

Nonlinear Analysis of Flat Steel Frame Structure With Semi-rigid Connection Under Static Load

Hong Son Nguyen, Trung Thanh Pham, Van Quan Tran, Quang Hung Nguyen



Abstract: The paper aims to analyze the elasticity-plasticity of a flat steel frame structure with a beam-column connection that is a semi-rigid connection; three-spring semi-rigid link models (two linear displacement springs and one rotating displacement spring); It includes hard areas at the beam-column position, with an ideal flexible-plastic material model. Based on that, it developed algorithms by PTHH method using step-by-step applying load method and programmed to determine displacement-internal force when make semi-rigid nonlinear connection in MATLAB programming language; consider the melting conditions of the section, including the influence of moments, vertical force and shear force. Make a numerical illustration with a two stage-one span steel frame.

Keywords: Nonlinear analysis, flat steel frame, sime rigid connection, static load

displacement spring. The Timoshenko bar element associated with the rigid zone with extremely stiffness by these springs is called the super-element, as shown in Figure 1. The characteristic line of the connection ($M-\theta$) is a curve, linearization in the design standard [5] as a polygon. Most commonly, bilinear or triangular are shown in Figure 2a.

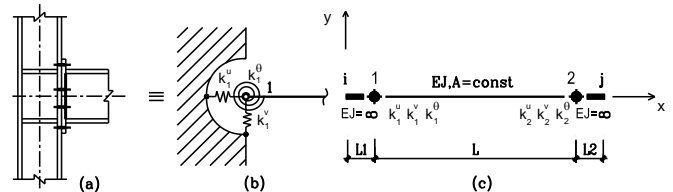


Figure 1. Semi-Rigid Connection Model And Super-Element

I. INTRODUCTION

The problem of elastic-plastic analysis of semi-rigid flat steel frame structure (Semi-Rigid Connections) to find the displacement and internal force mentioned by some authors [1] by step-by-step applying load method but it is only stopped at the model of connecting a spring for the displacement of the angle and the plastic melting condition of the section only due to the bending moment component. However, at the cross section and connection of flat steel frame there are always three internal forces: moments, vertical force and shear force. The relationship between force and displacement at the connection is modeled by three springs (a displacement spring and two straight displacement springs). These internal forces all affect to the melting status of sections [2], [4] and are included in plastic joints.. Below, present the problem of analyzing the elasticity-plasticity of steel frame with three-spring semi-rigid connection model, melting condition of the section, including the effect of all three internal forces and elastic materials, assume: ideal plasticizer. Problem solved by finite element method .

A. Semi-rigid connection model, characteristic line of the connection

Modeling a semi-rigid connection with three springs, in which two linear displacement springs and one angular

B. Characteristic lines of semi-rigid connection and bar material models

When elastic-plastic analysis, the characteristic line of the semi-rigid connection here is chosen as the linear triangle with the flexibility as shown in Figure 2.b. The material model for the bar structure is chosen as the ideal plastic elastic model as shown in Figure 2.c.

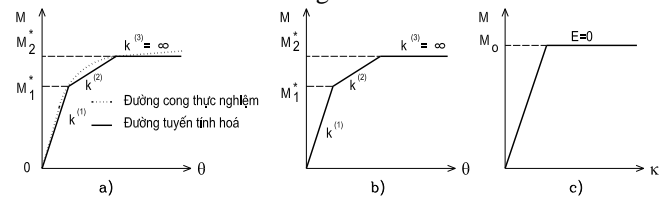


Figure 2. Characteristic line ($M-\theta$) of the connection and ($M-\kappa$) of the beam element

II. THEORY

A. Basic equation of finite element method, displacement model

In the case of linear problems, balance equations take the form:

$$KU = R \tag{1}$$

where:

$$K = \sum_{e=1}^n K_e^L = \sum_{e=1}^n L_e^T K_e L_e ;$$

$$R = \sum_{e=1}^n R_e^L = \sum_{e=1}^n L_e^T R_e ;$$

$$K_e = T_e^T H_e^T \bar{K}_e^* H_e T_e ;$$

Revised Manuscript Received on October 30, 2019.

* Correspondence Author

Hong Son Nguyen*, Hanoi Architectural University, 100000 Ha noi, Vietnam.

Van Quan Tran, University of Transport Technology, 100000 Hanoi, Vietnam

Trung Thanh Pham, Vietnam Institute for Bulding Science and Technology, 100000 Hanoi, Vietnam

Quang Hung Nguyen, Thuyloi University, Hanoi, Vietnam.

© The Authors. Published by Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP). This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>)

$$\mathbf{R}_e = \mathbf{T}_e^T \mathbf{H}_e^T \bar{\mathbf{R}}_e^* \quad (2)$$

with:

$\bar{\mathbf{K}}_e^*$, $\bar{\mathbf{R}}_e^*$ - The stiffness matrix and the load vector of the bar element with the two semi-rigid ends in the local coordinate system;

\mathbf{H}_e - the relational matrix between the super-element bar and the semi-rigid double-ends element;

\mathbf{T}_e - The matrix transforms from local coordinates to the common coordinate system;

\mathbf{K}_e , \mathbf{R}_e - stiffness matrix and the load vector of the super element in the common coordinate system;

\mathbf{L}_e - Super-element positioning matrix in the matrix of structure;

\mathbf{U} - displacement vector of the structure.

The matrices and load vectors of super beam elements are formed from matrices and load vectors of semi-rigid double-end connection beam elements. The making of matrix and node force vectors by unit displacement method, can be summarized by the following formulas:

B. The load and stiffness matrix of the element bars with two semi-rigid ends

1) Stiffness matrix $\bar{\mathbf{K}}_e^*$

Stiffness matrix $\bar{\mathbf{K}}_e^* = [\bar{k}_{ij}^*]$, ($i, j=1;6$), The elements of this stiffness matrix are:

$$\bar{k}_{11}^* = \bar{k}_{44}^* = \frac{EA}{L} \frac{1}{1 + EA(k_1^u + k_2^u)/L}$$

$$\bar{k}_{14}^* = \bar{k}_{41}^* = -\frac{EA}{L} \frac{1}{1 + EA(k_1^u + k_2^u)/L}$$

$$\bar{k}_{22}^* = \bar{k}_{55}^* = \frac{12i}{L^2} \frac{A_1 + A_2}{\bar{\Phi}(A_1 + A_2) + 12iA_1A_2}$$

$$\bar{k}_{23}^* = \bar{k}_{32}^* = \frac{6}{L} \frac{1 + 2ik_2^0}{\bar{\Phi}(A_1 + A_2) + 12iA_1A_2}$$

$$\bar{k}_{25}^* = \bar{k}_{52}^* = \frac{12i}{L^2} \frac{-(A_1 + A_2)}{\bar{\Phi}(A_1 + A_2) + 12iA_1A_2}$$

$$\bar{k}_{26}^* = \bar{k}_{62}^* = \frac{6}{L} \frac{1 + 2ik_1^0}{\bar{\Phi}(A_1 + A_2) + 12iA_1A_2};$$

$$\bar{k}_{33}^* = \frac{\bar{\Phi} + 6 + 12ik_2^0}{\bar{\Phi}(A_1 + A_2) + 12iA_1A_2}$$

$$\bar{k}_{35}^* = \bar{k}_{53}^* = -\frac{12i}{L} \frac{A_2}{\bar{\Phi}(A_1 + A_2) + 12iA_1A_2};$$

$$\bar{k}_{36}^* = \bar{k}_{63}^* = \frac{-\bar{\Phi}}{\bar{\Phi}(A_1 + A_2) + 12iA_1A_2};$$

$$\bar{k}_{56}^* = \bar{k}_{65}^* = -\frac{6}{L} \frac{1 + 2ik_1^0}{\bar{\Phi}(A_1 + A_2) + 12iA_1A_2}$$

$$\bar{k}_{66}^* = \frac{\bar{\Phi} + 6 + 12ik_1^0}{\bar{\Phi}(A_1 + A_2) + 12iA_1A_2}$$

where:

$$\bar{\Phi} = \frac{12i}{L^2} (k_1^v + k_2^v) + \frac{12i}{L} \frac{\mu}{GA} - 2$$

$$A_1 = \frac{1}{2i} + k_1^0$$

$$A_2 = \frac{1}{2i} + k_2^0; i = \frac{EI}{L}$$

Stiffness matrix $\bar{\mathbf{K}}_e^*$ of the bar element when the ends bar 1 or 2 has another connection just need to replace the softness k_1^0 or k_2^0 otherwise when the connection is stiff and is equal to infinity when the joint is aligned.

2) Load vector $\bar{\mathbf{R}}_e^*$

$$\bar{\mathbf{R}}_e^* = \{\bar{V}_1^*, \bar{M}_1^*, \bar{V}_2^*, \bar{M}_2^*\}^T,$$

where:

$$\bar{V}_1^* = -\frac{1}{iL^3} \frac{iL^2(A_1 + A_2)A_4 + (c_1A_1 - c_2A_2)A_0}{A_3(A_1 + A_2) + A_1A_2} - \frac{R(L-z)}{L}$$

$$\bar{M}_1^* = +\frac{1}{iL^2} \frac{A_2(c_2A_0 - iL^2A_4) + LA_0A_3}{A_3(A_1 + A_2) + A_1A_2}$$

$$\bar{V}_2^* = +\frac{1}{iL^3} \frac{iL^2(A_1 + A_2)A_4 + (c_1A_1 - c_2A_2)A_0}{A_3(A_1 + A_2) + A_1A_2} - \frac{Rz}{L}$$

$$\bar{M}_2^* = -\frac{1}{iL^2} \frac{A_1(c_1A_0 + iL^2A_4) + LA_0A_3}{A_3(A_1 + A_2) + A_1A_2}$$

With

$$A_3 = \frac{k_1^v + k_2^v}{L^2} - \frac{1}{6i} + \frac{\mu}{GAL}$$

$$A_4 = -Rz \frac{k_1^v + k_2^v}{L^2} + \frac{Rk_1^v}{L} + \frac{\mu R}{GA} \left(\frac{L-2z}{2L} \right)$$

A_0 , R , c_1 , c_2 , z respectively the area of the bending moment diagram, the force of the load, the center distance of the bending moment diagram to the ends 1, 2 of the bar and the distance of the force combination R to the ends 1; The above values can be found in the structural handbook.

3) Relationship matrix \mathbf{H}_e

Relationship matrix \mathbf{H}_e between the stiffness matrix (load vector) of the super beam element with the stiffness matrix (load vector) of the semi-rigid double-ends beam element in the local coordinate system is determined by the following formula:

$$\mathbf{H}_e = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & L_1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -L_2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

C. Solution

In the case of an ideal flexible plastic material and nonlinear semi-rigid, the balance equation (1) becomes a nonlinear equation (7). The displacement and internal forces state depend on the state of the connection at the ends of the bar.

$$\mathbf{K}(\mathbf{U})\mathbf{U} = \mathbf{R} \quad (7)$$

To solve (7) the author uses step-by-step applying load method [1] to determine the displacement and internal force after linearization of the connection properties by straight lines with melting.

Stiffness matrix $\mathbf{K}(\mathbf{U})$ Depends on the state of the connection $k^{(i)}$ ($i=1,3$) At the ends of the element and the state of the bar when forming the plastic joint, it means that it depends on the internal force of the bar.

D. Conditions of plastic melting of the connection

The characteristic line of the semi-rigid connection is chosen as the triplet and is determined by the following 5 parameters: moments M_1^* , M_2^* , $k^{(1)}$, $k^{(2)}$, $k^{(3)}$. The state of the connection and the melting conditions are identified through the following mathematical signs:

(1) The two ends of bar i and j are elastic when:

- + $|M_i| < M_1^*$ and $|M_j| < M_1^*$, respectively i and j have associated connection softness $k^{(1)}$;
- + $|M_i| < M_1^*$ and $M_1^* \leq |M_j| < M_2^*$, correspondingly i have connection softness $k^{(1)}$, j have connection softness $k^{(2)}$;
- + $M_1^* \leq |M_i| < M_2^*$ and $|M_j| < M_1^*$, correspondingly i have connection softness $k^{(2)}$, j have connection softness $k^{(1)}$;
- + $M_1^* \leq |M_i| < M_2^*$ and $M_1^* \leq |M_j| < M_2^*$, correspondingly i have connection softness $k^{(2)}$, j have connection softness $k^{(2)}$.

(2) correspondingly i elastic, correspondingly j plastic melting when: $|M_j| \geq M_2^*$,

- + $|M_i| < M_1^*$, correspondingly i have connection softness $k^{(1)}$;
- + $M_1^* \leq |M_i| < M_2^*$, correspondingly i have connection softness $k^{(2)}$.

(3) correspondingly i plastic melting, correspondingly j elastic when: $|M_i| \geq M_1^*$,

- + $|M_j| < M_1^*$, correspondingly j have connection softness $k^{(1)}$;

+ $M_1^* \leq |M_j| < M_2^*$, correspondingly j have connection softness $k^{(2)}$.

(4) Two ends of bar i and j melting when: $|M_i| \geq M_2^*$ and

$$|M_j| \geq M_2^*$$

E. Conditions of plastic melting of the section

Conditions of plastic melting of the bar section, including the influence of internal forces, for the section I is determined as follows [4]:

- Include the effect of moment (M) and vertical force (N):

$$\frac{|M|}{M_o} = 1 \text{ when } 0 \leq \frac{|N|}{N_o} \leq 0,15.$$

$$\text{and } \frac{|M|}{M_o} = 1,18 \left(1 - \frac{|N|}{N_o} \right) \text{ when } 0,15 < \frac{|N|}{N_o} \leq 1.$$

- Include the effect of moment (M) and shear force (V):

$$\frac{|M|}{M_o} = 1 + \frac{0,25t_w h_w^2 / b_f t_f h_f}{1 + 0,25t_w h_w^2 / b_f t_f h_f} \sqrt{1 - \left(\frac{V}{V_o} \right)^2} \quad (12)$$

where:

$$M_o = \sigma_o W_o$$

$$N_o = \sigma_o A$$

$$V_o = 0,65\sigma_o A_w$$

with symbols σ_o , W_o , A and A_w respectively the melting limit, the flexural module, the section area and the area of the bell section of the bar.

III. RESULTS AND DISCUSSION

On the basis of the above algorithm, the author has set up a Static Analysis of Steel Frame (SASF) program to analyze the plasticity of semi-rigid connection steel structures which are nonlinear linearized, the conditions of plastic flow dependence internal force: moment (M), vertical force (N) and shear force (V).

Considering the double-stage frame with one span as shown in Figure 3a, there is nonlinear semi-rigid connection for rotational displacement, softness of linear displacement is zero with the case of plastic melting conditions taking into account the influence of internal forces.

Nonlinear Analysis of Flat Steel Frame Structure With Semi-rigid Connection Under Static Load

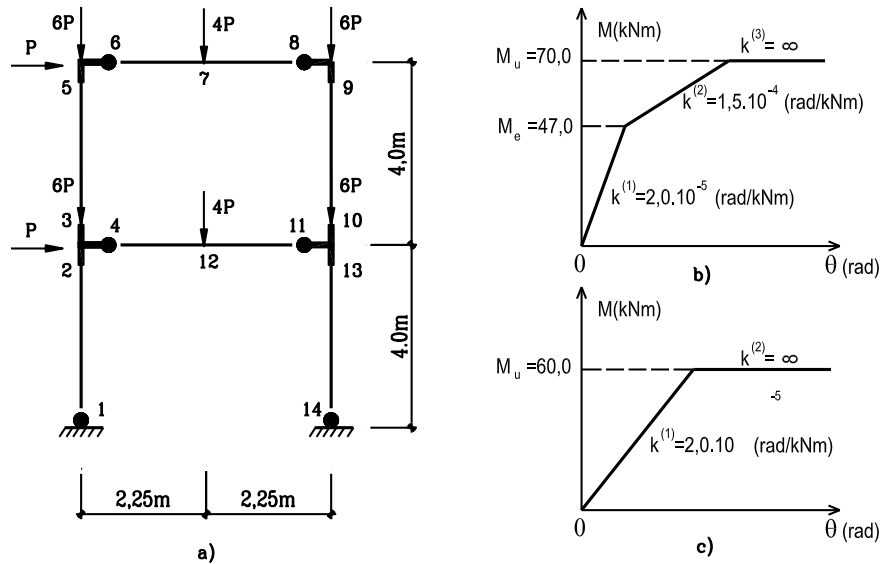


Figure 3. Schematic of a single-stage frame with two-span and non-linear semi-rigid connection

The parameters are as follows:

- Steel with elastic modulus $E=2,1 \cdot 10^5 \text{MPa}$, Poisson's coefficient $\nu=0,3$, $f_y=248 \text{MPa}$, Ideal plastic-elastic material model; Bearing load $P=16,0 \text{kN}$.
- Columns and beams with wing size (200x6)mm, bell plate (180x6)mm;
- The column-beam connection properties line is as shown in Figure 3b and the column foot as shown in Figure 3c.

Tab. 1. Result of pinned moment $M(\text{kNm})$ when analyzing the plastic structure of steel frame with semi-rigid linear and nonlinear connection

pinned	Linear elastic material			Ideal elastic-plastic material		
	Linear connection	Nonlinear connection	Difference (%)	Linear connection	Nonlinear connection	Difference (%)
1	31.5264	32.1914	-2.109	32.8913	33.1350	-0.741
2	13.7093	13.0134	5.076	13.9377	13.2623	4.846
3	-10.0444	-10.6568	-6.097	-9.8678	-10.4560	-5.961
4	-8.0981	-7.5066	7.304	-7.1988	-6.9022	4.120
5	42.5035	43.1882	-1.611	42.8538	43.3724	-1.210
6	37.1887	36.5350	1.758	35.2453	35.1584	0.247
7	34.6113	34.0348	1.666	32.9358	32.8494	0.262
8	44.4593	45.0566	-1.343	45.0588	45.4367	-0.839
9	-5.7477	-4.6589	18.943	-6.2779	-5.1772	17.533
10	-37.7856	-41.6481	-10.222	-39.6368	-42.5943	-7.462
11	-71.0369	-70.0000	1.460	-67.4064	-67.4064	0.000
12	6.2919	5.7263	8.989	5.3975	5.1226	5.093
13	44.4960	44.4925	0.008	44.6729	44.6250	0.107
14	-42.5722	-43.1446	-1.345	-43.1126	-43.4835	-0.860

Note: Flexible moment of the section $M_o=67,4064 \text{kNm}$
 The moment is taken at the ends of the element, the pinned balancing at the top of the super element satisfies.

Results of elastic-plastic analysis in Table 1 for cases of semi-rigid connection are linear and nonlinear.

From the results received, the following comments can be made:

If the material is linearly elastic, when analyzing with the nonlinear semi-rigid link, the number 11 pinned reaches the maximum number of moments of the connection. The

difference in pinned moment between two cases of linear and nonlinear semi-rigid steel frame analysis can reach 18,943%;

If the material is ideal elastic-plastic and nonlinear or linear semi-rigid connection, pinned 11 achieves the plastic moment value of the section. The difference in pinned moment between two cases of linear and nonlinear semi-rigid steel frame analysis reaches 17.533%.

IV. CONCLUSION

With SASF software, results in displacement-internal force when analyzing the plastic structure of linear and nonlinear semi-rigid steel frame. Supplemented the ability of non-linear structure of plastic structure with nonlinear connection that popular software like SAP2000 and ETAB have not mentioned. The value of the pinned moment in the problem of elastic-plastic analysis of linear and nonlinear semi-rigid steel structure is quite different. The use of linearized nonlinear semi-rigid connection properties will result in the displacement and internal force state close to the actual working of the beam-column link.

REFERENCES

1. Cao Van Mao (2005), "Flat frame texture analysis with rigid pinned and soft connection" Technical doctoral thesis, Ha Noi.
2. Bruneau M., Uang C., Whittaker A. (1998), Ductile Design of Steel Structures, Mc Graw-Hill, New York.
3. Bathe K.J. (1996), Finite Element Procedures, Prentice-Hall International, Inc
4. Chen W.F., Sohal I. (1994), Plastic Design and Second Order Analysis of Steel Frame, Springer-Verlag, New York Inc.
5. CEN (1992): Eurocode 3 (1992), Design of Steel Structures-Part I-I-General Rules and Rules for Building, CEN ENV.
6. Jaspert J.P. (1990), "Etude de la semi-rigide des noeuds-poutre et son influence sur la resistance et la stabilite des ossatures en acier", Universite de Liege.