cost is very low which significantly save more money over the long run. The key advantage of BEVs is absolutely 0% CO₂ emission vehicle by D. Berndt in [10]. As concern to environmental conditions, the BEVs are much more preferable in the most populated Cities.

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The viable batteries in the electric vehicles consist of Valve Regulated Lead Acid (VRLA), Nickel-Cadmium (Ni-Cd), Nickel-Zinc (Ni-Zn), Nickel-Metal Hydride (Ni-MH), Zinc/air (Zn/Air), Aluminium/Air (Al/Air), Sodium/Sulfur(Na/S), Sodium/Nickel Chloride (Na/NiCl₂), Lithium-Polymer (Li-Polymer), Lithium-Ion (Li-Ion) types by C. C. Chan in [4]. Therefore, the batteries are needed to be analysed mathematically with the equivalent circuits and the respective modelling equations to choose the selective battery for the best performance of the electric vehicle [6].

The battery modelling equation is majorly carried in terms of Depth-of-Discharge (DOD) or State-of-Charge (SOC). The DOD of the battery is very essential to know the driving range of the vehicle with respect to the drive cycle chosen. The drive cycles varies based on the road conditions of the Nation/State/City. In this paper, the Lead-Acid (LA) and Lithium-ion (Li-Ion) batteries are modelled and simulated with the US and EU drives cycle. The open - circuit voltage of the Lead-Acid battery is given as

$$E = N * [2.15 - DOD * (2.15 - 2.00)] \quad (1)$$

Similarly, the open - circuit voltage of the Lithium - Ion battery, where it is not as simple as Lead - Acid battery, is given as

$$E = N * [(-3.268 * DOD^4) + (9.1629 * DOD^3) + (10.847 * DOD^2) + (5.876 * DOD) - (0.6717 * DOD^2) - (0.8515 * DOD) + 4.1495]$$

(2)

Where, N is the number of cells in the battery. The battery current is calculated from the battery power P_{batt} with the equation as

$$I = \frac{E - \sqrt{E^2 - 4RP_{batt}}}{2R} \quad (3)$$

Where, R is the internal resistance of the cell. If the vehicle is in regenerative mode, the P_{batt} is negative. Now, the equation of battery current is

$$I = \frac{-E + \sqrt{E^2 - 4RP_{batt}}}{2R} \quad (4)$$

The battery capacity is reduced quickly if the current drawn is more. This phenomenon is significant for BEVs, as they draw the current at higher values. So, it is significant to forecast the effect of current on capacity. To know this, let us consider the Peukert model of battery behaviour. The Peukert capacity is given by the equation as

$$C_p = I^k * t \quad (5)$$

where, k is the Peukert's Coefficient of the battery and t is the time in hours that last with constant current I. The valuable charge removed from the battery is recorded in step time of Δt is given as

$$\text{Loss of charge} = \Delta t * I^k \quad (6)$$

The CR_n is the overall charge removed from the battery by the n^{th} step of simulation is given as

$$CR_{n+1} = CR_n + \frac{\Delta t * I^k}{3600} \quad (7)$$

At the n^{th} step, the DOD of a battery is the ratio of charge removed to the battery capacity and is given as

$$DOD_n = \frac{CR_n}{C_p} \quad (8)$$

The range of the vehicle can be estimated with the simplified equation as

$$\text{Range} = \frac{90\% \text{ of battery charge}}{DOD(\%) \text{ per cycle}} * \text{Distance per cycle} \quad (9)$$

III. TRACTIVE EFFORT

Tractive effort is the force required to propel the electric vehicle forward, pass on to the ground through drive wheels. The force propelling the vehicle forward has to accelerate the vehicle by overcoming the rolling resistance force, aerodynamic drag force and also need to provide force to overcome the vehicle's weight component acting towards down the slope. The schematic diagram of forces acting on a car moving up a slope is shown in Fig. 1. The various forces contribute to total tractive force required are as follows by James Larminie *et.al* in [3]:

A. Rolling Resistance Force (F_{rr})

The rolling resistance is mainly due to hysteresis losses in vehicle tyres. This value, is approximately constant, hardly depends on vehicle speed. The rolling resistance force is proportional to weight of the vehicle (m). The mathematical equation is given as

$$F_{rr} = \mu_{rr} * m * g \quad (10)$$

where, m is the mass of the vehicle in Kg, g is the gravitational constant 9.8, μ_{rr} is the coefficient of rolling resistance. This μ_{rr} value depends on kind of tyre and value of tyre pressure. Typical value is 0.015 for a radial tyre and can be reduced about 0.005 with specially designed tyres of electric vehicles.

B. Aerodynamic Drag (F_{ad})

The aerodynamic drag force is due to frictional action of vehicle body moving through the air. This value mainly depends on frontal area, shape side mirrors, air passages etc., of the vehicle. The mathematical equation is

$$F_{ad} = \frac{1}{2} * \rho * A * C_d * v^2 \quad (11)$$

where, ρ is air density in Kg/m³, A is the frontal area in m², v is the velocity in m/sec and C_d is the drag coefficient. With good vehicle design, the C_d can be reduced to great extent. Typical value of saloon car is 0.3 but it can be reduced up to 0.19 with other EV designs.

C. Hill Climbing Force (F_{hc})

The hill climbing force is required to push the vehicle to up a slope it is significant to be determined. This is simply the component of the vehicle weight which acts along the slope. The mathematical equation is

$$F_{hc} = m * g * \sin \psi \quad (12)$$

where, ψ is slope angle with respect to ground level in degrees.

D. Acceleration Force

As the vehicle acceleration (a) is varied then an additional force is needed to be applied. According to Newton's third law, the force required for the linear acceleration of the vehicle is given as

$$F_{la} = m * a \quad (13)$$

To extract more accuracy, it is necessary to consider the force required to turn the rotating parts faster. That is, the angular or rotational acceleration is considered in addition to linear acceleration. The mathematical equation of angular acceleration is given as

$$F_{ra} = I \frac{G^2}{\eta_p r^2} \quad (14)$$

where, I is the rotor's moment of inertia of the motor in Kgm², G is the gear ratio,

r is the radius of the tyre in metres and η_g is the efficiency of the gear system. Therefore, the total tractive effort force required to drive the vehicle is the sum of all these forces. Mathematically, it is expressed as

$$F_{te} = F_{rr} + F_{ad} + F_{hc} + F_{ia} + F_{wa} \quad (15)$$

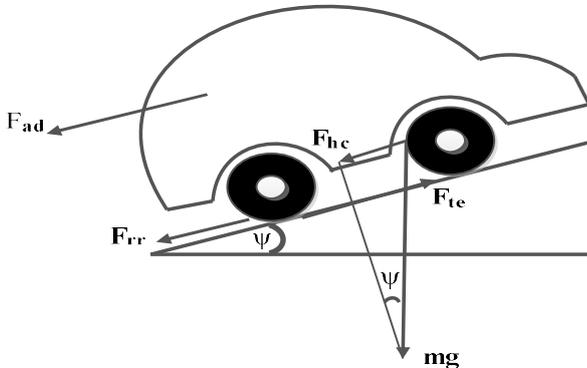


Fig. 1. The forces acting on a car moving up a slope

IV. ENERGY FLOW

In order to estimate the range of BEV, it is necessary to calculate the energy required to drive the vehicle for each second of the drive cycle. The energy flow in a classic BEV is shown in Fig.2. With the tractive effort calculation and drive cycle velocity, the power can be evaluated by product of tractive force and the velocity given by James Larminie *et al* in [3].

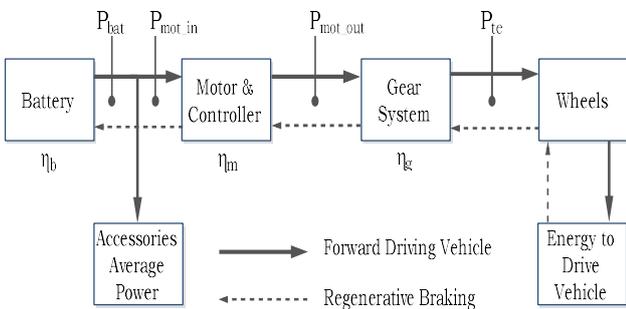


Fig. 2. Energy flows in BEV during forward motoring and regenerative braking

The energy required to drive the vehicle for 1 second is same as the power and is given as

$$\text{Energy required for each second} = P_{te} = F_{te} * v \quad (16)$$

The required energy to drive the vehicle for second is calculated with the help of energy flow diagram with various efficiencies. The gear efficiency η_g is considered constant because most of the EVs use only one gear. Since, the efficiency is high with simple gear system. The motor efficiency η_m will vary with speed (ω), torque (T) and size of the motor. The mathematical equation is given as

$$\eta_m = \frac{T * \omega}{T * \omega + k_c * T^2 + k_i * \omega + k_w * \omega^3 + C} \quad (17)$$

where, k_c is the coefficient of copper loss, k_i is the coefficient of iron losses, k_w is the coefficient of windage losses and C is the constant losses. Due to inefficiency of the motor, controller and gear systems, the motor power is not equal to traction power and the electrical input power needed by the motor is greater than mechanical output power of the motor are given by

$$P_{mot_out} = \frac{P_{te}}{\eta_g} \quad (18)$$

$$P_{mot_in} = \frac{P_{mot_out}}{\eta_m} \quad (19)$$

If the motor is under braking or to slow down the vehicle (regenerative mode), the efficiency works in the opposite way. Therefore, the electric power from the motor is decreased and are given as

$$P_{mot_out} = P_{te} * RgR * \eta_g \quad (20)$$

$$P_{mot_in} = P_{mot_out} * \eta_m \quad (21)$$

where, RgR is the regenerative ratio proportion to motor braking. The equations (16) to (21) will give the electrical power and mechanical power to or from the motor. Apart from this major power required from the electric

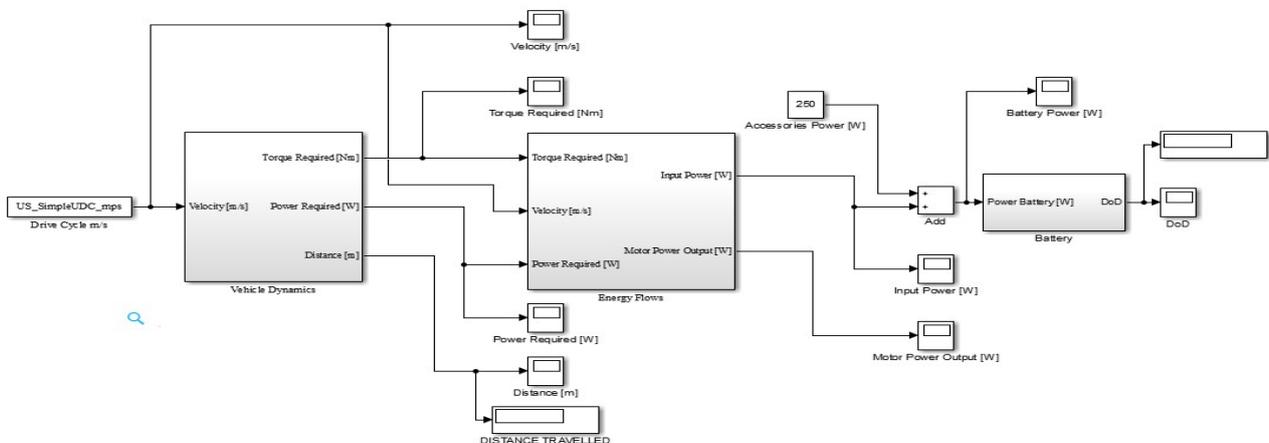


Fig. 3. Mathematical model of Battery Electric Vehicle (BEV) implemented in MATLAB-SIMULINK

propeller, other systems in the vehicle like focus lights, side light indicators, air-conditioners, accessories like FM, audio players, mobile charges etc., will consume the electrical power. So in addition to motor power, the average power required to the accessories will give the total electrical power required from the battery and is given as

$$P_{batt} = P_{mot} + P_{ac} \quad (22)$$

V. DRIVE CYCLE

One of the major problems of EV is the distance that it can travel with energy source capacity by J. Fenton in [8]. In any case, this range calculation is difficult based on the battery efficiency. So, to estimate the range of the vehicle, two possible ways methods are considered. One is with constant velocity as it is simple but impractical, because no vehicle on the road can be driven at constant speed. The second one is with a profile of ever-changing speeds as this is practical and complex. These cycles are large in number and are correspond to realistic driving pattern in diverse conditions. During the process of computation the speed is changing continuously then correspondingly other parts of the vehicle also varies, which makes it complex analysis [5].

There are different driving cycles were developed such as urban driving and highway driving. The urban cycles are Los Angeles (LA-4), Federal Urban Driving Schedule (FUDES) that lasts for 1500 Seconds, Simplified Federal Urban Driving Schedule (SFUDS) is lasts only in 360 Seconds, European Urban Driving Cycle (EUDC) which is simpler and with periods of constant velocity. The highway cycles are Federal Highway Driving Schedule (FHDS) is having 765 second samples, which is having unrealistic maximum speed then the US06 standard is most widely used now given by James Larminie *et.al* in [3].

VI. SIMULATION RESULTS

By using the mathematical equations of battery system, tractive system, motor and respective energy flows system, the battery electric vehicle was modelled and implemented in the MATLAB-SIMULINK as shown in Figure 3. The chosen simulation data specifications of the LA and Li-Ion batteries and three phases Induction motor are given in Table I and

Table II respectively. The chosen design specifications of the REVA Car are considered for simulation purpose is listed in Table III.

The simulation results of the range estimation of the REVA car under different drive cycles with chosen specifications are listed in the Table IV. The tabulated simulation data presents that the lithium-ion battery function superior than lead acid battery under the both EU and SFU drive cycles. In either drive cycles, the lithium-ion battery vehicle can run about 30Km (approximately) more than lead-acid battery vehicle of battery capacity. As this result supporting, the EVs with lithium-ion battery are more economical for long run duration.

The electric vehicle under the different parameter values such as rolling resistance coefficient (U_{rr}), frontal area of vehicle (A), aero dynamic drag coefficient (C_d) are considered

for range estimation. The simulation results are tabulated in table V. As per earlier simulation data, these results are suggesting that the lithium-ion performance is much better than lead-acid battery even when the design parameters are varied. It is also noted that, the change in speed range is much influenced by the change in rolling resistance coefficient and frontal area of the vehicle than drag coefficient value. So, design of EV should be made with more precise calculation of most affecting parameters.

The EU drive cycle and SFU drive cycle used for simulation of BEV are shown in Fig. 4 and Fig. 5 respectively. It is noted the SFU drive cycle is more realistic than EU drive cycle. So, for the selected SFU drive cycle based on the tractive effort mathematical equations, the torque and power required to drive the vehicle are shown in Fig. 6 and Fig. 7 respectively. The battery power simulated output is shown in Fig. 8.

Table-I: Specifications of Battery

Parameter	Lead Acid battery	Lithium-Ion Battery
Number of cells (each of 6V)	24	16
Capacity (in Ah)	200	200
Peukert's Coefficient	1.12	1.05

Table-II: Specifications of Machine

Parameter	Poly Phase Induction Motor
Copper loss coefficient (k_c)	0.9
Iron loss coefficient (k_i)	0.03
Windage loss coefficient (k_w)	0.000005
Constant loss	600

Table-III: Specifications of Vehicle Chassis

Parameter	REVA Car	Range of variation
Rolling resistance coefficient, U_{rr}	0.005	0.015-0.005
Frontal area, A	1.9992	0.5 - 6
Drag coefficient, C_d	0.6	0.18 - 0.7
Curb weight (with battery pack)	700	***
Pay load	227	***
Gear ratio, G	37	***
Gear efficiency, η_g	0.95	***
Regenerative Braking, RgR	0.3	***

Table-IV: Simulation results of the Range estimation of REVA Car under different drive cycles with chosen specifications

Parameter	REVA Car	European Urban Drive Cycle (EUDC)			Simplified Federal Urban Drive Cycle (SFUDC)		
		Distance Travelled per cycle (in Km)	Range of vehicle with 90% battery charge (in Km)		Distance Travelled per cycle (in Km)	Range of vehicle with 90% battery charge (in Km)	
			Lead Acid Battery	Lithium Ion Battery		Lead Acid Battery	Lithium Ion Battery
Rolling resistance coefficient, μ_{rr}	0.005	1.012	66.7742	95.7427	3.098	51.9702	80.0287
Frontal area, A	1.999						
Drag coefficient, C_d	0.6						
Curb weight (with battery pack)	700						
Pay load	227						
Gear ratio, G	37						
Gear efficiency, η_g	0.95						
Regenerative Braking, RgR	0.3						

Table-V: Simulation results of range estimation of Electric Vehicle with change in μ_{rr} , A, C_d Parameter values

S.No.	Rolling Resistance Coefficient, μ_{rr}	Frontal Area of Vehicle, A	Aero dyanamic Drag Coefficient, C_d	Distance Travelled per Cycle (in Km)	Lead Acid Battery		Lithium-Ion Battery	
					Depth of Discharge per cycle (%)	Range of vehicle with 90% battery charge (in Km)	Depth of Discharge per cycle (%)	Range of vehicle with 90% battery charge (in Km)
European Urban Drive Cycle (EUDC)								
1	0.005	1.9992	0.6	1.012	1.364	66.77	0.9513	95.74
2	0.01			1.012	1.521	59.88	1.055	86.33
3	0.015			1.012	1.682	54.15	1.161	78.45
4	0.005	2.5	0.6	1.012	1.433	63.56	0.9965	91.4
5		3		1.012	1.501	60.68	1.041	87.49
6		4		1.012	1.643	55.44	1.133	80.39
7	0.005	1.9992	0.4	1.012	1.276	71.38	0.8931	101.98
8			0.5	1.012	1.319	69.05	0.9221	98.77
9			0.7	1.012	1.364	66.77	0.9813	92.82
Simplified Federal Urban Drive Cycle (SFUDC)								
1	0.005	1.9992	0.6	3.098	5.365	51.97	3.484	80.03
2	0.01			3.098	5.954	46.83	3.861	72.21
3	0.015			3.098	6.579	42.38	4.255	65.53
4	0.005	2.5	0.6	3.098	6.094	45.75	3.926	71.02
5		3		3.098	6.843	40.75	4.389	63.53
6		4		3.098	8.441	33.03	5.338	52.23
7	0.005	1.9992	0.4	3.098	4.541	61.4	2.974	93.75
8			0.5	3.098	4.924	56.62	3.211	86.83
9			0.7	3.098	5.843	47.72	3.779	73.78

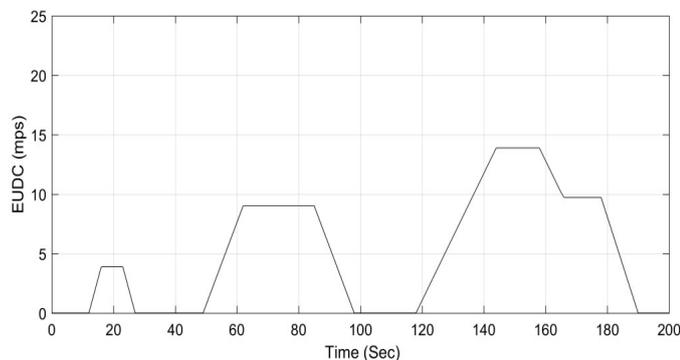


Fig. 4. European Urban Drive Cycle (EUDC)

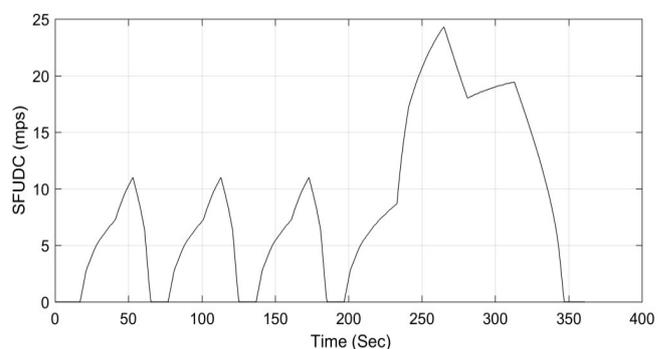


Fig. 5. Simplified Federal Urban Drive Cycle (SFUDC)

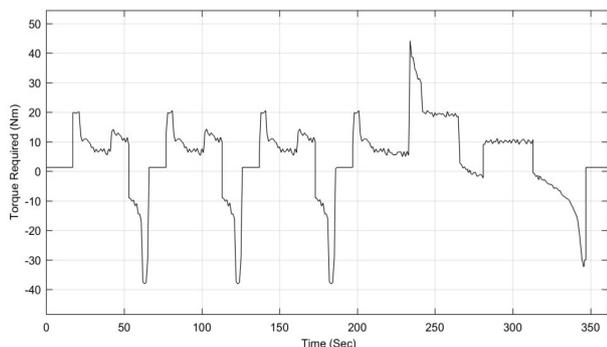


Fig. 6. Tractive Effort System Torque required under SFUDC per cycle

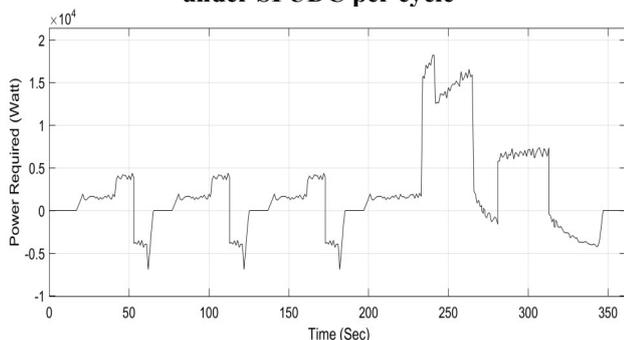


Fig. 7. Tractive Effort System Power required under SFUDC per cycle

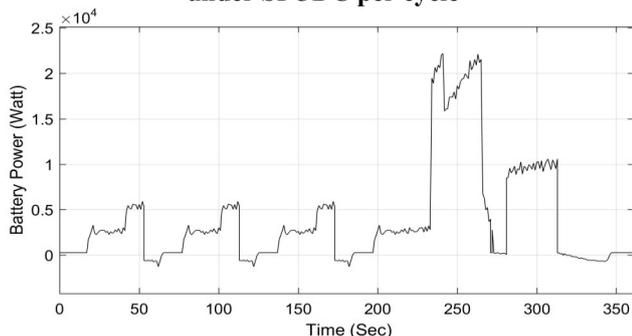


Fig. 8. Battery Power required under SFUDC per cycle

VII. CONCLUSION

This paper presents comprehensive analysis of the battery electric vehicle model. The simulation results assessment on the range of the vehicle for lithium ion battery are 95.74Km with EU Drive Cycle and 80.03 with SFU Drive Cycle and whereas for lead acid battery are 66.77Km with EU Drive Cycle and 51.97 with SFU Drive Cycle. Therefore, the results are suggesting that lithium ion battery can provide 30Km additional long range of drive than lead acid battery with same ampere-hour rating. The different values of the rolling resistance, frontal area and aero drag coefficients will influence the range of the vehicle as given by simulation results in Table V and is noted to be accountable for proper design of BEV. However this multidisciplinary subject computed for different parameters of chassis, machine, converters, batteries etc., with selective values of the vehicle design, the low cost – best performance EVs can be the future market.

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