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An Application of Linear Programming in the Estimation of Technical Efficiency of DMU

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Abstract: ***The main objective of this research article is to propose linear programming problems for estimating the technical efficiency of DMU. This research article deals with the Shepard's [1] input distance function and its properties are also evaluated. In addition to these extreme efficiency, efficiency but not extreme, weak efficiency and inefficient of a DMU are specifically examined here. In DEA the nature of returns to scale can be inferred. But we cannot quantify the returns to scale. The computations for the classification of RTS of a DMU are also derived in this discourse. In 2009, Barbara A. Mark et.al [2] in their paper, depicted an innovative method which is non-parametric to estimate technical efficiency. In 2011 S. Nuti et.al [3] inquired into the interrelations among technical efficiency scores, weighted per capita cost and overall performance. Gahe Zing Samuel Yannik et.al [4] used DEA to calculate technical assessment in banking sectors. In 2015 Smita Verma and others chosen a random sample of ten textile mills in India over the time period 2011-2013 and measures its technical efficiency using Data Envelopment Analysis.***

Index Terms: ***DMU (Decision Making Unit), input distance function, index set, DEA (Data Envelopment Analysis), VRTS (Variable Returns to Scale), IDF (Input Distance Function).***

I. INTRODUCTION

Efficiency measurement dates back to Farrell [6] who in his path breaking article introduced technical, allocative and cost efficiencies and their pictorial representations. Adding mathematical regour Charness et al. [7] proposed multiplier problems, input and output oriented, which can be readily transformed into linear programming problems. Banker et.al [8] formulated linear programming problems constructed axiomatically whose dual problems coincide with CCR multiplier problems. By the principle of duality the extreme values of the primal and dual objective functions are equal provided that both the problems are feasible. The BCC [9] problems are called the 'envelopment problems'. A decision making unit (DMU) under evaluation turns out to be efficient or inefficient. The efficient DMUs are of three types, extremely efficient, efficient but not extremely efficient and weakly efficient. For an extremely efficient unit, efficiency rating is unity and all input and output slacks assume zero values. Such decision making units are 'peerless'. Since the envelop is piecewise linear convex set, an extremely efficient DMU represents one of its vertices. If an efficient unit is efficient but not extremely efficient then its efficiency rating is unity. The input and output representation of such a unit belongs to the envelop, but it cannot represent a vertex. Two or more extremely efficient decision making units are its peers. For such DMU efficiency rating emerges to be unity and all input and output slacks are found vanishing.

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II. INPUT DISTANCE FUNCTION

In the contest of multiple inputs and multiple output scenarios, Shephard [1] introduced the concept of IDF which is inversely related to Farrell's input technical efficiency. The IDF is related to input level sets.

L(u) = {x: x produces u}, where $x \in R_+^m$ and $u \in R_+^s$

The structure imposed on *L(u)* forces *L(u)* to satisfy the following conditions.

1) $L(0) = R_+^m$, every input vector produces null output vector due to inefficiency.
2) $u_1 \geq u_2 \Rightarrow L(u_1) \subseteq L(u_2)$,
3) *L(u)* is closed set,
4) $\underset{u \to \infty}{Lt} L(u) = \varphi$, No input vector can produce infinite output vector
5) L(*u*) is convex set of inputs, if returns to scale are constant, $L(\lambda u) = \lambda L(u)$, $\lambda \geq 1$
6) L(u) satisfies the strong disposability of inputs. No cost is involved in disposing additional inputs due to inefficiency.

Shepard's IDF is defined as follows:

$$D_i(u_0, x_0) = [Min\ \{\lambda : \lambda x_0 \in L(u)\}]^{-1}$$
$$= [F_i(u_0, x_0)]^{-1}$$

(i) The IDF is inversely related to the Farrell's input technical efficiency measure.

(ii) $D_i(u_0, x_0) \geq 1$, $D_i(u_0, \lambda x_0) = \lambda D_i(u_0, x_0)$

$$D_i(u_0, \lambda x_0) = \left[Min\{\delta: \delta\lambda x_0 \in L(u_0)\}\right]^{-1}$$
$$= \left[\lambda^{-1} Min\{\lambda\delta: \lambda\delta\ x_0 \in L(u_0)\}\right]^{-1}$$
$$= \lambda \left[Min\{\eta: \eta x_0 \in L(u_0)\}\right]^{-1}$$
$$= \lambda\ D_i(u_0, x_0)$$

(iii) $u_0 \geq u_1 \Rightarrow D_i(u_0, x_0) \leq D_i(u_1, x_0)$

Let $u_0 \geq u_1$, $x_0 \in L(u_0) \Rightarrow$ x produces u_0
Where x_0 can also produce every output vector smaller than $u_0 \Rightarrow$ x produces u_1
$x \in L(u_0) \Rightarrow x \in L(u_1)$, $L(u_0) \subseteq L(u_1)$
Min $\{\lambda : \lambda\ x_0 \in L(u_0)\} \geq$ Min$\{\lambda : \lambda\ x_0 \in L(u_1)\}$

$$\left[Min\ \{\lambda:\ \ \lambda x_0 \in L(u_0)\}\right]^{-1} \leq \left[Min\ \{\lambda:\ \ \lambda x_0 \in L(u_1)\}\right]^{-1}$$

$D_i(u_0, x_0) \leq D_i(u_1, x_0)$

(iv) $x_1 \leq x_2 \Rightarrow D_i(u_0, x_1) \leq D_i(u_0, x_2)$

Let x_1, x_2 produce u_0, but $x_1 \leq x_2$
Min $\{\lambda : \lambda\ x_1 \in L(u_0)\} \geq$ Min$\{\lambda: \lambda\ x_2 \in L(u_0)\}$

$$\left[Min\ \{\lambda:\ \ \lambda x_1 \in L(u_0)\}\right]^{-1} \leq \left[Min\ \{\lambda:\ \ \lambda x_2 \in L(u_0)\}\right]^{-1}$$

$D(u_0, x_1) \leq D(u_0, x_2)$
$x_1 \leq x_2 \Rightarrow D(u_0, x_1) \leq D(u_0, x_2)$

Banker, Charnes and Cooper (BCC) [9], under the axioms of convexity, inefficiency and minimum extrapolation proposed a production possibility set where frontier is formed by linear hyper planes. Their input sets specification is as follows below:

$$L(u) = \left\{x: \sum_{j=1}^{n} \lambda_j x_j \leq x\ \ ,\ \sum_{j=1}^{n} \lambda_j u_j \geq u\ \ ,\ \sum_{j=1}^{n} \lambda_j = 1\ ,\ \ \lambda_j \geq 0\right\}$$

Where x_{ij} =input vector of j[th] DMU, u_{ij} = output vector of j[th] DMU and λ_j are some intensity parameters.

The technical efficiency of a DMU whose input and output vectors are respectively, x_0 and u_0, are evaluated through LPP given below.

$$\left[D_i(u_0, x_0)\right]^{-1} = Min\ \ \lambda$$

$$subject\ to\ \sum_{j=1}^{n} \lambda_j x_j \leq \lambda\ x_0 \quad (2.1)$$

$$\sum_{j=1}^{n} \lambda_j u_j \geq u_0, \sum_{j=1}^{n} \lambda_j = 1$$

If there are n DMUs one has to solve n linear programming problems. A DMU may be extremely efficient or with efficiency not at extreme level or with week efficiency or without efficiency..A DMU_0 is extremely efficient,

If λ^* = Min λ =1, $\lambda_0^* = 0$, $\lambda_j^* = 0$, $\forall$ j $\neq$ 0,All slacks vanish.

(i) A DMU_0 is efficient but not extremely efficient, if λ^* = Min λ =1, $\lambda_0^* = 0$ and $\lambda_j^* \neq 0$, for some j$\neq$ 0, all slacks vanish.

(ii) A DMU_0 is weakly efficient, if λ^* = Min λ =1, not all the slacks vanish.

(iii) A DMU_0 is inefficient, if λ^* = Min λ<1, if I denote the index set of the

DMUs, then I = E U E′ U F U N

Where E: Index set of extremely efficient DMUs

E′ : Index set of efficient but not extremely efficient DMUs
F: Index set of weakly efficient DMUs and N: Index set of inefficient DMUs.

III. THE EFFICIENT OF A DMU

The convexity constraint $\sum_{j=1}^{n} \lambda_j = 1$ models variable returns to scale. BCC formulated a DEA model, which is input oriented under the axioms of inefficiency, ray unboundedness and minimum extrapolation.

The LP model admits constant return to scale.

$$\lambda^* = Min\ \ \lambda \quad (3.1)$$
$$Subject\ to\ \lambda_1 x_{i1} + \lambda_2 x_{i2} + \ldots\ldots + \lambda_n x_{in} \leq \lambda_n x_{i0}$$

$\lambda_1 y_{n1} + \lambda_2 y_{n2} + \ldots\ldots + \lambda_n y_{rn}$ is not less than y_{r0}.

Where λ_j are non-negative.

The final solution of (3.1) leads to an optimal solution for which,

(i) $\sum_{j=1}^{n} \lambda_j^* = 1 \ \Rightarrow$ Constant returns to scale

(ii) $\sum_{j=1}^{n} \lambda_j^* \geq 1 \Rightarrow$ Non-increasing returns to scale

(iii) $\sum_{j=1}^{n} \lambda_j^* \leq 1 \Rightarrow$ Non-decreasing returns to scale.

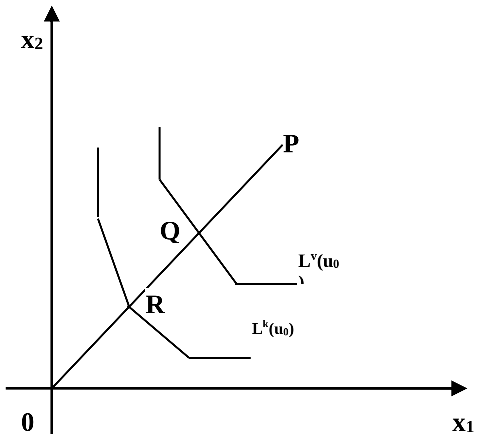

Fig (3.1)

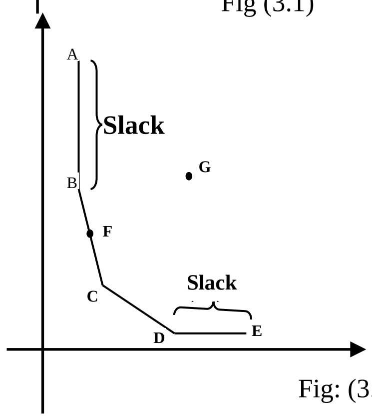

Fig: (3.2)

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The decision making units A, B, C, D and E determine unit output isoquant, that serves as a production frontier D, C, B are decision making units with extreme level efficiency is a decision making unit with efficiency but not at extreme level.

The decision making units A and E are with week efficiency. G is a decision making unit without efficiency. Identifying weak efficient as inefficiency, (2.1) can be reformulated as,

$$\lambda^* = Min\ \lambda - \sum_{i=1}^{m} s_i^- - \sum_{r=1}^{s} s_r^+$$

subject to

$$\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \lambda x_{i0} \quad , \qquad i = 1, 2, ..., m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{r0} \quad , \qquad r = 1, 2, ..., s$$

$$\sum_{j=1}^{n} \lambda_j = 1 \, , \lambda_j \geq 0$$

(3.2)

In the case of weak efficiency, we have, $1-\left(s_1^- + s_2^- + ... + s_m^-\right) - \left(s_1^+ + s_2^+ + ... + s_s^+\right)$ is less than unity. Since in the case of weak efficiency at least one slack emerges with a non-zero value, the input level sets $L^k(u_0)$ and $L^v(u_0)$ admit respectively constant, variable returns to scale.

$L^v(u_0) \subseteq L^k(u_0)$

To achieve pure technical efficiency, the producer should operate at Q; his input pure technical efficiency is,

$$D_i^v(u_0, x_0) = \frac{OP}{OQ} = \left[Min\ \left\{\lambda : \lambda x_0 \in L^k(u_0)\right\}\right]^{-1}$$

To achieve scale efficiency as well as technical efficiency the DMU shall operates at R.

$$D_i^k(u_0, x_0) = \frac{OP}{OQ} = \left[Min\ \left\{\lambda : \lambda x_0 \in L^k(u_0)\right\}\right]^{-1}$$

$$D_i^k(u_0, x_0) \geq D_i^v(u_0, x_0)$$

$$\left[D_i^k(u_0, x_0)\right]^{-1} \leq \left[D_i^v(u_0, x_0)\right]^{-1}$$

In data envelopment analysis the nature of returns to scale can be inferred. But we cannot quantify the returns to scale.

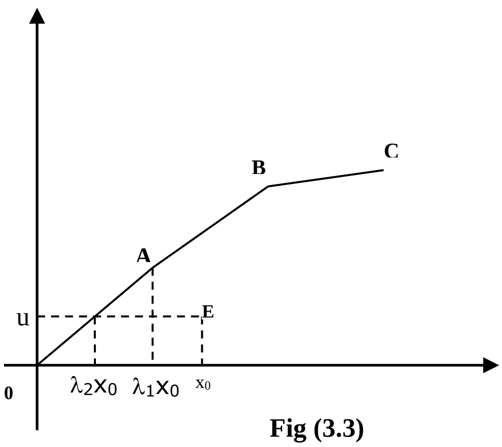

Fig (3.3)

The decision making units A, B and C constitute the piecewise linear production frontier. The segment OA that emanates from the origin reflects constant returns to scale. The segments AB and BC model decreasing returns to scale [10, 11, 12]. The model in figure (3.2) admits non increasing RTS. The suitable LPP:

$$Min\ \ \lambda$$

$$such\ \ that$$

$$\sum_{j=1}^{n} \lambda_j x_j \leq \lambda x_0$$

$$\lambda_1 u_1 + \lambda_2 u_2 + ... + \lambda_n u_n \geq 0$$

$\lambda_1 + \lambda_2 + ... + \lambda_n$ is not greater than one λ_j are not less than zero.

$$j \in \{1, 2, ...n\}$$

DMU that operates at E(x_0,u_0) is inefficient. Its input x_0 is horizontally projected on to the frontier one admitting VRTS and the other NIRTS.

$$\lambda_1^{-1} = D_i^v(x_0, u_0)$$

$$\lambda_2^{-1} = D_i^{NI}(x_0, u_0)$$

$_0$ and produces output x_0, its horizontal projection on to the VRTS frontiers shows its returns to scale are decreasing.

$$\lambda_1 = \left[D_i^v(x_0, u_0)\right]^{-1}$$

$$\lambda_2 = \left[D_i^k(x_0, u_0)\right]^{-1}$$

$$\lambda_1 < \lambda_2 \Leftrightarrow \left[D_i^v(x_0, u_0)\right]^{-1} < \left[D_i^k(x_0, u_0)\right]^{-1}$$

$$D_i^v(x_0, u_0) > D_i^k(x_0, u_0)$$

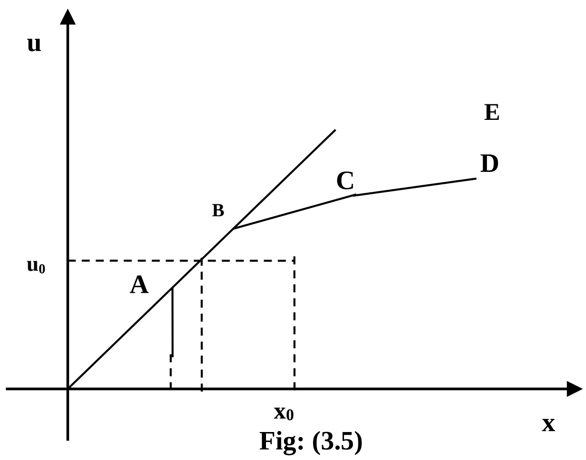

Fig: (3.5)

For DMU E returns to scale are constant

$$D_i^k(x_0, u_0) = D_i^v(x_0, u_0) = D_i^{NI}(x_0, u_0)$$

To classify returns to scale of a DMU$_0$, we compute,

$D_i^v(u_0, x_0)$, $D_i^{NI}(u_0, x_0)$ and $D_i^k(u_0, x_0)$

If (i) $D_i^v(x_0, u_0) > D_i^{NI}(x_0, u_0)$, RTS are increasing

(ii) $D_i^v(x_0, u_0) > D_i^k(x_0, u_0)$, RTS are decreasing

(iii) $D_i^v(u_0, x_0) = D_i^{NI}(u_0, x_0) = D_i^k(u_0, x_0)$ RTS are constant

IV. CONCLUSION

In the above research paper LPPs are formulated to estimate the technical efficiency of a DMU and the pure technical and scale efficiency are achieved. The expressions by which the returns to scale of a DMU are classified are proposed. In addition to these the concept of input distance function is defined and its properties re discussed.

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