

# A New Cost Effective and Reliable Interconnection Topology for Parallel Computing Systems



Pradyumna Kumar Tripathy, Ranjan Kumar Dash, Chitta Ranjan Tripathy

**Abstract:** Topology of the of interconnection network is one of the most important considerations in the design of parallel systems as it is the backbone network over which the different components of the computer communicate with each other. The properties of the topology such as connectivity, reliability, cost, fault tolerance, diameter and bisection width determine the flawless-less data transmission between the source and the sink nodes. In this paper, we propose a new hybrid interconnection network topology called TOR-CUBE (TC) which is a product of two classical popular interconnection topologies namely hypercube and torus. Further, we show the construction and characteristics of the proposed interconnection topology. We also presented some basic important properties and formulated two routing algorithms for TC. Our results show that the proposed interconnection topology has high connectivity, lesser diameter, low cost, fault tolerant, scalable and low average distance. The different reliability measures of TC are computed and found to be better as compared with its counterpart topologies and the parent topologies.

**Keywords :** Interconnection Topology, Parallel Systems, Reliability, Routing Function

## I. INTRODUCTION

The interconnection network plays a vital role in the design of distributed systems as it defines the means of data exchange among many standalone processing units. The interconnection network also presents the underlying architecture i.e. topology used for these systems. Hence, the performance of these systems mainly depends on their interconnection network. Thus, in order to design a highly reliable distributed system, it must be ensured that the used interconnection network should be highly reliable i.e. it should work satisfactorily for a specific period of time even occurrence of some failures like link and/or computing units.

Further, the use of the interconnection network in these systems should be economical. One way to measure the cost of the interconnection network is the product of its degree to diameter. In other words, it can be said that highly reliable as well as economical interconnection network must be deployed to design such systems. Some examples of Interconnection network include Hypercube and its variants, Torus, Mesh, etc. The hypercube (HC) has been used in [1-2, 20, 28-31] due to its properties like small diameter, strong connectivity regularity, symmetry, recursive construction, partitionability, fault tolerance, and reliability. Many variants of hypercube have been proposed in the past either to enhance its reliability or to decrease its cost. The variant includes cube connected cycles [23], twisted hypercube [3], folded hypercube [5], crossed cube [6, 8], exchanged hypercube [4, 41], incomplete crossed cube [5], banyan hypercube [15], etc. The hypercube is also a good choice of researchers to propose many hierarchical interconnection networks viz. hierarchical hypercubes, hierarchical crossed cubes, hierarchical cube networks, extended hypercube, etc. [30]. However, the main problem associated with hypercube and its variants is their implied constraint to scale. The prominent topologies to provide a high degree of scalability are Mesh and torus [24-27]. These networks are found to be used widely in the area of scientific calculations, flow dynamics, structural analysis, etc. However, the large diameter of mesh network prohibits its use in the design of a massively parallel computer system [9-10]. In addition to this, high cost and difficulty in set-up and maintenance of this network further make it difficult to implement in real life parallel machines. In comparison to mesh, Torus has high connectivity and more reliable. Hence, it is used in the design of various parallel machines such as Alpha 21364-based HPGS 1280 [4], Cray XT3 [11] and XT4 [12], X1E [13], Cray X1-E vector computer [14]. Moreover, IBM Blue Gene family of massively parallel computers are some remarkable examples of mixed Radix 3D Tori [30-32]. In order to decrease very large diameter of the torus, a new class of recursively structured torus connections networks called recursive diagonal torus (RDT) has been proposed in [22]. The discussion carried out so far reveals that the torus has a fixed degree but has a relatively large diameter ( $2n$ ). While, the diameter of the hypercube is comparatively small, but its node connectivity increases logarithmically with the size of the network. Under such conditions taking the product of two standard topologies is a potential process of constructing a new interconnection network [16-19].

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\* Correspondence Author

**Pradyumna Kumar Tripathy\***, Dept. of Computer Science and Engineering, Silicon Institute of Technology, Bhubaneswar, India. Email: pradyumnatripathy@gmail.com

**Ranjan Kumar Dash**, Dept. of Information Technology, College of Engineering and Technology, Bhubaneswar, India. Email: drranjandash@gmail.com

**Chitta Ranjan Tripathy**, Dept. of Computer Science and Engineering, VSSUT, Burla, India. Email: cr.tripathy1@gmail.com

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The cross product of interconnection networks is observed to be outperforming traditional topologies with different structural properties [11]. Hence, the problem of designing a scalable, cost effective, highly reliable, fault tolerant network with the lower diameter and strong connectivity with less hardware complexity is yet a challenge. This motivates us to propose a new hybrid interconnection network called TOR-CUBE (TC) which inherits the properties of two popular interconnection topologies: hypercube and torus. The proposed topology (TC) exploits most of the important properties of hypercube and torus. The present work focuses and analyzes the different design factors of TOR-CUBE (TC) and compares its topological properties with that of other important interconnection topologies.

## II. PROPOSED INTERCONNECTION TOPOLOGY: TOR-CUBE (TC)

The proposed interconnection network uses an n-dimensional hypercube as its base. Additional links from a torus interconnection network are added to the base hypercube interconnection network to form an n-dimensional TOR-CUBE. e.g. For a 3D TC, first a 3D hypercube (HC) is taken (Fig. 1), and then extra 4 links from the torus interconnection topology are added to it. The extra links are connected to the adjacent opposite nodes in torus pattern (Fig. 2). The proposed topology is scalable and therefore is most suitable for large scale parallel processing (Fig. 3).

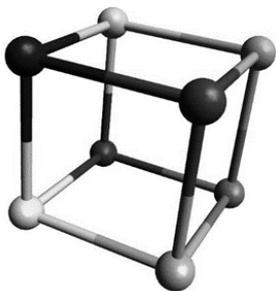


Fig. 1

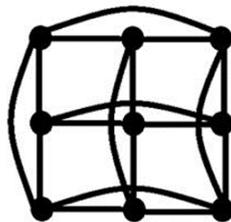


Fig. 2

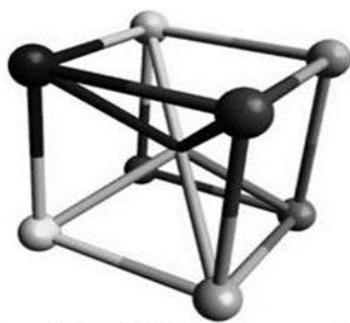


Fig. 3

Fig. 1: 3-D Hyper Cube Topology

Fig. 2: 3-D Torus Topology

Fig. 3: 3-D TOR-CUBE Topology

## III. TOPOLOGICAL PROPERTIES OF TOR-CUBE INTERCONNECTION NETWORK

The TOR-CUBE is viewed as an undirected graph  $G(V, E)$ , where  $V$ , is the node set representing the processing

elements and  $E$  represents the set of edges representing the connecting links between the processing elements.

The different topological properties of TOR-CUBE are presented in this Section.

### A. Nodes

The number of nodes of n-dimensional TOR-CUBE is  $2^n$ .

### B. Links

A link denotes a communication edge between two processing elements in a network. The nodes  $u$  and  $v$  are said to be adjacent if a link  $l = (u, v) \in E$ .

**Theorem1:** If  $L_n$  is the total number of links of TOR-CUBE having dimension 'n' then the  $L_n$  can be computed from the following recursive relation.

$$\begin{cases} L_n = 2 \times L_{n-1} + 2^{n-1} & ; \text{ for } n > 3 \\ L_n = 2^{n+1} & ; \text{ for } n = 3 \end{cases}$$

*Proof:* For the dimension  $n = 3$ , the number of links of the TC is  $2^{n+1}$ . However, the number of additional links that have to be added to scale the TC towards its next dimension is  $2^{n-1}$ . Again, for every increase in dimension, the number of links are doubled which leads to the total number of links of TC to be computed as  $2 \times L_{n-1} + 2^{n-1}$ .

### C. Degree

The degree of an interconnection network is the degree of the node which has a maximum number of links connected to it. The degree of TOR-CUBE is  $(n + 1)$  since every node is connected to exactly  $n + 1$  of its neighbor nodes.

### D. Diameter

The diameter of graph  $G(D(G))$ , is the maximum of the minimum distances between any two distinct vertices  $d(u, v)$  of  $G$ . The is measured in terms of a number of distinct hops between any two nodes. It determines the number hops a node will take to broadcast a message to all other nodes in the network. Therefore, it should be kept small.

**Theorem4:** The diameter of the TOR-CUBE is  $(n - 1)$ .

*Proof:* Every node can communicate with every other node in  $n - 1$  hop distance only. For example, for the dimension  $n = 3$ , every node can communicate with its four other adjacent nodes leaving the rest three nodes unvisited. The remaining nodes can be communicated with one more hop distance. For  $n > 3$ , every node will take the same number of hops within the TC and one more hop in between the TC.

### E. Cost

For a symmetric interconnection network, the cost is defined as the product of the degree and the diameter of the network. This is an important factor is broadly used in performance evaluation.

Mathematically,

$$\text{Cost} = \text{degree} \times \text{diameter}$$

Therefore, the cost of TOR-CUBE (TC) can be computed as,

$Cost = (n + 1)(n - 1) = n^2 - 1$ , where  $n$  is the dimension of the network.

For example, a TC of dimension  $n = 3$  has the cost computed to be 8, whereas it is 9 in the case of Hypercube and 8 in case of Torus.

However, the number of links in Torus is much larger as compared to TC. So, the hardware complexity reduces with a decrease in a number of links with TC as compared to torus by keeping the cost in-tact.

**F. Bisection Width**

It is defined as the minimum number of links, the removal of which will cause two distinct and equal sub networks.

**Theorem5:** The Bisection width of the TOR-CUBE is

$$\begin{cases} 2^n & ; \text{if } n = 3 \\ 2^{n-1} & ; \text{if } n > 3 \end{cases}$$

*Proof:* When  $n = 3$ , minimum  $2^n$  number of augmented edges are to be removed to divide the network into two distinct sub-networks. However, when  $n > 3$ , the minimum number of links that are required to be removed are the links connecting the TC with dimension  $(n - 1)$ . Therefore, the bisection width can be computed as  $2^n$  if  $n = 3$  and  $2^{n-1}$  if  $n > 3$ .

**G. Average Distance**

In practice, the average distance describes the performance of the interconnection network. The average distance of the interconnection network is the summation of distances of all nodes from a specified node over the entire number of nodes. The average distance of an interconnection network is mathematically represented as:

$$\frac{\sum_{i=1}^N \text{distance from node } i \text{ to node } j}{\text{Total number of nodes}}$$

**Theorem7:** The average distance of the TOR-CUBE is

$$\frac{1}{2^n} [(l_1)_n + \sum_{k=2}^{n-1} (k \times (l_1)_n)]$$

*Proof:* When  $n = 3$ , four of the nodes of the TC is reachable in one hop distance and the other three of the nodes are reachable in two hop distances. Likewise, if  $n = 4$ , five nodes would be reachable in one hop distances, the other seven can be achieved in two hop distances and the next three can be achieved in three hop distances. From the above results, the following sequence can be inferred in tabular form (Table-I) where the  $l_i$  represents the  $i^{th}$  hop distance.

Table-I: Hop-wise distance for different dimensions of TC

Dimensions/ Hops	n=3	n=4	n=5	n=6	n=7
$l_1$	4	5	6	7	8
$l_2$	3	7	12	18	25
$l_3$		3	10	22	40
$l_4$			3	13	35
$l_5$				3	16
$l_6$					3

From the Table-I it can be deduced that,  
 $(l_1)_n = n + 1$  and  $(l_k)_n = (l_k)_{n-1} + (l_{k-1})_{n-1}$   
For example, for the dimension  $n = 3$ ,

$$\begin{aligned} (l_1)_3 &= 4 \text{ and } (l_2)_3 = 3 \\ \text{So, the average distance} &= \frac{(l_1)_n + (l_2)_n + \dots + (l_{n-1})_n}{2^n} \\ &= \frac{1}{2^n} [(l_1)_n + \sum_{k=2}^{n-1} (k \times (l_1)_n)] \end{aligned}$$

The average distance so computed for the TC is also less as compared to the HC. As the distance between any two nodes in TC is less as compared to HC and the total number of nodes in the TC is the same as HC, hence, the average distance of TC is less than the HC as well.

**H. Message Traffic Density**

An effective interconnection network must have extensive bandwidth to incorporate the consequential traffic in order to keep the traffic density minimum. If every node is sending message to another node at distance 'd' apart, then it is required to analyze the performance of the network in managing the message traffic. The message traffic density is defined by  $\frac{\text{Averagedistance} \times \text{Number of nodes}}{\text{Total number links}}$ .

So, the Message traffic density of TOR-CUBE is given by 
$$\frac{[(l_1)_n + \sum_{k=2}^{n-1} k((l_1)_n)]}{2l_{n-1} + 2^{n-1}}$$

**IV. ROUTING IN TOR-CUBE**

The routing process is to select paths to send a message from a designed node to any other destination node in a network. The message is forwarded from the source node to a neighboring node one step closer to the destination and the process is repeated till it reaches the destination node. The routing path must be minimal, adaptive, starvation free and deadlock-free [21]. For constructing a 3D TC, a 3D-hyper cube of 8 nodes is taken and then 4 additional links are added to it, as it is there in a 3D-torus topology. Every node will communicate to its first three neighbors according to a conventional 3D hypercube routing algorithm. Along with the above three neighbors, every node can also communicate to one or more additional adjacent nodes considering the following routing algorithm.

**A. Routing Algorithm**

The algorithm consists of scanning the opposite node of the source node i.e.  $opp(N_i)$  where  $N_i$  is the source node. Then, it searches the adjacent of the respective opposite node of the source node i.e.  $adj(opp(N_i))$ . The searching is terminated after reaching the destination node.

**Proposed Routing Function**

The routing function of TC is given as:

$$[n_i \leftrightarrow adj(opp(N_i))] \text{ for } i = 0, 1, 2, 3 \dots 2^n - 1$$

Function  $opp(N_i)$

1. Find out the value of  $K$  for which  $4k \leq i \leq 4(k + 1)$  Condition holds
2.  $i \in [4 \times k, 4 \times (k + 1)]$
3. if  $i + 2 < 4(k + 1)$  then  $return(N_{i+2 \bmod N})$  else  $return(N_{i-2 \bmod N})$  end



end

Function  $adj(N_i)$

return  $N_{i+4}$

end

**B. Explanation of the Algorithm**

The stepwise illustration of the above-mentioned routing algorithm is presented below:

1. When,  $N_0=0$ , the following condition holds only for the value of  $k=0$ ,  $4k \leq i \leq 4(k + 1)$
2.  $i+2=0+2=2$
3. So, the function  $opp(N_0)$  returns 2 as the opposite node of 0.

Thus,  $opp(N_0) = 2$

Similarly, the function  $adj(opp(N_0))$  returns 6 i.e.

$adj(opp(N_0)) = 6$

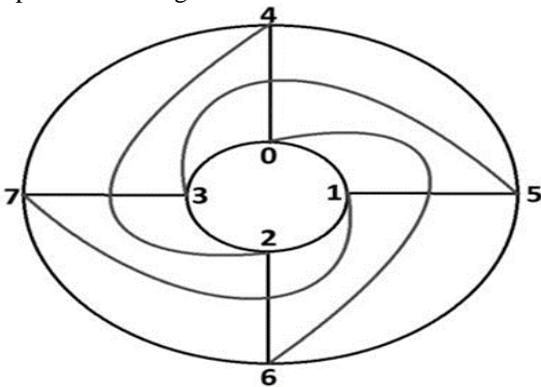
$opp(N_0) = 2, adj(opp(N_0)) = 6$

So, node 0 can communicate directly with nodes 1, 3, 4, 6.

Similar calculations for the rest of the node can be performed which is summarized below:

Node	Nodes directly connected
0	1, 3, 4, 6
1	0, 2, 5, 7
2	1, 3, 4, 6
3	0, 2, 5, 7
4	0, 2, 5, 7
5	1, 3, 4, 6
6	0, 2, 5, 7
7	1, 3, 4, 6

From, the above results, the spiral view of a 3D-TOR-CUBE is presented in Fig.4.



**Fig. 4: Spiral View of 3-D TOR-CUBE**

Fig. 4 shows a spiral view of a 3D-TOR-CUBE. From Fig. 4 it is quite clear that node 0 can communicate with node 1, 3 and 4 as it is in case of a hypercube. However, unlike hypercube, node 0 can communicate with one more adjacent node directly by using the proposed routing algorithm. Here, in the example, the opposite of node 0 is 2 and the adjacent of node 2 is node 6. Therefore, node 0 will directly communicate with node 6. Similarly, the node opposite to 1 is 3 and the adjacent of node 3 is node 7. Therefore, node 1 can directly communicate with node 7 along with the nodes 0, 2 and 5. In a similar fashion, every node can communicate with all its four neighboring nodes.

**V. BROADCASTING IN TOR-CUBE**

In a parallel system, broadcasting is a method of transferring a message to all nodes simultaneously. It's an All-to-all communication in which every sender transmits

messages to all its adjacent receivers within a group. It is the most general communication method in which, the transmitted packet will be received by every node on the network. It may be performed as all-scatter or all-broadcast where, each node performs its own scatter depending on the messages are distinct or same.

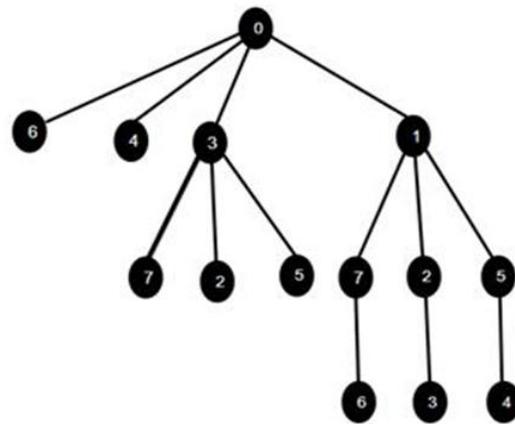
**A. Broadcast Algorithm**

The broadcast procedure of TC is presented below where  $u$  is the source node and  $n$  is the dimension of the network.

**Broadcast Algorithm( $u, n$ ):**

- Step 1.  $v_i = nebh(u)$ , where,  $i = 1, 2, 3, 4$  (since each node has 4 neighbor),  $nebh(u)$  generates the nodes which are neighbor of node  $u$
- step2. For each  $v_i$
- Step 2.1. Send message from  $u$  to  $v_i$
- Step 3. For each  $v_i$
- Step 3.1. Send message from  $v_i$  to neighbor ( $v_i$ )
- Step 4. Continue Step3 until all the nodes receive the message.
- Step 4. End

The broadcasting procedure of TOR-CUBE is presented in Fig. 5 where the starting node is considered as 0.



**Fig. 5: Broadcast view of 3-D TOR-CUBE**

**B. Illustration**

Let us take an example of broadcasting of a message in a 3D-TOR-CUBE. The message is broadcast from the source node 0 to all other nodes in the network (Fig. 5). The nodes that are reachable from the source node 0 are nodes 1, 3, 4 and 6. Further, node 1 after getting the message can broadcast it to its all other 3 neighbors nodes i.e 2, 5 and 7. Similarly, the other nodes adjacent to node 0, will also send the message to their neighbor nodes in the same step. In the next step, all the nodes in the network will get the message with only 3 hop distances.

**VI. RELIABILITY ANALYSIS OF TOR-CUBE**

Reliability is the probability that a system should perform all its required functions over a period of time under some predefined constraints.



It represents the probability of success. Network reliability is the probability that all the nodes of the network are connected at least through one path.

Here, all the nodes and links are considered to be identical with their failure rates statistically independent and exponentially distributed.

For simplicity, both the node reliability and the link reliability are assumed to be 0.9 for estimating the different reliability measures of the TOR-CUBE. The network reliability of TOR-CUBE for  $n = 3$  is computed to be 0.9573384 and for  $n = 4$  network reliability is found to be 0.946593. Similarly, the two-terminal reliability of 3D and 4D TOR-CUBEs are computed to be 0.9874081 and 0.9793002 respectively. Considering the k-set to be {1, 3, and 4}, the k-terminal reliability of 3D and 4D TOR-CUBEs are

evaluated to be 0.9813806 and 0.9779117 respectively. The graphs are plotted between mission time and different reliability measures viz. network reliability, two-terminal reliability, and k-terminal reliability by taking different interconnection networks into consideration.

From Fig. 6, Fig. 7 and Fig 8, it is quite clear that the proposed topology TC yields better reliability value in all the cases as compared to other regular topologies. The reliability value of Torus is almost near to the TC; however, the number of links, cost, diameter and an average distance of TC are less as compared to the Torus.

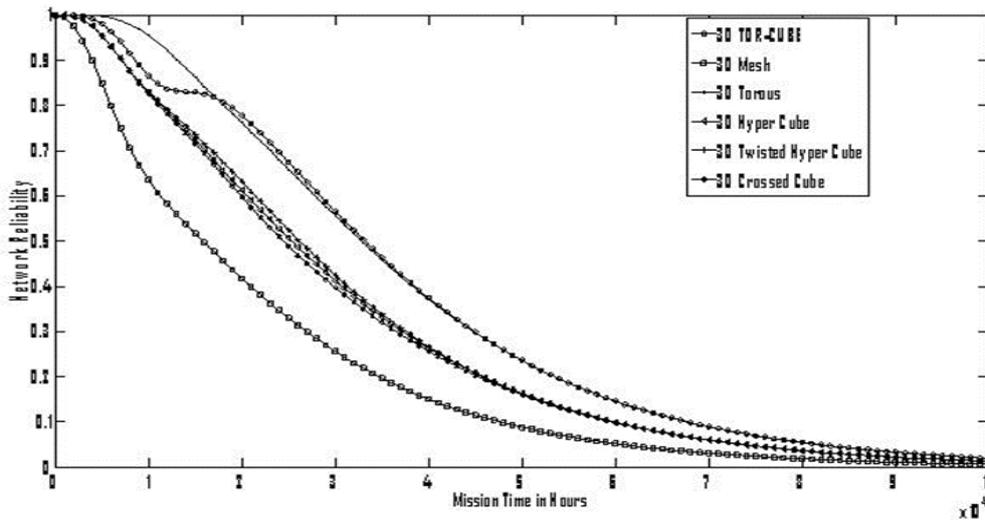
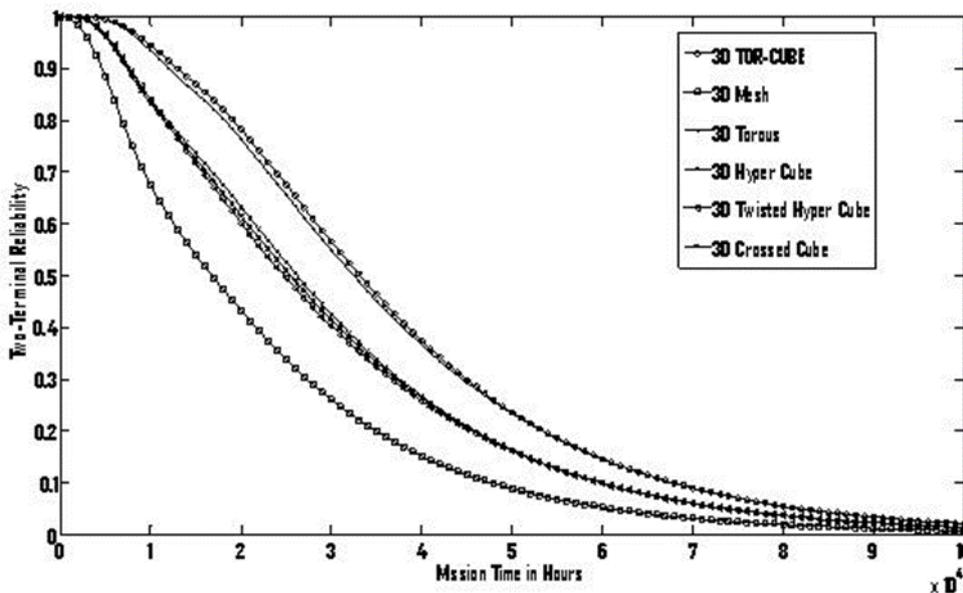


Fig. 6: Network Reliability Vs Mission Time for  $\lambda = 0.0005$



Two-Terminal Reliability Vs Mission Time for  $\lambda = 0.0005$

Fig. 7:

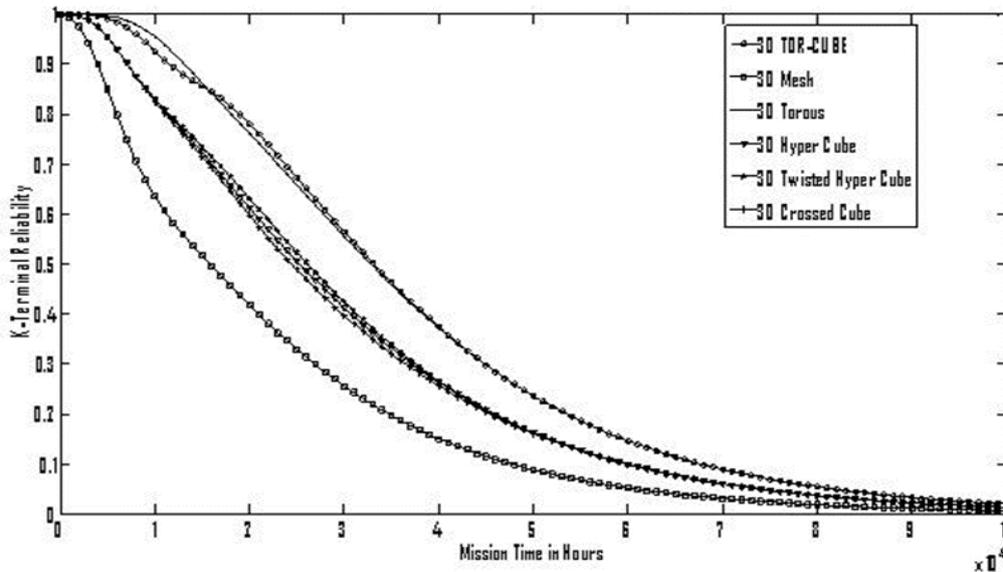


Fig. 8: K-Terminal Reliability Vs Mission Time for  $\lambda = 0.0005$

VII. FAULT TOLERANCE IN TOR-CUBE

The performance improvement can be obtained if the interconnection network is more fault tolerant in nature. The fault tolerant attribute of an interconnection network ensures that the network can able to deliver all its required services in-spite of some failures of components in the system [7]. It may be static or dynamic in nature. A k-fault tolerant system is a system which can tolerate up to k-number of link failures without hampering the network performance. A network is considered to be disconnected if any of its node is isolated. It purely depends on the degree of the network or the connectivity of the network.

In TC, for all the processing elements, the node degree is  $(n + 1)$ . This indicates that the network can tolerate up to  $n$  -faults.

A. Fault Diameter of TOR-CUBE

In a parallel system, one or more of links/nodes may become out of order over the time, resulting in a new layout or topology. This new topology has modified diameter referred to as fault-diameter often denoted as  $f_d$ , where  $d$  represents the number of defective links associated with a specific node. Therefore, if the degree of the interconnection network is  $d$ , then  $f_d = \infty$ , as the node is inaccessible from the rest of the nodes of the network. The fault diameter must be close to the original diameter.

**Theorem 8:** For a TOR-CUBE  $(n, l)$ , the fault diameter  $f_d$  is given by  $f_d = n$

*Proof:* In  $TC(n)$ , a message originating at any node can travel through  $(n + 1)$  paths. In case, link failure occurs, the message travels through one more node. This results in an increase in diameter by unity.

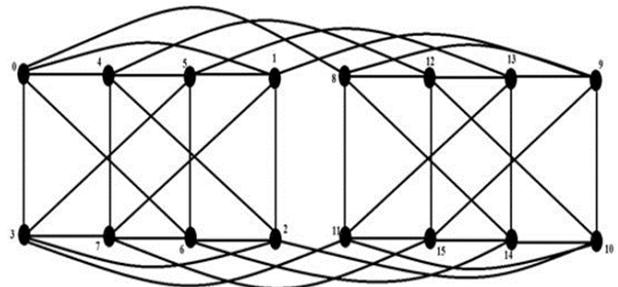
So, the *Diameter of the fault network* = Original diameter + 1 =  $n$

VIII. EXTENSIBILITY OF TOR-CUBE

The extensible property of TOR-CUBE is the same as the hypercube and can be extended with an increase in dimension without disturbing the construction of the basic module. The node connectivity of TC is increased by one for every extension of dimension. A 4D-TC can be built by joining two 3D-TOR-CUBEs at their respective node positions as in case of the hyper cube (Fig. 9). Therefore, the extensible property is very flexible as it is there in case of the hyper cube.

Fig. 9: A 4D TOR-CUBE by extending the 3D TOR-CUBE

A. Illustration



Let us take an example: (Refer Fig. 7)

For  $n = 3$ , the number of 3D-TC =  $n - 2 = 1$

For  $n = 4$ , the number of 3D-TC =  $n - 2 = 2$

For joining the two 3D-TCs, the links to be added between the two cubes are as follows

$(N_i \leftrightarrow N_{i+8})$  for  $i = 0, 1, 2, \dots, 7$

Similarly,  $n = k$ , The links required between the node sets can be formulated as  $(N_i \leftrightarrow N_{i+2^n})$ .

Fig. 9 shows the extensible property TOR-CUBE. Here, a 4D-TC can be constructed by extending a 3D-TC.

The corresponding nodes are joined by taking two 3D-TCs. The links that are connected between the nodes are (0,8), (1,9), (2,10), (3,11), (4,12), (5,13), (6,14) and (7,15).

**IX. RESULTS AND DISCUSSIONS**

This section evaluates different parameters of TOR-CUBE (TC) and a comparison is made with other networks. Different topological properties are compared in Table-II. The parameters that are considered for comparison are degree, diameter, cost and bisection width. From the Table-II, it can be observed that the degree of TC is more as compared to a hyper cube, crossed cube, dual cube and folded dual cube. The mesh connected computer, torus, and cube-connected cycles have a constant degree. More the degree reflects, the better is the connectivity among the nodes or the processing elements. Folded hypercube has the same degree as TC. The diameter of a network determines the shortest path between any two vertices. Therefore, a network with fewer diameters is preferable. As far as the diameter is considered, the TC offers the shortest diameter as compared to the many other kinds of networks like hypercube, mesh connected computer, torus, connected cycles, and Dual cube. Although the folded dual cube has the same diameter as TC, the degree of folded dual cube is less as compared to that of the TC.

The degree, diameter, and cost are the three desirable and important properties of any interconnection network, where one always needs a higher degree network with the low diameter and low cost. From the Table-II, it can be observed that the proposed topology TOR-CUBE offers the lowest cost as compared to many kinds of the network such as hypercube, mesh-connected computer, torus, and cube connected cycles. Similarly, while comparing the bisection width, the proposed topology TOR-CUBE requires  $2^n$  number of links to be discarded to make the network disconnected for dimension 3 and  $2^{n-1}$  number of links to be discarded for  $n > 3$ . The proposed network is more reliable than the parent networks as it has more node disjoint paths with increase values of n. The average distance and fault tolerance of the proposed network TC are also found better compared to its parent networks.

The cost of different interconnection networks mentioned in Table- II are calculated by multiplying their degree and diameter and shown in Table- III

Table-II: Comparison of topological properties ( $N$  is the number of nodes and  $n$  is the dimension of the networks)

Networks	No. of Nodes	Degree	Diameter	Bisection Width
Hypercube	$2^n$	$n(=\log_2 N)$	$n(=\log_2 N)$	$2^{n-1}$
Folded hypercube		$n + 1$	$\lfloor \frac{n}{2} \rfloor$	$2^{n-2}$
Mesh-connected computer (N XN)	$N^2$	4	$2(\sqrt{N} - 1)$	$\sqrt{N}$
Torus (N XN)	$N^2$	4	$\sqrt{N}$	$2\sqrt{N}$
Cube-connected cycles		3	$O(\log N)$	$O(N/\log N)$
Dual cube		$\frac{(n+1)}{2}$	$n + 1$	$2^{n-2}$
Folded dual cube		$\frac{(n+3)}{2}$	$n - 1$	$2^{n-1}$
TOR-CUBE	$2^n$	$n + 1$	$n - 1$	$2^n$ for $n = 3$ $2^{n-1}$ for $n > 3$

From the Table -III, it can be observed that Folded hypercube, Dual cube, Folded Dual cube and TOR-CUBE have minimal cost i.e. 8.

A graph between degree and dimension of different kinds of networks are plotted and presented in Fig.10. It shows that the degree of the proposed interconnection network TOR-CUBE is more than its parent topologies and also more than other some other important network topologies. The crossed cube has the same degree as hypercube, mesh connected network has the same degree as Torus, for which they are not included in the graph. As the degree of TC is found to be more, therefore, the node connectivity, fault

tolerance, and reliability of the network increases. Similarly, a graph is also plotted between diameter and dimension considering different network topologies and presented in Fig. 11. The figure shows that the TC has a lesser diameter as compared to its parent topologies and also lesser from some other important network topologies. Folded dual cube has the same diameter as TC and has not been plotted in the graph. As the TC offers lesser diameter, therefore reaching of packets from any node to any other node will be faster which yields in routing and broadcasting even simpler.

Table III: Comparison of cost of different popular interconnection networks

Networks	Cost	Example
Hypercube (HC)	$n^2$	Degree of 3D HC=3 Diameter of 3D HC= 3 Cost= degree X diameter=9
Folded hypercube (FHC)	$(n + 1) \times \left\lfloor \frac{n}{2} \right\rfloor$	Degree of 3D FHC=4 Diameter of 3D FHC= 2 Cost= degree X diameter=8
Mesh-connected computer (N XN) (MCC)	$8(\sqrt{N} - 1)$	Degree of 3D MCC=4 Diameter of 3D MCC= 6 Cost= degree X diameter=24
Torus (N XN)	$4\sqrt{N}$	Degree of 3D Torus=3 Diameter of 3D Torus= 4 Cost= degree X diameter=12
Cube-connected cycles (CCC)	$3(\log N)$	Degree of 3D CCC=3 Diameter of 3D CCC= 3 Cost= degree X diameter=9
Dual Cube (DC)	$\frac{(n+1)}{2} \times (n + 1)$	Degree of 3D DC=2 Diameter of 3D DC= 4 Cost= degree X diameter=8
Folded Dual Cube (FDC)	$\frac{(n+3)}{2} \times (n - 1)$	Degree of 3D FDC=4 Diameter of 3D FDC= 2 Cost= degree X diameter=8
TOR-CUBE (TC)	$(n + 1) \times (n - 1)$	Degree of 3D TC=4 Diameter of 3D TC= 2 Cost= degree X diameter=8

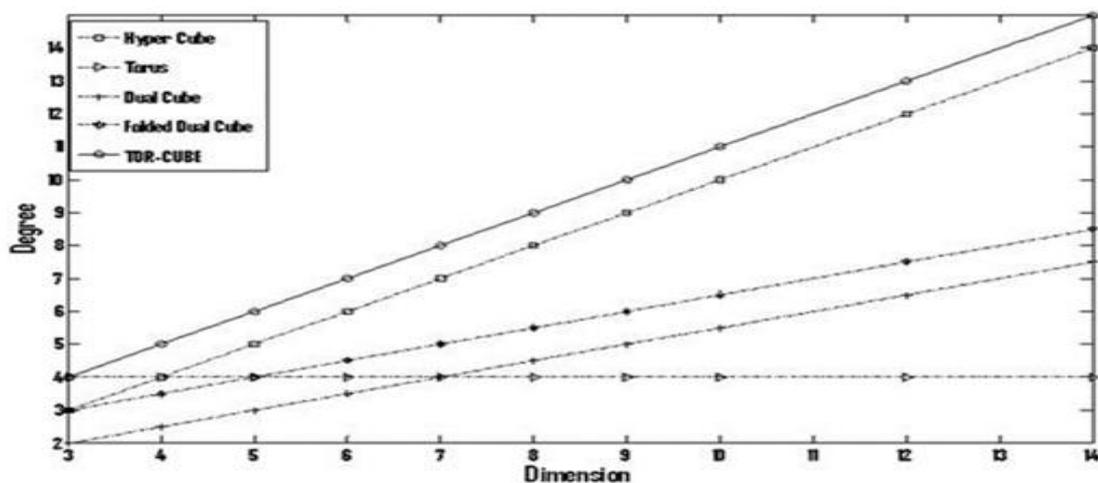


Fig. 10: Degree Vs Dimension

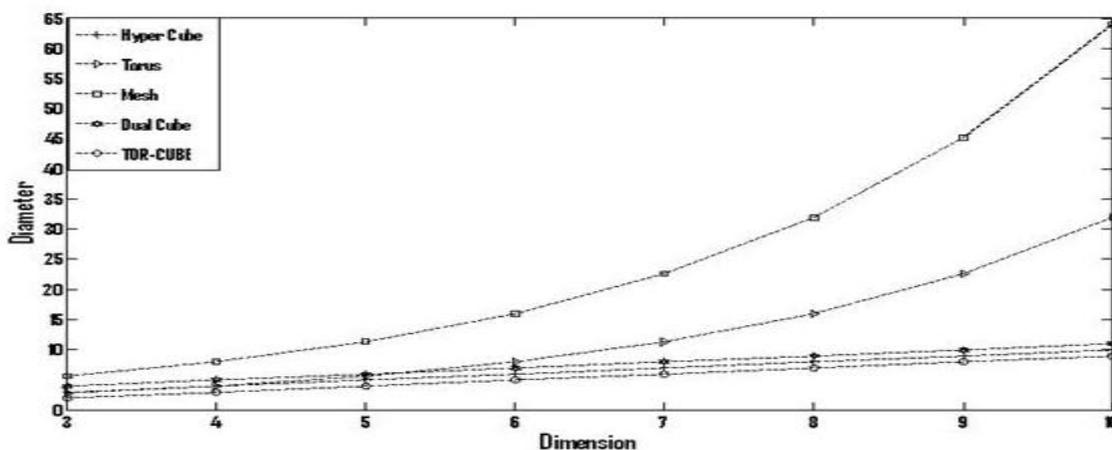


Fig. 11: Diameter Vs Dimension

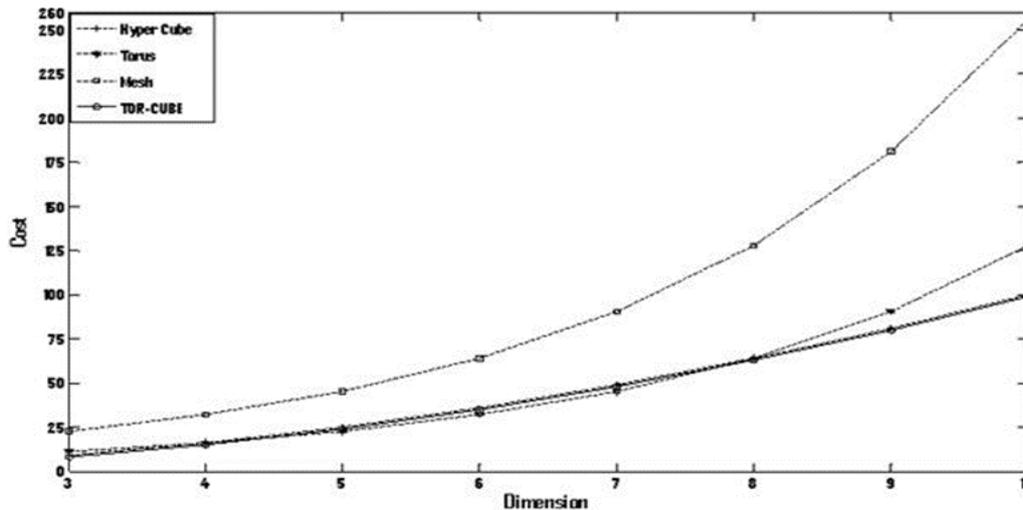


Fig. 12: Cost Vs Dimension

The effect of cost with dimension is also taken for consideration. A graph is plotted between the dimension and cost of different networks and is presented in Fig. 12. The graph depicts that TC has less cost not only from its parent topologies but also from some other important topologies. Therefore, the hardware complexity for the design of TC is also economical, reasonable and feasible.

**X. CONCLUSIONS**

A new interconnection topology namely TOR-CUBE (TC) is proposed in this paper. The proposed topology is inherited from two base class topologies: hypercube and torus. A routing algorithm is also proposed for the exchange of information among the processing elements. The various topological properties of the proposed topology are analyzed and estimated. A detailed comparison of TOR-CUBE against other important interconnection networks is presented and discussed. From this comparison, it can be observed that the proposed interconnection network is more suitable for parallel computer architecture because of its high connectivity, lesser diameter, low cost, better bisection width, more fault tolerant and low average distance. The different reliability measure of the TC like network reliability, two-terminal reliability and k-terminal reliability are evaluated and compared against that of other interconnection networks of interest. From these comparisons, it can be concluded that the proposed interconnection network is highly reliable and cost effective with a better degree of scalability. The work carried out in this paper may further be extended to propose a new hierarchical interconnection network using TOR-CUBE as the base network.

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Dependability, Reliability and Fault-tolerance of Parallel and Distributed system.

### AUTHORS PROFILE



**Dr. Pradyumna Kumar Tripathy** has completed his M.Tech. and Ph.D. in Computer Science from Utkal University, India in 2007 and 2015 respectively. He is currently working as Associate Professor in the Department of Computer Science & Engineering at Silicon Institute of Technology, Bhubaneswar, India. His research interests include Reliability Analysis of Interconnection Networks, Parallel Distributed Systems and Topological Optimization of Interconnection Networks, Data Analysis.



**Dr. Ranjan Kumar Dash** received the MCA degree from F.M. University in 2001. He got his Ph.D. in Computer Science from Sambalpur University in the year 2008. He is now working as Reader in Department of Information Technology, College of Engineering & Technology, Bhubaneswar, Odisha, India. His research areas are Reliability of parallel & Distributed system, Analysis of & Design of algorithm.



**Prof.(Dr.) Chitta Ranjan Tripathy** received the B.Sc. (Engg.) in Electrical Engineering from Sambalpur University and M.Tech. Degree in Instrumentation Engineering from Indian Institute of Technology, Kharagpur. He got his Ph.D. in the field of Computer Science & Engineering from Indian Institute of Technology, Kharagpur. He is currently working as a Professor, Department of Computer Sc. & Engg., UCE, Burla, Odisha, India. He has more than 150 publications in different national and international Journals and Conferences. His Research interest includes