Abstract: The accurate design of spur gear drive has a tremendous impact on size, weight, transmission and machine performance. Also, the demand for lighter gears is high in power transmission systems, as they save material and energy. Hence this paper presents an enhanced method to solve a two stage spur gear optimization problem. It consists of a mathematical model with a nonlinear objective function and 11 constraints. A two stage spur gear is considered. To obtain minimum volume of spur gear drive is objective of the problem. The considered design variables are: Module, number of teeth, base width of the gears and, shaft diameter and power. Besides considering regular mechanical constraints based on American Gear Manufacturers Association (AGMA) requisites, six more additional critical constraints on contact ratio, load carrying capacity, power loss, root not cut, no involute interference and line of action are imposed on the drive. Nature inspired optimization algorithms, namely, Simulated Annealing (SA), Firefly (FA) and MATLAB solver fmincon are used to find solution in MATLAB environment. Simulation results are analyzed, compared with literature and validated.

Keywords: Gear optimization, Spur gear drive, AGMA, Nature inspired algorithms

1. INTRODUCTION

The simplest of all gears is the spur gear. They offer considerable precision and high power transmission efficiency than any other gears. Hence, they are favourites in industrial machines. In this arrangement, two meshing gears are mounted on parallel shafts. In spur gears, teeth are cut and arranged parallel to the axis of gear. A spur gear and a typical two stage spur gear train are shown in Figure 1 and Figure 2. Normally, spur gears design is a very intricate task, as it involves many empirical formulas, graphs, tables and the use of number of linear functions and discrete variables. Minimizing volume of gear is of great interest, since gear trains of many high performance power transmissions systems such as automotives, aero space and machine tools require low volume. Hence it is desirable to develop low volume, quiet and more reliable gear designs. In such situation, it is attractive for the designer to prefer optimization [28] in order to obtain optimum solutions that are computationally efficient through reliable algorithms [23]. A fair amount of research has been done in mechanical gear design optimization using various algorithms in single and two stage gear pairs by many scholars [1~5]. Single gear pair optimization accounted optimum bending and contact stresses, displacement on the gear tooth pertaining to space needs, transmitted power, weight, profile of tooth and material. Mendi et al. [1] optimized volume of gearbox by Genetic Algorithm (GA). The reduction in volume was 1.47% compared to analytical method. Thompson et al [2] performed minimum volume and surface fatigue life optimization for two and four stage gears trade off analysis. Chong et al. [3] employed GA to get minimum geometric volume of a two stage gear train and plain planetary gear train. They achieved 40% reduction in pitch diameter and face width, and 3% error reduction in gear ratio. Chong et al[18] solved a Multi-criteria optimization problem. They considered cylindrical gear pairs to decrease gear size and vibrations. Gologlu and Zeyveli [4] reduced volume of the gear train in a two stage gear drive preliminary design. Tudose et al. [5] automated a two stage transmission design taking into account helical gear, including shafts, bearings and housing. He applied a two phase evolutionary method for solving the same. Abiud and Ameen [19] established an optimum design procedure for a two stage spur gear system. Buiga and Tudose [35] presented mechanized optimal design for a two-stage helical coaxial gear drive using Genetic Algorithms (GAs). The problem had mass of the speed reducer as objective function, 17 mixed design variables and 76 non-linear constraints. They observed GA offers better design solutions than traditional design method. Marjanovic [36] presented a realistic move towards gear train optimization. They formulated a mathematical model for spur gears and solved the same by his original software developed. Golabi et al [27] got minimum volume of two and three stage gear drive using a MATLAB program. Different optimization algorithms such as GA, SA, Ant-Colony Optimization (ACO), PSO and Neural Network were used for design optimization of gear problems. Yokota et al [20] used an improved GA to get optimum weight of the gear pair taking gear bending and torsional strength of the shaft. Savsani V et al [6] obtained optimum weight for spur gear train by...
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involving additional constraints based on PSO. Zhang, et al. [21] optimized spur gear reducer design based on GA. Li et al. [7] solved multi-objective optimization design of reducer through an adaptive GA. Sanghvi et al [15] determined the optimum volume of a two stage gear train and optimum load delivering capacity of the gear by Non Sorted Genetic Algorithm (NSGA-II). Barbieri et al. [8] investigated modifications spur gear pair profile by GA. Zhang et al. [9] established a numerical model for getting optimum volume design of a gear drive by GA and MATLAB toolbox. He got best optimal solution with higher efficiency, gear design quality and reduced production cycle. Marjanovic et al [11] obtained optimal gear box design taking into account suitable materials, gear ratio and position of shaft axis. Marcelin [12] examined the performance of GA in gear optimization problem. Bonori et al [13] solved an optimization problem taking into account micro geometric modifications. Faggioni et al [14] performed global optimization in order to reduce vibration based on profile modifications. Ananthapadmanabhan et al [33] dealt with optimum power and dimensions of the gear box concurrently. They considered scoring criterion number, flash temperature and minimum film thickness as constraints. William et al [10] focussed on transmission error, harmonic locations and the detection of surface and bending fatigue of damages. Vadim [34] presented a gear box structure optimization graphically with given number of output vertives, and minimized total number of vertives. Patel et al [16] designed an optimum two stage spur gear for maximum power transmission of the gear train using NSGA - II. They considered standard mechanical constraints and two tribological constraints (wear and scuffing). They proved that the inclusion of tribological constraints increase the efficiency, at the cost of a small increase in gear box volume. C. Gologlu and M. Zeyveli [26] optimized gear volume of two stage helical gear using GA without considering the shafts volume. However the authors [16, 26] did not consider certain critical mechanical constraints. Cited literature shows various researches on gear optimization problems. Nevertheless, literature reveals a gap as follows: 1.Not much research had been done on spur gear optimization using nature inspired algorithms, namely, SA and FA. 2. Most of the researchers had used either conventional or less efficient techniques. 3. Certain important mechanical constraints viz. contact ratio, load carrying capacity, no involute interference, root not cut, power loss, line of action which are very important for today’s high performance power transmission systems had not been considered. To overcome this, and to accomplish the task of optimum design, this work, besides considering all standard mechanical constraints, includes six other additional mechanical constraints viz. contact ratio and contact ratio, load carrying capacity, no involute interference, root not cut, power loss and line of action for improved power transmission. This work also uses powerful and superior nature inspired algorithms, namely, SA and FA [17] along with MATLAB solver fmincon. AGMA instructions are followed for regular mechanical constraints [17]. Thus this work bridges the gap and shows the mechanical engineer a way out to the problems in design of spur gears drives. This work is based on C. Gologlu and M. Zeyveli [26] and Patel et al [16], but it provides an enhanced optimum gear design through its precise approach by accommodating certain new constraints. The following are the novel things included as additional constraints in this paper.

1. Contact Ratio (CR) – It is the standard number of teeth gets in touch with as the gear rotates jointly. It is necessary that tooth profiles be proportional so that a second pair of mating teeth comes into contact before first pair is out of contact. CR must be between 1.2 and 2 for spur gear [31]. The greater the CR, the quieter and smoother are the operations of the gear [30].

2. Condition of root not cut – This happens when the contact between pinion tooth gear teeth occurs lower than the base circle of pinion on the non-involute portion of the flank [16].

3. Load carrying capacity- It is a functional characteristic of the gear. In this work, the optimum values for minimum volume obtained in this research are used to find the load carrying capacity at both the stages [15]. The constraint condition is formulated in such a way that, this calculated value should be more than the minimum allowed load carrying capacity of the gear [29].

4. Power loss - Power loss in the gears is primarily of two forms viz. load dependant and load independent losses. The total power losses due to meshing of gears, loss in bearings and in the seals should be less than 1.2% - 2.2% of input power. The range of percentage power loss value is obtained from the results of [16].

5. No involute interference - This happens when the pinion tooth get in touch with gear tooth wherein the contact occurs under the base circle of pinion on the non involute portion of the flank [33]. Accordingly constraints are laid.

6. Line of action - To ensure even rotation, the arc of action must be larger than the line of action [31]. Accordingly constraints are formulated.

Popular nature inspired algorithms are used in this research. They are much simpler and powerful enough to solve any design problems [22], [23], [24], and [25]. They also give faster convergence and greater computational speed. SA and FA show better robustness and global searching capability. FA is superior to GAs and PSO as fireflies come together more intimately around each optimum point [25]. This is how the work is organized. The mathematical formulation of the optimization problem for two stage spur gear drive involving relevant constraints is presented in Section 2. In Section 3, the nature inspired optimization algorithms SA, FA and MATLAB solver fmincon are presented. In Section 4 the results and discussion is presented. Last of all, conclusion drawn and scope of the work is offered.

II. MATHEMATICAL FORMULATION OF SPUR GEAR DRIVE OPTIMIZATION PROBLEM

The specifications and input parameters are given in Table 1.
Table 1 Specifications, coefficients and inputs for the spur gear drive

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>VALUE</th>
<th>PARAMETERS</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transferred power, (P)</td>
<td>5kW</td>
<td>Temperature factor, $Y_0$</td>
<td>1</td>
</tr>
<tr>
<td>Usage</td>
<td>Electricity motor</td>
<td>Pitting Stress cycle life factor, $Z_{SN}$</td>
<td>1</td>
</tr>
<tr>
<td>Total gear ratio, $i_{tot}$</td>
<td>11</td>
<td>Bending strength stress cycle life factor, $Y_N$</td>
<td>1</td>
</tr>
<tr>
<td>Input pinion speed, $n$ (rpm)</td>
<td>1440</td>
<td>Hardness ratio for pitting resistance, $Z_{HM}$</td>
<td>1.005</td>
</tr>
<tr>
<td>Pressure angle, $\phi$ (deg)</td>
<td>20</td>
<td>Reliability factor, $Y_Z$</td>
<td>0.814</td>
</tr>
<tr>
<td>Gear material</td>
<td>SCM415, 60HRC</td>
<td>Size factor, $K_S$</td>
<td>1</td>
</tr>
<tr>
<td>Gear surface roughness, $R_a (\mu_m)$</td>
<td>0.4</td>
<td>Gear power loss factor, $H_p$</td>
<td>0.1</td>
</tr>
<tr>
<td>Shafts materials</td>
<td>SAE1060</td>
<td>Average coefficient of friction, $\mu_{max}$</td>
<td>0.2</td>
</tr>
<tr>
<td>Safety factor for pitting, $S_{FS}$</td>
<td>1.2</td>
<td>Coefficient of friction in bearing, $\mu$</td>
<td>0.001</td>
</tr>
<tr>
<td>Safety factor for bending, $S_F$</td>
<td>1.2</td>
<td>Geometry factor for pitting resistance, $Z_{H}$</td>
<td>0.35</td>
</tr>
<tr>
<td>Shaft design safety factor, $S_H$</td>
<td>1.5</td>
<td>Bending strength geometry factor, $Y_1$</td>
<td>1</td>
</tr>
<tr>
<td>Overload factor, $K_o$</td>
<td>1</td>
<td>Safety factor for pitting, $S_P$</td>
<td>1.2</td>
</tr>
<tr>
<td>Velocity factor, $K_v$</td>
<td>1.39</td>
<td>Pitting resistance surface condition factor, $Z_{SN}$</td>
<td>1</td>
</tr>
<tr>
<td>Load distribution factor, $K_H$</td>
<td>1.3</td>
<td>Rim thickness factor, $K_B$</td>
<td>1</td>
</tr>
</tbody>
</table>

A. Design Problem

To obtain a compact gear with optimum design, a two stage gear drive suggested in [16] is considered. Regular mechanical and other additional constraints are taken into account. Superior techniques are used to solve problem. Design variables, objective function and the mechanical constraints shall be discussed now.

B. Design Variables

The design variables for problem are: Module at stage 1 and 2 are ($m_1$, $m_2$). Number of teeth on pinion and gear at stage 1 and 2 are $(z_1, z_2)^T$ ($z_2, z_3$) ($z_3, z_4$), Face width at stage 1 and 2 are ($d_{w1}$, $d_{w2}$) respectively. The diameter of the shafts at input, intermediate and output are $d_{s1}$, $d_{s2}$, $d_{s3}$ respectively. The design variable function of the spur gear drive is defined as follows:

$$F(x) = \frac{F(m_1, m_2, z_1, z_2, z_3, z_4, b_1, b_2, d_{s1}, d_{s2}, d_{s3})}{F(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11})}$$

(1)

Bounds of design variables are:

- $1.5 \leq m_1 \leq 10, 1.5 \leq m_2 \leq 10$;
- $17 \leq z_1 \leq 100$;
- $35 \leq z_2 \leq 145$; $17 \leq z_3 \leq 100$;
- $35 \leq z_4 \leq 145$ $25 \leq b_1, b_2 \leq 100$;
- $15 \leq d_{s1}, d_{s2}, d_{s3}, \leq 80$.

All the bound values of design variables are taken as continuous.

C. Objective Function

To obtain minimum volume of spur gear drive is the objective of the problem. In this paper, volume equations (1), (2), (3), and (4) are from [16] and equation (5) from [27] is as follows:

Total volume of gears is $V_{total} = V_{gear} + V_{shaft} + V_{shell}$

$$V_{gear} = \sum_{i=1}^S \frac{\pi}{2} d_i^2 b_i - \frac{\pi}{4} \left[ d_{s1}^2 b_1 + d_{s2}^2 b_2 + d_{s3}^2 b_3 \right]$$

(2)

Volume of shafts is:

$$V_{shaft} = \sum_{i=1}^{S+1} \frac{\pi}{4} d_{s1}^2 + \frac{\pi}{4} d_{s2}^2 L_{in}(b_{2i-2} - b_{2i-1}) + \frac{\pi}{4} d_{s3}^2 SL_{out}$$

(3)

where $L_{in}$, $L_{out}$ are the lengths of the input and output shafts, given by,

$$L_{in} = 2d_{s1}, L_{out} = 2d_{s2}$$

(4)

where $d_{s1}$ and $d_{s2}$ are the diameter of the shafts at Stage 1 and 2 respectively and $S =$ Number of Stages, and $i$ varies from 1 to 2.

Volume of the shell is:

$$V_{shell} = (w+\delta) \cdot (h-2d)$$

(5)

where, $L, h, t$ are the width, length, height and thickness of the gear box.

D. Constraints

Regular mechanical constraints as well as additional constraints are considered in this work. They shall be discussed one by one.
1. **Bending of the gears**

   The bending stress and contact stresses formulated according to [16] are as follows:

   
   \[
   F_t K_w K_p K_{e_d} \frac{1}{2} \frac{K_{h} K_{e_b}}{Y_f} - \frac{F_p V_N}{Z_H f_{t} f_{z}} \leq 0,
   \]

   where,

   \(F_t\) is transmitted load (N) and \(m_{e_d}\) is transverse module (mm),

   \(\sigma_{BP}\) is allowable bending stress number (N/mm²).

2. **Pitting resistance of the gears**

   The contact stress constraint is devised according to [16].

   \[
   Z_E \sqrt{F_t K_w K_p K_{e_d} \frac{K_{h} K_{e_b}}{Z_H f_{t} f_{z}} - \frac{\sigma_{BP} Z_N Z_w}{Z_H f_{t} f_{z}}} \leq 0,
   \]

   \(d_{e_d}\) is effective pitch diameter of the pinion (mm),

   \(\sigma_{BP}\) is allowable contact stress number (N/mm²) and \(Z_E\) is elastic coefficient (N/mm²)⁰.⁵

3. **Shaft diameter constraint**

   This constraint is suggested in [16].

   \[
   \left[ \frac{3.2 S_{PS}}{\pi} \sqrt{\left( \frac{T}{S_y} \right)^2 + \left( \frac{M}{S_p} \right)^2} \right]^{1/3} - d_z \leq 0
   \]

   \(S_y, S_e\) are yield strength and endurance limit of the shaft material (N/mm²) respectively, \(T\) is torque transmitted by the shaft, (Nmm) and \(M\) is maximum bending moment on the shaft (Nmm).

4. **Interference**

   In order to stay away from interference between the gears, as per [16] least number of teeth on pinion is:

   \[
   \frac{2}{z_p + 2} \sin^2 \phi \left( \frac{x_g}{z_p} + \frac{x_g}{x_p} \right) \frac{z_g}{z_p} \left( 1 + 2 \frac{z_g}{x_p} \sin^2 \phi \right) - z_p \leq 0
   \]

   where \(z_g, z_g\) are number of teeth on pinion and gear.

5. **Diameter of the pinion**

   It should always be less than the mating gear [16].

   \[
   d_p < d_g
   \]

6. **Gear face width**

   According to [16] it must be limited to:

   \[
   3 \pi m \leq b \leq 5 \pi m
   \]

7. **Interference of gear and next shaft**

   To counter interference between the gear and next shaft, the constraint adopted from [27] and [16] is used. In general for all gears, it is given by,

   \[
   d_{2i} < (d_{2i+1} + d_{2i+2})
   \]

8. **Teeth on gear**

   It should be less than the maximum teeth as per [16].

   \[
   Z_g - Z_{g_{max}} \leq 0
   \]

9. **Total volume of gear**

   It ought to be less than maximum gear box volume [16]. Total volume of gears,

   \[
   V_{Total} < (V_{max})
   \]

10. **Power loss in the gear**

    The range of power loss percentage is calculated from the results of power loss values for various power inputs found in [16].

    It must be between 1.2 and 2.2% of input power. This is given by,

    \[
    P_{loss} - 1.2 \% (P) \leq 0
    \]

    Where \(P_{loss} = P_{mu} H\nu + \mu Fv + 7.69 \times 10^{-6} d_{2}^2 n\)

    \(F\) is bearing load (N), \(\nu\) is peripheral speed (m/s).

11. **Contact ratio**

    It should be above 1.4 and normally below 2 according to [30]. It is given by,

    \[
    1.4 \leq m_p \leq 2
    \]

    \(m_p\) = Contact ratio, given by,

    \[
    m_p = \frac{r_p - r_g}{r_p - r_g - C - \Delta C}
    \]

    where \(r_p, r_g\) are contact ratio, pitch circle radii of pinion and gear (mm) \(a_p, a_g\) are addendum of pinion and gear (mm) and centre distance (mm), \(C = r_p + r_g\) and \(P_d\), diametral pitch of the gear (mm).

12. **No involute interference**

    To ensure this, the required condition [33] is given by,

    \[
    \sqrt{(r_g + a_g)^2 - (r_p \cos \phi)^2} - C \sin \phi \leq 0
    \]

13. **Minimum Modulus**

    To achieve this, the required condition is to be as per [16].

    \[
    1.5 \leq m \leq m_{max}\]

14. **Condition of root not cut for pinion**

    The minimum number of teeth for avoiding interference and prevent undercutting, the condition given below as per [31] should be satisfied.

    \[
    \frac{2}{\sin^2 \phi} \leq Z_p \leq Z_{max}
    \]
15. Load carrying capacity
It should be more than minimum value of the gear [15]. It is given by,
\[ F_{metn} - F_1 \leq 0 \] (21)
where deformation factor \( C = 11860 \theta \), is the sum of error between meshing teeth, \( \theta = 0.025 \).

III. METHODS OF SOLUTION
Nature inspired methods viz. SA and FA [23, 24, 27] and MATLAB solver fmincon are applied for solving this spur gear reducer optimization problem.

A. Nature inspired algorithms
Supremacy of contemporary techniques is in their imitation of the best characteristics from nature, principally biological systems [30]

1. Simulated Annealing (SA)
It is a primitive and an admired metaheuristic algorithm [27]. It imitates annealing process in metal cooling. It is a memory less search technique. Initially a solution is found out randomly, followed by a second one nearby the first and now the difference \( \Delta E \) is given by equation (23).
\[ \Delta E = \Delta f = f_{t+1} - f_t = f(X_{t+1}) - f(X_t) \] (23)
If the objective function value of the second one is lesser, it is accepted as current solution search continues from there. Otherwise it is accepted with a probability \( e^{-\Delta E/kT} \) where \( k \) is the Boltzmann's constant. Thus one iteration of the SA is over. SA has ability to tide over local minima. The initializing parameters and settings of SA used for this research are: Initial temperature, \( T_{init} = 1.0 \); Final stopping temperature, \( T_{min} = 1 \times 10^{-10} \); Min value of the function, \( F_{min} = -1 \times 10^{+100} \); Maximum number of rejections, \( max_{rej} = 500 \); Maximum number of runs, \( max_{run} = 150 \); Maximum number of accept, \( max_{accept} = 50 \); Initial search period, \( initial_{search} = 500 \); Boltzmann constant \( k = 1 \); Energy norm (eg, \( E_{norm} = 1 \times 10^{-8} \) \( E_{norm} = 1 \times 10^{-5} \); etc.

2. Firefly Algorithm (FA)
FA was introduced by Xin-She Yang at Cambridge University in 2007 [22], [25]. FA has three rules: i) All fireflies are considered as unisex in order to see that attraction between fireflies happen; ii) Attraction of fireflies is directly proportional to their glow. In the absence of any brighter fireflies in close proximity, a firefly moves arbitrarily; and iii) Glow of a firefly is proportional to its value of fitness function. The degree of pull of a firefly is given by the equation (24):
\[ \beta = \beta_o e^{-r^2} \] (24)
where \( \beta, \beta_o \) are the degrees of pull of a firefly at a distance \( r \), and \( r = 0, r \) is the distance between any two fireflies, and \( \gamma \) is a light absorption coefficient. The distance \( r \) between firefly \( i \) and firefly \( j \) located at \( X_i \) and \( X_j \) respectively is Euclidean distance in (25):
\[ r = ||X_i - X_j|| = \sqrt{\sum_{k=1}^{n}(X_i^k - X_j^k)^2} \] (25)
The movement of the dull firefly \( i \) towards the light firefly \( j \) in terms of the dull fly's updated location is calculated by equation (26): For implementation of FA, in this work, values of the parameters used are: 20 fireflies, Number of iterations = 250, \( \alpha = 0.5, \gamma = 1 \) and \( \beta_o = 0.2 \). These parameters have been preferred after cautious adjustment to suit for solving spur gear optimization.

B. MATLAB solvers
These solvers can be used for obtaining optimum solutions for unconstrained, linear programming, quadratic programming, and nonlinear programming problems. fmincon
It is built in of MATLAB optimization Toolbox. It finds the minimum scalar function of multiple variables, in a region subject to linear constraints and bounds [15]. It is one of the most flexible Nonlinear Programming solvers. It is helpful for solving constrained non-linear optimization.

The structure for fmincon is \( x_{opt} = \text{fmincon} \{ \text{fun}, x0, A, b, Aeq, beq, lb, ub \} \) where \( \text{fun} \) is the name of an M-file that
Table 2 Comparison of optimal values by different algorithms of this work and literature

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>POWER 5 KW</th>
<th>SA</th>
<th>FA</th>
<th>FMINCON</th>
<th>LITERATURE [26]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Module of stage 1 (mm), $m_1$</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>2.25</td>
<td></td>
</tr>
<tr>
<td>Module of stage 2 (mm), $m_2$</td>
<td>3.02</td>
<td>3.00</td>
<td>3.18</td>
<td>3.5</td>
<td></td>
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<tr>
<td>No. of teeth of pinion stage $1, z_1$</td>
<td>17</td>
<td>17</td>
<td>17</td>
<td>22</td>
<td></td>
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<tr>
<td>No. of teeth of gear stage 1, $z_2$</td>
<td>47</td>
<td>47</td>
<td>47</td>
<td>79</td>
<td></td>
</tr>
<tr>
<td>Number of teeth of pinion stage 2, $z_3$</td>
<td>19</td>
<td>20</td>
<td>19</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>No. of teeth of gear stage 2, $z_4$</td>
<td>76</td>
<td>79</td>
<td>75</td>
<td>64</td>
<td></td>
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<tr>
<td>Face width of stage 1 (mm), $b_{11}$</td>
<td>38.20</td>
<td>37.14</td>
<td>50</td>
<td>46.77</td>
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<tr>
<td>Face width of stage 2 (mm), $b_{22}$</td>
<td>33.58</td>
<td>34.84</td>
<td>50</td>
<td>67.10</td>
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<tr>
<td>Diameter of the shaft of stage 1, $d_{11}$</td>
<td>21.18</td>
<td>27.14</td>
<td>30</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Diameter of the shaft of intermediate stage, $d_{s2}$</td>
<td>44.29</td>
<td>38.33</td>
<td>30</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Diameter of the shaft of stage 2, $d_{22}$</td>
<td>33.34</td>
<td>38.64</td>
<td>30</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Volume (mm$^3$), $V$</td>
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<td>4466154</td>
<td>4962524</td>
<td>4513290</td>
<td></td>
</tr>
<tr>
<td>% volume reduction</td>
<td>+7.20</td>
<td>+1.03</td>
<td>-9.95</td>
<td>-</td>
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</tr>
<tr>
<td>Time (seconds)</td>
<td>1.236</td>
<td>6.324</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Iterations/generations</td>
<td>3187</td>
<td>100</td>
<td>63</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

describes the function, preceded by an "@" sign; $x\theta$ is an initial value for the optimizer; $A, b$ describe a linear inequality constraint $A \cdot x \leq b$ on the solution. $Aeq, beq$ label a linear equality. In this research, the default values of the fmincon are taken, for all the parameters as well as convergence criteria, maximum iterations and the gradients for optimization.

IV. RESULTS AND DISCUSSION

The results given by SA, FA and MATLAB solver fmincon is given in Table 2. Optimal design parameters values given by the algorithms of this work and literature are compared [26]. From Table 2, it is observed that optimal values by SA, FA and fmincon for $m_1, m_2, z_1, z_2$ (1.5, 3 mm and 17 and 47) are same. However, for other parameters slightly different optimal values are obtained. The highest optimum volume value (4962524.74mm$^3$) is by fmincon solver and the lowest value (4188500.43 mm$^3$) by SA. As for the value of $z_3$ of spur gear in stage 2, SA, FA and fmincon give nearly the same value (19, 20, 19 ) whereas for $z_4$, SA and fmincon give almost same value( 76 and 75).

![Fig. 3 Comparison of the results obtained by various algorithms and literature](image)

Different values of number of teeth in stage 1 $d_{s1}, d_{s2}$ are given by SA and FA (21mm, 44mm and 27mm, 38mm). The highest value of teeth in both the stages is by FA (47, 79). Figure 3 shows the graphical representation of the results obtained by various algorithms and fmincon. However in terms of computational speeds, SA is much faster than FA algorithm (1.236s and 6.324s).
In this research, SA gives the desirable minimum values of volume (4188500 mm$^3$). As the SA algorithm give the best value it is more suitable for solving the problem. Results also show that all the algorithms of this work are faster and computationally efficient. Figure 5 shows the optimized values of design variables given by fmincon. It is a current point versus optimum fitness value graph. In Figure 4, function value versus iteration and Fitness value versus Generations by fmincon are shown. From Figure 4, it is observed that at zeroth iteration function value is 4.9 x10$^7$ mm$^3$ and it decreases upto 2.9x10$^7$ mm$^3$ at tenth iteration and 1.2x10$^7$ mm$^3$ at fortieth iteration and afterwards it keeps on decreasing upto 5x10$^6$ mm$^3$ and then remains constant at 4962524.74 mm$^3$ for the next iterations. Optimal design variables values and all objective functions by fmincon are presented in Table 2. Table 3 shows there is 7.20% reduction in volume of spur gear drive is achieved by SA, as compared to the literature. At this point, it is worthwhile to mention that this reduction in volume of spur gear drive is achieved after including critical additional mechanical constraints. A representation of the same is also presented in graph in Figure 6. As SA gives the best results, it is decided to compare the optimum values of design parameters with literature [26]. Considerable percentage change in optimal design parameters values is also noted. It is observed that a decrease in value of 33.33% in $m_1$, 13.71% in $m_2$. Interestingly, there is considerable change of values noted in other parameters as well.

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>LITERATURE</th>
<th>SA</th>
<th>% CHANGE IN VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Module of stage 1- $m_1$ (mm)</td>
<td>2.25</td>
<td>1.50</td>
<td>+33.33</td>
</tr>
<tr>
<td>Module of stage 2- $m_2$ (mm)</td>
<td>3.5</td>
<td>3.02</td>
<td>+13.71</td>
</tr>
<tr>
<td>Face width of stage 1 , $b_1$ (mm)</td>
<td>46.77</td>
<td>38.20</td>
<td>+18.32</td>
</tr>
<tr>
<td>Face width of stage 2 , $b_2$ (mm)</td>
<td>67.10</td>
<td>33.58</td>
<td>+49.95</td>
</tr>
<tr>
<td>Diameter of the shaft of stage 1 , $d_{s1}$ (mm),</td>
<td>Not considered</td>
<td>21.18</td>
<td>-</td>
</tr>
<tr>
<td>Diameter of the shaft of intermediate stage, $d_{s2}$ (mm),</td>
<td>Not considered</td>
<td>44.29</td>
<td>-</td>
</tr>
<tr>
<td>Diameter of the shaft of stage 2 , $d_{s3}$ (mm),</td>
<td>Not considered</td>
<td>33.34</td>
<td>-</td>
</tr>
</tbody>
</table>

Fig. 4 Function value Vs Iteration by MATLAB fmincon solver

Fig. 5 Current point Vs Number of variables by MATLAB fmincon solver

Fig. 6 Comparison of the percentage change in optimal parameter values obtained by SA algorithm and literature
V. CONCLUSION

There is a 7.20% reduction in volume of spur gear drive achieved by SA as compared to literature. It is worthwhile to mention that this reduction in volume has been achieved after including six additional constraints on contact ratio, load carrying capacity, power loss, root cut not, no involute interference and line of action, which are extremely important in power transmission. It is also observed that there is a considerable percentage change value in all the optimal design parameters. There is a significant reduction in volume achieved (7.20%) for 5 kW power input. A decrease in value of 33.33% in $m_1$ and 13.71% in $m_2$ is also achieved. SA algorithm gives the best optimal result. Nature inspired algorithms SA, FA used in this work are computationally faster. Further this work is readily applicable and suitable for optimization of similar mechanical gears drives employed in industries.

REFERENCES


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