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Isomorphism and Automorphism in Closed Kinematic Chains

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Abstract: ***Effective mechanisms are the need of growing engineering world, the roots of which were found long back in the history of simple machines, but in today's scenario it changes the pace of development of any country by the use of robots, numeric control machines, automated transfer lines, space exploration programs defense systems etc. for this sound and effective mechanisms are required everywhere. Hence demand for the development new effective mechanisms increases day by day. At the starting phase in the designing of mechanisms structural requirements obtained from the functional requirements, but if there is some error in choosing the unique functional requirement to be converted into the structural requirement there may be a chance that the mechanism will not work or if it works it may be an isomer of some other kinematic chain in that situation it is a loss of effort, time of the designer and also the loss of money. The present work facilitates the designer to check the isomorphism and automorphism during the conceptual phase of designing the kinematic chain.***

Keywords: Automorphism; Isomorphism; Kinematic Chains; Mechanisms.

I. INTRODUCTION

The kinematic chain developed by the designer for obtaining the mechanisms must be unique meaning thereby that if two or more chains are already available with the same link assortment, the newly developed chain with the same link assortment will not be functionally same if it is functionally equivalent to any of the chains with the same link assortment the chains are the functional isomers. This problem in the design of kinematic chains is known as isomorphism Furthermore after sincere efforts of the designer if an isomer free kinematic chain is developed the next step is to develop the unique mechanisms from the kinematic chain. Ideally the number of mechanisms are equal to the number of links in a kinematic chain, but it can be a matter of chance that more than one mechanisms developed from the chain will be functionally equivalent i.e. the mechanisms are isomer of each other this phenomenon is known as automorphism which is essential to be known for getting the true count of distinct mechanisms obtainable from the developed kinematic chain.

The six link kinematic chains with the following link assortment {2(3)-4(2)} are namely Stephenson's and Watt's chain are not the isomer of each other but these chains exhibits automorphism phenomenon and it will be discussed in the subsequent sections.

The various methods for the isomorphism detection is available in literature as Davies and Crossley (1966) [1] have suggested the methods of visual inspection, which uses the expertise of the design engineer and the chains with a limited number of links can only be inspected by this method. Uicker and Raicu (1975) [2] devised the approach based on adjacency matrix and its characteristic polynomial obtained by the graphs of the kinematic chains, but this method needs large calculation time and later it was found by Mruthyunjaya (1987) [3] that the method of characteristic polynomial involves lengthy calculations also gives false test of the isomorphism for two sets of non-isomorphic kinematic chains having ten link as they are having same characteristic polynomials, which was claimed by Uicker and Raicu [2] as a condition for isomorphism. Mruthyunjaya (1984) [4,5,6] introduced binary coding for the kinematic chains. Rao and Raju(1991) [7] gives hamming number method which works satisfactorily without any failure. But, there are the cases where the primary hamming string does not work,it requires the secondary hamming string. Genetic algorithm is used by Rao (2000) [8] for isomorphism test and also for the obtainable distinct inversions of a chain, but it requires the fitness test up to third generation strings of the chains. Water flow analogy proposed by Sarkar and Khare (2004) [9] works for isomorphism detection up to ten links kinematic chains. A test for isomorphism which makes the use of Eigen value and eigenvectors of was presented by Cubillo and Wan [10] in 2005. Ding and Hufang [11] proposed some rules for relabeling the vertices canonically of topological perimeter graph and writing the canonical adjacency matrices for the kinematic chains for isomorphism detection but it is not a trivial task. In 2012 Yang et al [12] introduced the incident matrix method to test the isomorphism in kinematic chains, but it is difficult to form the incidence matrix. Sunkari and Schmidt [13] put a question mark on the ability of the available spectral methods for isomorphism detection in 2006. Rizvi et al devised a method for isomorphism which works on fuzzy similarity index in 2014 [14]. The phenomenon of automorphism is still to be a topic of investigation due to the less available literature

II. . MATRIX REPRESENTATION OF KINEMATIC CHAINS

Kinematic chains can be modeled mathematically by representing them by matrices.

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Various matrices are available in the literature like adjacency matrix, degree matrix, incidence matrix, joint-joint matrix etc. the author had also developed some matrices in his previous work chain identification matrix [15], link identity matrix [16], inversion adjacency matrix [17]. In this paper a modified form of chain identification matrix is used to determine isomorphism and automorphism in kinematic chains.

A. Chain identification matrix and link level modification

Chain identification matrix of the selected kinematic chain is obtained as explained below

a) $CI_{i,j} = 0$ when i^{th} and j^{th} link are adjacent links.
b) $CI_{i,j} = n$,where 'n' is the number of links connected to both i^{th} and j^{th} links commonly.
c) $CI_{i,j}$ = degree of that link, if i = j
d) $CI_{i,j} = 0$ if i^{th} and j^{th} link are not commonly connected to any other link of the chain.

The link level modification is done for obtaining a new link level matrix by changing the value of the diagonal element of the selected link to zero in the chain identification matrix; hence there are 'n' link level matrices available for each chain.

III. ISOMORPHISM AND AUTOMORPHISM DETERMINATION

Isomorphism among kinematic chains means that the two or more kinematic chains with same link assortment are also functionally same where as automorphism means a kinematic chain will have two or more functionally similar inversions. For determining the number of distinct inversions the link structural invariant **(*LSI*)** should be calculated for every link of the chain, by adding the absolute Eigen values of the modified chain identification matrix, the value of **"*LSI*"** for each distinct link is different and same for identical links, upon fixing identical links gives same type of mechanisms. The groups of identical mechanisms are termed as automorphs thus the numbers of distinct mechanisms are identified.

"The two kinematic chains are isomorphic if there is one to one correspondence between the links of the two kinematic chains".

The link correspondence between the two chains will be established by examining the values of link structural invariant ***(LSI)*** i.e. if the values of the **"*LSI*"** of both chains are same for their corresponding links; the two chains are isomorphic or otherwise. The chain structural invariant ***(CSI)*** is obtained by summing all the values of **"*LSI*"** non-isomorphic chains have distinct value of ***"CSI"*** and the same value depicts isomorphic chains.

IV. APPLICATION OF MODIFIED CHAIN IDENTIFICATION MATRIX

A. Applying the matrix for Watt's and Stephenson's chain

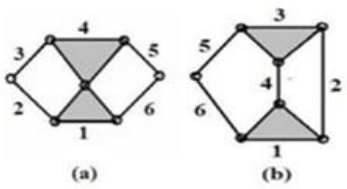

Fig.1.Watt's and Stephenson's chain

The CI matrices for both the chains shown in fig .1 are written as

$$\mathbf{CI1a} = \begin{vmatrix} 3 & 0 & 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 & 0 & 1 \\ 2 & 0 & 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 3 & 0 & 2 \\ 2 & 0 & 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 & 0 & 2 \end{vmatrix}$$

$$\mathbf{CIb} = \begin{vmatrix} 3 & 0 & 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 2 & 1 & 1 \\ 2 & 0 & 3 & 0 & 0 & 1 \\ 0 & 2 & 0 & 2 & 1 & 1 \\ 1 & 1 & 0 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 & 0 & 2 \end{vmatrix}$$

Modified **CI1a** matrix for link1 of the chain which is shown in fig.1 (a) is obtained by replacing the diagonal element

CI1a (1, 1) = 0.

$$\mathbf{CI1aL1} = \begin{vmatrix} 0 & 0 & 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 & 0 & 1 \\ 2 & 0 & 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 3 & 0 & 2 \\ 2 & 0 & 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 & 0 & 2 \end{vmatrix}$$

The Eigen values (E1) for Modified **CI1a** matrix for link1 i.e. **CI1aL1** matrix, are obtained by using the MATLAB and the value of the link structural invariant **"*LSI*"** is obtained for watt's chain shown in fig.1.(a) are

E1= (-1.7016 0.1716 1.0000 1.0000 4.7016 5.8284)
***LSI_1*= 14.4031**

Similarly the Modified **CI1b** matrices for other links in the chain are obtained by replacing the diagonal elements

CI1a (i, i) = 0 where i = 2,3,4,5 and 6 and the link structural invariants **"*LSI*"** and ***"CSI"*** are obtained as explained in section III.

***LSI_2*= 14.0000, *LSI_3* = 14.0000, *LSI_4*= 14.4031,**
***LSI_5*= 14.0000, *LSI_6*= 14.0000**

The chain structural invariant ***$CSI_{1\,(a)}$* = 88.7809**

Upon examining the ***LSI*** values of chain shown in fig-1(a) it is seen that the inversions obtained by fixing link 1 and link 4 exhibits automorphism and the inversions obtained by links 2, 3, 5 and 6 also exhibits automorphism which indicates that only two distinct mechanisms are possible from this chain.

Similarly the values of structural invariants **"*LSI*"** and ***"CSI"*** for Stephenson's chain shown in fig.1 (b) are obtained

***LSI_1*= 13.6221, *LSI_2*= 14.6528, *LSI_3*= 13.6221, *LSI_4*= 14.6528, *LSI_5*= 13.6091, *LSI_6*= 13.6091**.

The chain structural invariant ***$CSI_{1\,(b)}$* =83.7680**

Upon examining the ***LSI*** values of chain shown in fig.1 (b) the inversions obtained by fixing the links shows automorphism in the following pairs of inversions (1^{st} & 3^{rd}), (2^{nd} & 4^{th}) and (5^{th} & 6^{th}) therefore only three distinct mechanisms can be available for this chain.

The ***LSI*** values of kinematic chains shown in fig .1 (a) and (b) are not in one to one correspondence and the chain structural invariants ***$CSI_{1\,(a)}$*** and ***$CSI_{1\,(b)}$*** are different for both the kinematic chains which indicates that the both six link chains i.e. watts and Stephenson's chains with the link assortment **{2(3)-4(2)}** are non-isomorphic but exhibits automorphism.

B. Applying the matrix for ten link single degree of freedom kinematic chains

The CI matrices for both the chains shown in fig .2 are written as

$$\mathbf{CI2a} = \begin{vmatrix} 4 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 1 & 0 & 0 & 1 & 1 & 2 & 0 \\ 2 & 0 & 3 & 0 & 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 & 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 & 1 & 1 & 0 & 1 \\ 2 & 0 & 0 & 1 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 2 & 2 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 2 & 2 & 1 & 0 \\ 0 & 2 & 0 & 2 & 0 & 0 & 1 & 1 & 3 & 0 \\ 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 2 \end{vmatrix}$$

$$\mathbf{CI2b} = \begin{vmatrix} 4 & 1 & 1 & 0 & 0 & 2 & 1 & 1 & 1 & 2 \\ 1 & 3 & 1 & 2 & 0 & 0 & 1 & 1 & 2 & 1 \\ 1 & 1 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 2 & 1 & 3 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 & 1 & 1 & 1 & 0 & 1 \\ 2 & 0 & 0 & 1 & 1 & 3 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 2 & 2 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 2 & 2 & 1 & 0 \\ 1 & 2 & 0 & 1 & 0 & 0 & 1 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 & 1 & 0 & 0 & 0 & 1 & 3 \end{vmatrix}$$

The chain identification matrices **CI2a** and **CI2b** for the chains shown in fig.2 written above used to obtain the modified chain identification matrices for all links by replacing the diagonal elements **CI2a** (i, i) = 0, and **CI2b** (i, i) = 0 where i= 1,2,3,4,5,6,7,8,9 and 10 and the link structural invariants ***"LSI"*** and ***"CSI"*** are obtained as explained in section 3.The values of link structural invariants for chain shown in fig.2 (a) are

LSI_1**=25.5761,** LSI_2**=26.2323,** LSI_3**=26.3223,** LSI_4**= 25.6540,** LSI_5**=25.9032,** LSI_6**=25.5018,** LSI_7**=26.7044,** LSI_8**=26.7044,** LSI_9**= 26.4379,** LSI_{10}**=26.1850**.

The value of chain structural invariant of the chain shown in fig. 2(a) is $CSI_{2(a)}$ **= 261.2214**

The values of link structural invariants for chain shown in fig.2 (b) are

LSI_1**=25.5761,** $LSI2$**=26.4379,** LSI_3**=26.1850,** LSI_4**=25.6540,** LSI_5**=25.9032,** LSI_6**=25.5018,** LSI_7**=26.7044,** LSI_8**=26.7044,** LSI_9**=26.2323,** LSI_{10}**=26.3223.**

The value of chain structural invariant for chain fig.2(b) is $CSI_{2(b)}$ **= 261.2214**

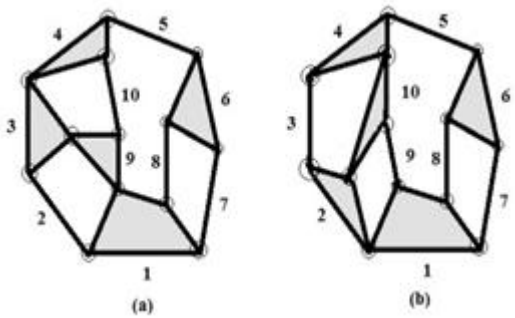

Fig. 2. A pair of isomorphic chain with ten links

The method discussed in this paper shows that chains shown in fig.2 (a) and (b) are isomorphic because the values of link structural invariants ***"LSI"*** are in correspondence with each other and the values of chain structural invariant (***CSI)*** is equal for the both kinematic chains.

Kong in1999 [18] proved that these chains are isomorphic but doesn't tell anything about automorphism.

The link structural invariant LSI_7 & LSI_8 have same values as 275.8184 which indicate the automorphism between the 7th & 8th inversions hence only nine distinct mechanisms are possible from these chains.

C. Applying the matrix for ten link three degree of freedom kinematic chains

A pair of 10-links 3-degree of freedom chains shown in fig.3, which are non-isomorphic and the characteristics polynomial approach fails while examining this pair of chains as indicated by He, P.R. & Li, Q. (2003) [19] and Mruthyunjaya and Balasubraminium (1987) [3].

$$\mathbf{CI3a} = \begin{vmatrix} 3 & 0 & 2 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 3 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 1 \\ 2 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 3 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 2 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 2 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 2 \end{vmatrix}$$

$$\mathbf{CI3b} = \begin{vmatrix} 2 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 3 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 1 \\ 2 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 3 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 3 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 2 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 2 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 2 \end{vmatrix}$$

the chain identification matrices **CI3a** and **CI3b** are written above for the chains shown in fig.3 are used to obtain the modified chain identification matrices for all links by replacing the diagonal elements **CI3a** (i, i) = 0, and **CI3b** (i, i) = 0 where i= 1,2,3,4,5,6,7,8,9 and 10 and the link structural invariants ***"LSI"*** and *"**CSI"*** are obtained as explained in section 3.The values of link structural invariants for chain shown in fig.3 (a) are

LSI_1**=23.8487,** LSI_2**=23.8670,** LSI_3**=24.5819,** LSI_4**=23.8670,** LSI_5**=23.5960,** $LSI6$**=23.9091,** LSI_7**=23.0710,** LSI_8**=23.9091,** LSI_9**=23.5960,** LSI_{10}**=23.4641.**

The value of chain structural invariant for chain shown in fig.3 (a) *is* $CSI_{3(a)}$ **= 237.7099**

The values of link structural invariants for chain shown in fig.3 (b) are

LSI_1**=24.5819,** LSI_2**=23.8670,** LSI_3**=24.5819,** LSI_4**=23.8670,** LSI_5**=22.5844,** LSI_6**=23.9091,** LSI_7**=23.0912,** LSI_8**=23.9091,** LSI_9**=22.5844,** LSI_{10}**=23.4641.**

The value of chain structural invariant for chain fig.3(b) is $CSI_{3(b)}$ **= 236.4401**

The structural invariant ***CSI*** for both the chains are different which indicates that the chains shown in fig. 3(a) and (b) are

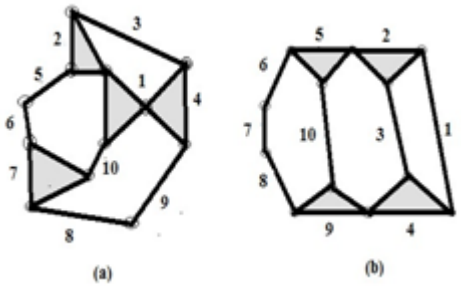

Fig.3 A pair of non-isomorphic ten link three degree of freedom chains

non-isomorphic the same result was obtained by He, P.R. & Li, Q.(2003) [19] and Mruthyunjaya and Balasubraminium (1987) [3]. For chain shown in fig.3(a) the structural invariants ***LSI*** shows the following sets of inversions shows automorphism (2 & 4), (5 & 9) and (6 & 8) hence only seven distinct mechanisms are possible from chain shown in fig.3(a).For chain shown in fig.3 (b) the structural invariants ***LSI*** shows the following sets of inversions shows automorphism (1 & 3), (2 & 4), (5 & 9) and (6 & 8) hence only six distinct mechanisms are possible from chain shown in fig.3 (b).

V. RESULTS AND DISCUSSIONS

The results of the present study are summarized in Table-I. To develop an inversion from a kinematic chain any one link is to be made ground and two other links are to be selected one as input link and the other link as output link. So each time selecting a different link as ground link will give the different inversion from that chain, meaning thereby from a kinematic chain the number of obtainable distinct inversion are equal to the number of links but it is not possible all times due to the presence of automorphs. As in the case of Watt's shown in the fig.1(a) contains six links but he distinct inversions obtained from this chain are only two not six, this is due to the presence of four automorphs and also in case of Stephenson's chain shown in fig.1(b) instead of six inversions it gives three distinct inversions and three automorphs. The table-I shows that the total automorphs of two six link chains are seven and the distinct inversions are five.

VI. CONCLUSIONS

The method proposed here is a reliable, simple and efficient method to check isomorphism and automorphism. The method has been found to be successful in checking isomorphism and automorphism in all known 8-links kinematic chains with 1-F which are 16 in number. All 40 chains with 9-links and 2-F are identified. 230 kinematic chains of 10-links, 1-F and 98 chains with 10 links having 3-F are also identified. The advantage is that by using MATLAB software Eigen values are very easy to compute. The CI matrices can be formed with very easily just by seeing the sketch of the chain. The method presented here very simple in nature and can be used to check automorphism and among the chains single and multi degree of freedom chains.

Table-I : Summary of results

S.No	No of links in the chain	No of chains	Degree of freedom	Distinct inversions	Automorphs
1.	6	2	1	5	7
2.	8	16	1	71	57
3.	9	40	2	254	106
4.	10	230	1	1834	466
5.	10	98	3	684	296

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