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Wind Speed using Improved Mixture Weibull Distribution

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Abstract: ***Wind is random in nature both in space and in time. Several technologies are used in wind resource assessment (WRA).The appropriate probability distribution used to calculate the available wind speed at that particular location and the estimation of parameters is the essential part in installing wind farms. The improved mixture Weibull distribution*** $MWbl(2,3)$ ***is proposed model which is the mixture of two and three parameter Weibull distribution with parameters including scale, shape, location and weight component. The basic properties of the proposed model and estimation of parameters using various methods are discussed.***

Index Terms**:** ***Improved Mixture Weibull Distribution, Maximum likelihood method, Parameter Estimation, Wind speed.***

I. INTRODUCTION

In this paper we discuss about the application of advanced Weibull distribution models in wind energy. Enormous number of authors accept Weibull distribution as the best fit in wind resource assessment[1]. A few mixture distributions are available in literature but we have taken only the mixture of Weibull distribution because of flexibility[2].

Several authors have used Two and three parameter Weibull distribution for wind sites having single peak distribution. But this does not suit all the locations with varying wind speed data. In this case we have to implement the mixture Bimodal Weibull distribution [3] . A few areas experience calm wind speed for several days of the whole year which is not included in the estimation of average wind speed often which leads to some drawback in the computation part. So we have considered calm wind speed also in improved mixture Weibull distribution $MWbl(2,3)$ [4]**.** In section two of this paper the improved mixture Weibull distribution is defined. In the next section we have the description of the sites. In the fourth chapter the distribution of wind speed is given. In fifth chapter we have the computation of the parameter using MLE method continued with wind power equation. The objective of the study is to develop a new mathematical formulation for computing wind speed.

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II. WEIBULL DISTRIBUTION

A. Two Parameter Weibull Distribution

The probability density function of the two parameter Weibull distribution is given by

$$f(v;\alpha,\beta)=\frac{\alpha}{\beta}\left(\frac{v}{\beta}\right)^{\alpha-1}\exp\left[-\left(\frac{v}{\beta}\right)^{\alpha}\right]\quad for\quad v>0 \qquad (1)$$

α and β are the shape and scale parameters respectively. The unit of wind speed is (m/s) [5].

The cumulative distribution function (cdf) [6]is defined by $F(v)=1-\exp\left[-\left(\frac{v}{\beta}\right)^{\alpha}\right]$ (2)

B. Three Parameter Weibull Distribution

The PDF of the three parameter Weibull distribution is given by (3)

$$f(v;\alpha,\beta,\gamma)=\frac{\alpha}{\beta}\left(\frac{v-\gamma}{\beta}\right)^{\alpha-1}\exp\left[-\left(\frac{v-\gamma}{\beta}\right)^{\alpha}\right]\quad for\quad v>0$$

where γ is the location parameter.

The associated cumulative distribution function is

$$F(v)=1-\exp\left[-\left(\frac{v-\gamma}{\beta}\right)^{\alpha}\right] \qquad (4)$$

C. Improved Mixture Weibull Distribution

The proposed Improved mixture Weibull distribution $MWbl(2,3)$ is a mixture of two and three parameter Weibull distributions with a weight component[7]. A random variable V that is denoted as v_i with mixing parameter is said to have a improved mixture Weibull distribution $MWbl(2,3)$ with probability density function

$$ff_{(2,3)}(v;\alpha_1,\beta_1,\alpha_2,\beta_2,\omega,\gamma_0)=\omega f(v;\alpha_1,\beta_1)+(1-\omega)f(v;\alpha_2,\beta_2,\gamma_0)$$

$$=\omega\left\{\frac{\alpha_1}{\beta_1}\left(\frac{v}{\beta_1}\right)^{\alpha_1-1}\exp\left[-\left(\left(\frac{v}{\beta_1}\right)^{\alpha_1}\right)\right]\right\}$$

$$+(1-\omega)\left\{\frac{\alpha_2}{\beta_2}\left(\frac{v-\gamma_0}{\beta_2}\right)^{\alpha_2-1}\exp\left[-\left(\left(\frac{v-\gamma_0}{\beta_2}\right)^{\alpha_2}\right)\right]\right\} \qquad (5)$$

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Cumulative Distribution function:

The cumulative distribution function of $MWbl(2,3)$ is given by

$$FF_{(2,3)}(v;\alpha_1,\beta_1,\alpha_2,\beta_2,\omega,\gamma_0)=P(V\le v)$$

$$=\omega F(v;\alpha_1,\beta_1)+(1-\omega)F(v;\alpha_2,\beta_2,\gamma_0) \qquad (6)$$

$$=\omega\left\{1-\exp\left[-\left(\frac{v}{\beta_1}\right)^{\alpha_1}\right]\right\}+(1-\omega)\left\{1-\exp\left[-\left(\frac{v-\gamma_0}{\beta_2}\right)^{\alpha_2}\right]\right\}$$

III. WIND RESOURCE ASSESSMENT

Wind resource assessment at two sites are specifically considered for the study because of the variation in distribution of the wind.

A Site Specification

. The site is located in Tamil Nadu with Latitude $78^{\circ}7\ 46'\ N$ and Longitude $10^{\circ}\ 79\ 04'$ E.

Figure 1 : Thiruchirapalli

B Wind Source Data

The wind speed data are observed using an anemometer at 10m height above the sea level. The collected data are saved in the data logger for further process.

Hr	Dir/Speed (km/hr)	Hr	Dir/Speed (km/hr)
1	EAST 11.1	**13**	ESE 11.1
2	ENE 20.4	**14**	ENE 14.8
3	ENE 11.1	**15**	ESE 11.1
4	ENE 7.4	**16**	ENE 18.5
5	NE C	**17**	ENE 9.3
6	NE C	**18**	ENE 11.1
7	NE 14.8	**19**	ENE 9.3
8	NE 11.1	**20**	EAST C
9	NE 9.3	**21**	EAST 9.3
10	NNE 9.3	**22**	EAST 7.4
11	NNE 11.1	**23**	ESE 11.1
12	ENE 9.3	**24**	ESE 7.4

Figure 2. Wind speed data

C Site Specification -Sulur

The latitude $11^{\circ}0\ 65'\ N$ and longitude $77^{\circ}\ 13\ 12'$ E are the geo coordinates of Sulur. The measured wind speed at a height of 10 m is observed and the average wind speed for a particular day or month is calculated.. The sample wind speed or an average wind speed calculated month wise are given in the graph.

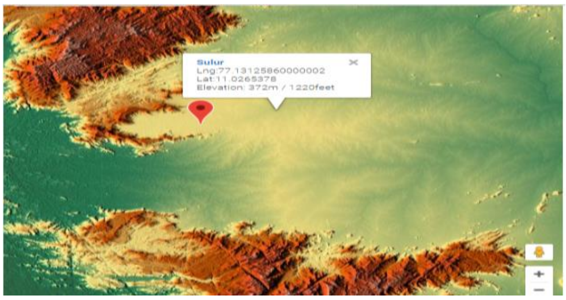

Figure. 3 : Sulur Elevation map

D Wind Source Data

A twelve month time series data (considering hourly averages over a 24 hr period is given) in **TABLE -1**.

Table 1 Wind Speed Data in 24 Hour Time-Series

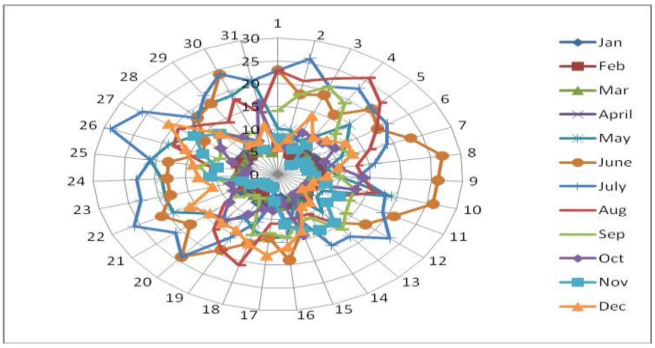

Figure. 4:

IV. WIND SPEED FREQUENCY

The observed wind speed plotted in a graph shows single peak frequency distribution [8].

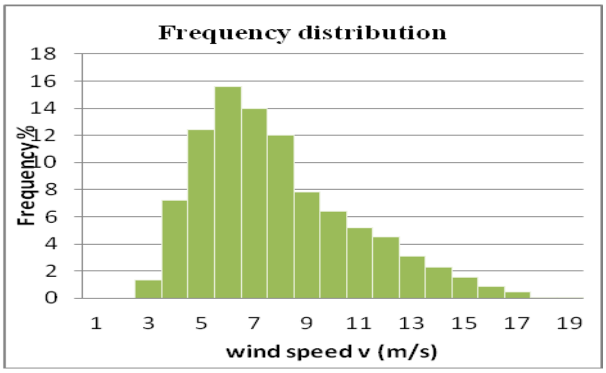

Figure. 5 wind speed observed at Tiruchirapalli

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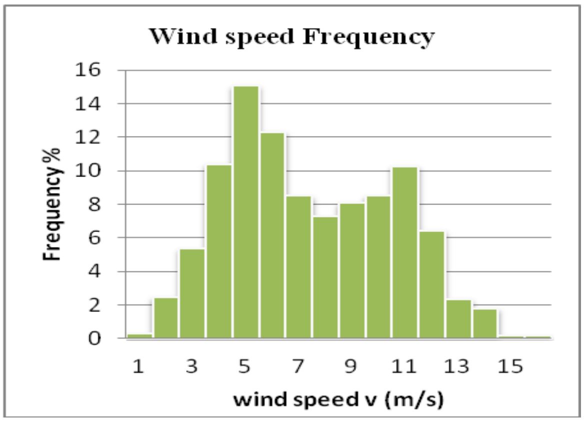

Figure 6 Frequency distribution of wind observed at Sulur

V. ESTIMATION OF PARAMETERS

There are several methods available in literature to estimate the parameters but in our study we discuss only about the maximum likelihood method. MLE is used because their unique properties are not shared by any other parameter estimation methods [9]**.** The scale and shape parameters are estimated using the following equations. The Weibull using the estimated parameters by maximum likelihood method, Least square method, Energy pattern factor method are compared with maximum likelihood method are shown in the graph.

$$\rightarrow \beta_1 = \sum_{i=1}^{n}\left(\frac{v_i^{\alpha_1}}{n}\right)^{\frac{1}{\alpha_1}} \quad or \quad \beta_1^{-\alpha_1} = \sum_{i=1}^{n}\left(\frac{v_i^{\alpha_1}}{n}\right) \tag{7}$$

$$\rightarrow \beta_2 = \sum_{i=1}^{n}\left(\frac{(v_i-\gamma_0)^{\alpha_2}}{n}\right)^{\frac{1}{\alpha_2}} \quad or \quad \beta_2^{-\alpha_2} = \sum_{i=1}^{n}\left(\frac{(v_i-\gamma_0)^{\alpha_2}}{n}\right) \tag{8}$$

$$\frac{\sum_{i=1}^{n} v_i^{\alpha_1}\ \log\ (v_i)}{\sum_{i=1}^{n} v_i^{\alpha_1}} - \frac{1}{\alpha_1} = \frac{\sum_{i=1}^{n}\log\ (v_i)}{n} \tag{9}$$

$$\frac{\sum_{i=1}^{n}(v_i-\gamma_0)^{\alpha_2}\ \log\ (v_i-\gamma_0)}{\sum_{i=1}^{n}(v_i-\gamma_0)^{\alpha_2}} - \frac{1}{\alpha_2} = \frac{\sum_{i=1}^{n}\log\ (v_i-\gamma_0)}{n} \tag{10}$$

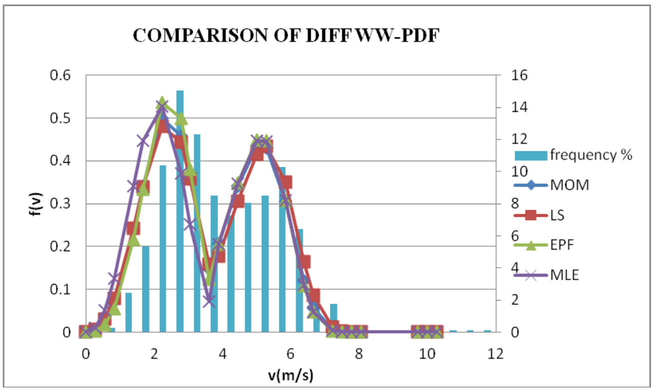

Figure 7

VI. WIND POWER DISTRIBUTION

Wind turbine which converts the wind energy to mechanical energy can be defined as **[10]**

$$P = \frac{1}{2}\,m|v|^2 \tag{11}$$

where v is the wind speed in (m/s) and A is the rotor area of the wind turbine blades.

$$\Rightarrow \quad P = \frac{1}{2}\rho A|v|^3 \tag{12}$$

Moreover the power produced by the turbine depends on the power coefficient C_p, [11] and $\eta \in (0,1)$ as efficiency constant [12].

The power output of the turbine can be written as

$$P = \frac{1}{2}C_p\eta\rho A|v|^3 \tag{13}$$

A. Proposition

The cumulative distribution function of power is defined by

$$FF_{p(2,3)}(v) = \omega\left\{1-\exp\left[-\frac{1}{\beta_1^{\alpha_1}}\left(\frac{2v}{\rho A}\right)^{\frac{\alpha_1}{3}}\right]\right\} + (1-\omega)\left\{1-\exp\left[-\frac{1}{\beta_2^{\alpha_2}}\left(\frac{2(v-\gamma_0)}{\rho A}\right)^{\frac{\alpha_2}{3}}\right]\right\} \tag{14}$$

And the probability density function corresponding to the cdf is

$$ff_{p(2,3)}(v) = \omega\left\{\exp\left[-\frac{1}{\beta_1^{\alpha_1}}\left(\frac{2v}{\rho A}\right)^{\frac{\alpha_1}{3}}\right]\right\}\frac{1}{\beta_1^{\alpha_1}}\,\frac{\alpha_1 v^{\frac{\alpha_1}{3}-1}}{3}\left(\frac{2}{\rho A}\right)^{\frac{\alpha_1}{3}}$$
$$+(1-\omega)\left\{1-\exp\left[-\frac{1}{\beta_2^{\alpha_2}}\left(\frac{2(v-\gamma_0)}{\rho A}\right)^{\frac{\alpha_2}{3}}\right]\right\}\frac{1}{\beta_2^{\alpha_2}}\,\frac{\alpha_2 (v-\gamma_0)^{\frac{\alpha_2}{3}-1}}{3}\left(\frac{2}{\rho A}\right)^{\frac{\alpha_2}{3}} \tag{15}$$

Proof :

$$FF_{p(2,3)}(v) = P(P \le v) = P\left(V \le \left(\frac{2v}{\rho A}\right)^{\frac{1}{3}}\right)$$

$$= \omega\left\{1-\exp\left[-\frac{1}{\beta_1^{\alpha_1}}\left(\frac{2v}{\rho A}\right)^{\frac{\alpha_1}{3}}\right]\right\} + (1-\omega)\left\{1-\exp\left[-\frac{1}{\beta_2^{\alpha_2}}\left(\frac{2(v-\gamma_0)}{\rho A}\right)^{\frac{\alpha_2}{3}}\right]\right\} \tag{16}$$

Differentiating both sides with respect to v, we get the pdf

$$ff_{p(2,3)}(v) = \omega\left\{\exp\left[-\frac{1}{\beta_1^{\alpha_1}}\left(\frac{2v}{\rho A}\right)^{\frac{\alpha_1}{3}}\right]\right\}\frac{1}{\beta_1^{\alpha_1}}\,\frac{\alpha_1 v^{\frac{\alpha_1}{3}-1}}{3}\left(\frac{2}{\rho A}\right)^{\frac{\alpha_1}{3}}$$
$$+(1-\omega)\left\{1-\exp\left[-\frac{1}{\beta_2^{\alpha_2}}\left(\frac{2(v-\gamma_0)}{\rho A}\right)^{\frac{\alpha_2}{3}}\right]\right\}\frac{1}{\beta_2^{\alpha_2}}\,\frac{\alpha_2 (v-\gamma_0)^{\frac{\alpha_2}{3}-1}}{3}\left(\frac{2}{\rho A}\right)^{\frac{\alpha_2}{3}}$$

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Thus the PDF is obtained.

B. Power Distribution

The wind speed that flows through the turbine producing maximum power between the cut-in and rated wind speed , having maximum efficiency between the rated and cut-off wind speed **[13]** is given by

$$V_{turbine} = \begin{cases} 0 & V_{turbine} < v_{cut-in} \\ V & v_{cut-in} \leq V \leq v_{rated} \\ v_{rated} & v_{rated} < V < v_{cut-off} \\ 0 & V \geq v_{cut-off} \end{cases} \quad (17)$$

TESTS	α_1	α_2	β_1	β_2
LS	3.38192	6.29637	2.66	5.41492
MLE	3.25216	6.3886	2.37929	5.26293
EPF	3.85618	6.40486	2.63343	5.26294
EM	3.51636	6.17992	2.64675	5.27318

Where v_{cut-in} is the wind speed at which the turbine starts operating and $v_{cut-off}$ is the highest wind speed at which the turbine shuts down. And these units are fixed by the manufacturer [14].

And $P_{turbine}$

$$= \begin{cases} 0 & V_{turbine} < v_{cut-in} \\ \frac{1}{2}\rho C_p \eta A V^3 & v_{cut-in} \leq V \leq v_{rated} \\ \frac{1}{2}\rho C_p \eta A {v_{rated}}^3 & v_{rated} < V < v_{cut-off} \\ 0 & V \geq v_{cut-off} \end{cases} \quad (18)$$

VII. RESULTS

The PDF drawn from the estimated parameters clearly shows that only improved Weibull parameters computed using Maximum likelihood method gives a best fit. γ_0 is the minimum wind speed experienced at that location.

VIII. CONCLUSION

An improved mixture Weibull distribution function plays a vital role in the assessment of wind resource and for the installation of wind turbines. A new mathematical formulation has been derived to estimate the wind experienced by the turbine. The improved mixture Weibull distribution is different from the other mixture Weibull distribution because of the location parameter. This method can be used in specific location where the site experiences wind speed which represents a bimodal frequency peak.

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