Conventional vs. Convolutional Windows for Reduction of Side Lobes

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Abstract: Convolutional windows analysis and performance comparison with traditional windows is main intent of the present paper. Convolutional windows are formed by convoluting the window by itself. These new class of windows are applied to a pseudo-LFM signal designed by using two stage piece wise linear frequency functions. Simulations were performed for the designed LFM signal with both the traditional and convolutional window functions are is observed that the convolutional windows yield better peak to side lobe level ratio (PSLR) values compared to traditional windows.

Keywords: Traditional, Convolution windows, LFM, PSLR

I. INTRODUCTION

Basically window is a mathematically limited function which exists within given interval and is zero valued anywhere else and is used to reduce the well known Gibbs oscillations caused by the abrupt truncation of a Fourier series [1]. A window function is a basic signal processing tool that is needed in many signal processing fields such as radar/sonar [2]. In most of these applications window function is assumed to have all the spectral power into extremely narrow band with zero sidelobes which is impossible both theoretically and practically [3]. Window function always has a mainlobe with sidelobes. No window is ideal and it should be selected based on the requirement of the application [4]. Thus, a method for designing window functions with flexible spectral characteristics is greatly needed [5]. Commonly used spectral characteristics of a window function include the mainlobe width (MW), the peak sidelobe level ratio (PSLR) and relative sidelobe attenuation which are closely related to the resolution and spectrum leakage [6]. The essential building block of pulse compression matched filtering is FFT. The FFT computation takes on the signal as periodic or repeats itself for each block of data. If the signal is non-periodic, it results in the leakage in frequency spectrum of the signal causing the spectrum to spread out over a wide frequency range which arise difficulty in identifying the exact frequency content of the measured signal [7]. So window function is employed to reduce the leakage factor. When a window function is multiplied to a non periodic signal it makes the signal to be periodic and suppress the side lobes to a certain extent [7]. Window weighting can be applied both in time/frequency domain, the former method is preferred to later as it produces low PSLR values [8]. Generally windows can be categorized into fixed and variable having all parameters fixed or variable. Designers must make trade-offs among the mainlobe width (MW), the peak sidelobe level ratio (PSLR) of windows by carefully adjusting these parameters [9]. Convolutional windows are formed by convolving the window with itself. High side lobe attenuation and flat top spectrum is obtained by time convolution of classical windows [10]. These windows are appropriate for harmonic amplitude evaluation in non synchronous sampling case. The convolutional windows from second to eighth order for rectangular window are derived in [11], [12]. If \( w(t) \) is the window function, \( n \)\textsuperscript{th} order convolution window is obtained by convolution of window by itself as given in Eq(1)

\[
W_n(t) = w(t)*w(t)*...*w(t)
\]

In this paper both the fixed and flexible convolutional window functions in time domain are used to reduce the side lobe values of the pseudo LFM signal designed using two stage piece wise linear frequency functions which are described below. Cauchy and power of cosine are variable parameter window functions and Papoulis and Parzen are fixed window functions.

1) Cauchy

\[
w(t) = \frac{1}{1+(\frac{t}{\alpha})^2} \quad 0 \leq |t| \leq \frac{T}{2}
\]

\[\alpha = 2.3, 4\]

2) Power of Cosine

\[
w(t) = \cos^p\left(\frac{\pi}{T}t\right) \quad n = \frac{(T-1) \cdot (T-1)}{2} \quad p = 2.5, 7
\]

3) Papoulis

\[
w(t) = \left(1 - \frac{t}{T/2}\right) \cos\left(\frac{\pi}{T/2}t\right) + \frac{1}{\pi} \sin\left(\frac{\pi n}{T/2}\right) \quad 0 \leq |t| \leq \frac{T}{2}
\]

4) Parzen

\[
w(t) = \begin{cases} 
1 - \theta \left(\frac{T}{T/2}\right) \left(1 - \frac{|t|}{T/4}\right) & 0 \leq |t| \leq \frac{T}{4} \\
2 \left(1 - \frac{|t|}{T/2}\right)^3 & \frac{T}{4} \leq |t| \leq \frac{T}{2}
\end{cases}
\]
II. PSEUDO-LFM SIGNAL

A pseudo LFM signal is designed and convolutional window functions are applied to this signal to reduce the autocorrelation (ACF) sidelobes. In this paper, pseudo LFM signal is generated using simple two-stage piece wise linear functions which are described below.

\[
F(\tau) = \begin{cases} 
\alpha_0 \tau & 0 \leq \tau \leq T_1 \\
B_1 + \alpha_1(\tau - T_1) & T_1 \leq \tau \leq (T_1 + T_2) 
\end{cases} \tag{2}
\]

Equation (2) shows the instantaneous frequency variation of modified LFM signal formed by concatenating two piece wise LFM functions with a sweep rate of \(\alpha_0\) in the first stage and \(\alpha_1\) in the second stage. The total pulse width of the chirp signal \(\tau\) is divided into two time slots with respective pulse widths \(T_1\) and \(T_2\). If \(B_1\) and \(B_2\) are the corresponding bandwidths of the first and second stage LFM functions, then the corresponding sweep rates can be defined as

\[
\alpha_0 = \frac{B_1}{T_1}, \quad \alpha_1 = \frac{B_2}{T_2}
\]

The corresponding phase variation of this concatenated NLFM function can be obtained by integrating (2)

\[
\phi(\tau) = \int F(\tau) \, d\tau = \begin{cases} 
\alpha_0 \frac{\tau^2}{2} & 0 \leq \tau \leq T_1 \\
B_1 \tau + \alpha_1 \left(\frac{\tau^2}{2} - T_1 \tau\right) & T_1 \leq \tau \leq T_1 + T_2 \end{cases} \tag{3}
\]

III. SIMULATION AND RESULTS

Simulations were done for \(B = 20\) MHz and \(T = 10\) µs with different combinations of \(T_1, B_1, T_2,\) and \(B_2\). All the possible combinations are examined by choosing different sweep rates. Amongst all the combinations, Fig. 1 shows the frequency variation of two-stage PWLFM function which achieved highest PSLR of -26.26 dB at the output of matched filter (MF) as shown in Fig. 2. This is achieved at specific values of \(B_1=2.24\) MHz, \(T_1=2.71\) µs, \(B_2=17.76\) MHz and \(T_2=7.29\) µs.

Fig. 2. MF output of pseudo LFM function

Fig. 3 to 5 shows the MF output of pseudo LFM signal with traditional and convolutional power of cosine window for different values of \(p\). It is clearly evident from the below figs that there is a significant reduction in PSLR values when convolution window is applied so as from -32dB to -91.12dB.

Fig. 3. MF output with power of cosine (\(p=2\)) window

Fig. 4. MF output with power of cosine (\(p=5\)) window
Fig. 6 to 8 shows the MF output of pseudo LFM signal with traditional and convolutional Cauchy window for different values of α. It is clearly evident from the below figs that the PSLR values are reduced when convolution window is applied so as from -37.08 dB to -57.05 dB.

Fig. 8. MF output with Cauchy (α=5) window

Fig. 9 and 10 shows the MF output of pseudo LFM signal with traditional and convolutional Papoulis and Parzen window. Table I summarizes the PSLR values of all the windows used.
Traditional vs. Convolutional Windows for Reduction of Side Lobes

<table>
<thead>
<tr>
<th>Windows</th>
<th>PSLR with Traditional windows</th>
<th>PSLR with Convolutional windows</th>
</tr>
</thead>
<tbody>
<tr>
<td>Papoulis</td>
<td>-41.56</td>
<td>-58.72</td>
</tr>
<tr>
<td>Parzen</td>
<td>-43.79</td>
<td>-62.89</td>
</tr>
<tr>
<td>Cauchy</td>
<td>(\alpha = 3)</td>
<td>-37.08</td>
</tr>
<tr>
<td></td>
<td>(\alpha = 4)</td>
<td>-50.28</td>
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<tr>
<td></td>
<td>(\alpha = 5)</td>
<td>-55.36</td>
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<tr>
<td>Power of Cosine</td>
<td>(p = 2)</td>
<td>-32</td>
</tr>
<tr>
<td></td>
<td>(p = 5)</td>
<td>-56.44</td>
</tr>
<tr>
<td></td>
<td>(p = 7)</td>
<td>-74.1</td>
</tr>
</tbody>
</table>

### IV. CONCLUSION

Convolutional windows are applied to pseudo LFM signal designed using two stage piece wise linear functions to reduce the side lobes and is compared with the traditional windows. In this paper fixed and variable parameter window functions are used. From the simulation results it is evident that convolutional windows reduce PSLR values better than the traditional widows. Amongst all the window functions used variable parameter power of cosine window yielded maximum PSLR of -91.12dB.

### REFERENCES