Image Segmentation using a Fuzzy Roughness Measure

Sheeja T. K., Sunny Kuriakose A.

Abstract: Measures of uncertainty are highly useful for determining the information content of a system. In this paper, new measures of information on fuzzy approximation spaces are introduced based on divergence measures of fuzzy sets. The proposed fuzzy rough uncertainty measure is used to develop an algorithm for histogram based foreground background segmentation of a grey level image and it is experimented with twelve standard test images. It is observed that the overlapping of the foreground background pixels in the images segmented using the proposed method is lesser than those produced by OTSU and FCM methods. The segmented images are compared using their root mean square error values.

Index Terms: Rough set, Fuzzy rough set, Uncertainty measure, Image segmentation.

I. INTRODUCTION

Image segmentation refers to partitioning of an image into different non-intersecting homogeneous regions. It is a basic process in image analysis. There are many different methodologies that use gray level histogram, spatial details, mathematical morphology, fuzzy set theoretic approaches, neural networks etc. which are successfully applied in image segmentation [3], [8], [7]. Gray level thresholding is a simple and popular method used for the binary segmentation of a gray level image. Some thresholding techniques are described in [5], [18]. OTSU and FCM methods are two classic methods used for image segmentation which are very popular in the image processing research domain [1], [14]. In OTSU method, the threshold is obtained by maximizing the inter-class intensity variance, whereas in FCM method the threshold is determined by optimizing an objective function defined using a measure of similarity between the pixels and the cluster centres.

The theory of rough sets was introduced by Z. Pawlak in early 1980’s [15]. It has become a potential tool to manage the uncertainty in data caused by incomplete information. Rough set theory along with its generalized versions have found uncertainty in data caused by incomplete information. Rough set concepts to image processing and conclusion is given in section 5. In this paper, new uncertainty measures on fuzzy approximation spaces are defined based on fuzzy divergence measures. Divergence measures provide an indication of the extent of difference between two fuzzy sets [13]. It is shown that these new measures are generalizations of some of the existing uncertainty measures on crisp and fuzzy approximation spaces. Further, a histogram based image thresholding algorithm using the fuzzy roughness measure is presented and experimented with different images. The segmented images of twelve standard test images namely, baboon, barbara, boat, bridge, cameraman, house, lena, mountain, peppers, rice, sailboat and zelda are compared with those of OTSU method and FCM method. It is observed that the overlapping of foreground background pixels in the images segmented using the proposed method is lesser than those produced by the OTSU and FCM methods. Further, the proposed algorithm produces images having the least root mean square error values in ten out of twelve cases. The remaining part of the paper is organized as follows: some basic concepts related to this paper are recalled in section 2, section 3 presents the new fuzzy rough uncertainty measures, section 4 describes an application of the proposed measure to image segmentation and conclusion is given in section 5.

II. BASIC CONCEPTS

Consider a non-empty finite set of objects denoted by $U$.

Let $\Theta$ be a fuzzy equivalence relation on $U$.

**Definition 2.1.** [9] The fuzzy equivalence classes of $\Theta$ are defined as $\forall u \in U$ as, $\mu_{\Theta}(u) = \Theta(u, v), \forall v \in U$.

**Definition 2.2.** [22] The fuzzy cardinality of a fuzzy subset $X$ of $U$ is given by, $|X| = \sum_{u \in U} \mu_X(u)$

**Definition 2.3.** [4] A weak fuzzy partition on $U$ is a collection $\{A_1, A_2, ..., A_k\}$ of fuzzy sets on $U$ such that $\inf (\max_i A_i (u)) > 0$ and $\sup (\min_i A_i (u)) < 1, \forall i, j$.

**Definition 2.4.** [13] A function $\delta : \mathcal{F}(U) \times \mathcal{F}(U) \rightarrow R$ is said to be a divergence measure if $\forall X, Y, Z \in \mathcal{F}(U)$,

1. $\delta(X, X) = 0$
2. $\delta(X, Y) = \delta(Y, X)$
3. $\max(\delta(X \cup Z, Y \cup Z), \delta(X \cap Z, Y \cap Z)) \leq \delta(X, Y)$

where, $\mathcal{F}(U)$ is the collection of all fuzzy sets on $U$.

**Proposition 2.5.** [13] For the fuzzy subsets $X, Y, Z, W$ of $U$ such that $X \subseteq Y \subseteq Z \subseteq W$, $\delta(Y, Z) \leq \delta(X, W)$

**Definition 2.6.** [15] Let $\theta$ be a crisp equivalence relation on $U$. Then, the lower approximation and the upper approximation of $X \subseteq U$ with respect to $\theta$ are respectively defined as

$\overline{\theta}(X) = \{u \in U : \{u\}_\theta \subseteq X\}$
and
$\underline{\theta}(X) = \{u \in U : \}$
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Definition 2.7. [16] The \( \theta \)-roughness of \( X \subseteq U \) is given by
\[
\rho_\theta(X) = 1 - \frac{\theta(x)}{\theta(x)}
\]

Definition 2.8. [20] Consider \( U/\theta = \{\theta_1, \theta_2, \ldots, \theta_k\} \). An information measure of \((U, \theta)\) is given by
\[
G(\theta) = -\sum_{i=1}^k \frac{|\theta_i|}{|U|} \log_2 \left(\frac{|\theta_i|}{|U|}\right)
\]

Definition 2.9. [4] The fuzzy rough lower and upper approximations of a fuzzy set \( X \) on \( U \) are given by
\[
\mu_\theta(X)(u) = \inf_{u \in \theta} \max \{1 - \theta(u, v), \mu_\theta(v)\}
\]
\[
\mu_\theta(X)(u) = \sup_{u \in \theta} \min \{\theta(u, v), \mu_\theta(v)\}
\]
respectively.

There are several intensive studies on the theory and applications of fuzzy rough sets [12], [17], [21]. A survey of the various approaches to fuzzy rough sets was conducted by L. Deer et al [2].

Definition 2.10. [11] Let \( U = \{u_1, u_2, \ldots, u_n\} \). The entropy of the fuzzy approximation space \((U, \theta)\) is given by,
\[
F(\theta) = -\sum_{i=1}^n \frac{1}{|U|} \log_2 \left(\frac{|U_i|}{|U|}\right)
\]

III. INFORMATION MEASURES ON FUZZY APPROXIMATION SPACES

Let \( U = \{u_1, u_2, \ldots, u_n\} \). Let \([u_i]_\theta\) represents a fuzzy equivalence class of \((U, \theta)\). That is, \([u_i]_\theta \subseteq U \to [0, 1], \mu_{[u_i]_\theta}(v) = \theta(u, v), \forall u, v \in U \). Also, let \( \bar{U} \) be the fuzzy set \( U \to [0, 1], \bar{U}(u_i) = 1, \forall u_i \in U \) and \( \bar{U} \) be the fuzzy set \( \bar{U} : U \to [0, 1], \bar{U}(u) = 0, \forall u \in U \). Let \( \delta(X, Y) \) be a divergence measure on \( F(U) \times F(U) \). For notational convenience, we assume that \( \log_2(0) = 0 \).

Definition 3.1. For the fuzzy approximation space \((U, \theta)\), the \( \delta \)-information entropy is defined as
\[
E_\delta(\theta) = -\frac{1}{|U|} \sum_{i=1}^n \frac{1}{|U_i|} \log_2 \left(1 - \frac{\delta([u_i]_\theta, [u_i]_\theta)}{\delta(\theta)}\right)
\]

Proposition 3.2 On a fuzzy approximation space \((U, \theta)\), if \( \theta(u_i, u_j) = 1, \forall u_i, u_j \in U \), then \( E_\delta(\theta) = 0 \).

Proof:
\[
\theta(u_i, u_j) = 1, \forall u_i, u_j \in U \Rightarrow [u_i]_\theta = \bar{U}, \forall u_i \in U \Rightarrow \delta(\bar{U}, [u_i]_\theta) = \delta(\bar{U}, \theta) = 0, \forall u_i \in U \Rightarrow \log_2 1 = \log_2 1 = 0, \forall u_i \in U
\]

Therefore, \( E_\delta(\theta) = 0 \).

Now, we will show that the information measure on a crisp approximation space defined by Wiermann [20] is a special case of the proposed \( \delta \)-information entropy measure.

Theorem 3.3 Consider a crisp approximation space \((U, \theta)\). Then, \( E_\delta(\theta) = G(\theta) \) if \( \delta(X, Y) = \sum_j |X(u_i) - Y(u_j)| \).

Proof:
Let \( U/\theta = \{\theta_1, \theta_2, \ldots, \theta_k\} \). Then, each \( u \in U \) is an element of exactly one equivalence class. We have,
\[
\theta(u_i, u_j) = \begin{cases} 1, & \text{if } u_i \in [u_i]_\theta \\ 0, & \text{otherwise} \end{cases}
\]

Hence, \( \delta([u_i]_\theta, [u_i]_\theta) = \sum_j |[u_i]_\theta| - |[u_i]_\theta| = \sum_j |[u_i]_\theta| - |[u_i]_\theta| = |[u_i]_\theta| - |[u_i]_\theta| = 0 \)

Therefore, \( E_\delta(\theta) = -\frac{1}{|U|} \sum_j \frac{1}{|U_i|} \log_2 \left(1 - \frac{\delta([u_i]_\theta, [u_i]_\theta)}{\delta(\theta)}\right) = 0 \)

Also, \( \delta(\bar{U}, \bar{U}) = \sum_j |\bar{U}(u_i) - \bar{U}(u_j)| = \sum_j |\bar{U}(u_i)| - \bar{U}(u_i)| = |\bar{U}(u_i)| - |\bar{U}(u_i)| = 0 \)

Thus, \( E_\delta(\theta) = 0 \).

Therefore, \( E_\delta(\theta) = -\frac{1}{|U|} \sum_{i=1}^n \frac{1}{|U|} \log_2 \left(1 - \frac{\delta([u_i]_\theta, [u_i]_\theta)}{\delta(\theta)}\right) = 0 \)

Let \( [u_i]_\theta = \theta_j \). Then, in the above sum, each term \( \frac{1}{|U|} \log_2 \left(\frac{|[u_i]_\theta|}{|U|}\right) \) is repeated \( \theta_j \) times. Hence, it follows that,
\[
E_\delta(\theta) = -\frac{1}{|U|} \sum_{i=1}^n \frac{|[u_i]_\theta|}{|U|} \log_2 \left(\frac{|[u_i]_\theta|}{|U|}\right) = G(\theta)
\]

The fact that the fuzzy rough entropy proposed by J. S. Mi et al. [11] is a special case of the proposed \( \delta \)-information entropy measure is asserted in the next theorem.

Theorem 3.4 On a fuzzy approximation space \((U, \theta)\), \( E_\delta(\theta) = G(\theta) \) for \( \delta(X, Y) = \sum_j |X(u_i) - Y(u_j)| \).

Proof:
\[
E_\delta(\theta) = -\frac{1}{|U|} \sum_{i=1}^n \frac{1}{|U|} \log_2 \left(1 - \frac{\delta([u_i]_\theta, [u_i]_\theta)}{\delta(\theta)}\right) = 0
\]

We have, \( E_\delta(\theta) = G(\theta) \).

Definition 3.6 The joint \( \delta \)-information entropy of the fuzzy equivalence relations \( \theta_1 \) and \( \theta_2 \) on \( U \) is defined as
\[
E_\delta(\theta_1 \cup \theta_2) = -\frac{1}{|U|} \sum_{i=1}^n \frac{1}{|U_i|} \log_2 \left(\frac{\delta([u_i]_{\theta_1 \cup \theta_2}, [u_i]_{\theta_1 \cup \theta_2})}{\delta(\theta_1 \cup \theta_2)}\right)
\]

Definition 3.7 The conditional \( \delta \)-information entropy of the fuzzy equivalence relation \( \theta_1 \) given \( \theta_2 \) is defined to be
\[
E_{\delta \theta_1}(\theta_2) = -\frac{1}{|U|} \sum_{i=1}^n \log_2 \left(\frac{\delta([u_i]_{\theta_1 \theta_2}, [u_i]_{\theta_1 \theta_2})}{\delta(\theta_2)}\right)
\]

Theorem 3.8 For two fuzzy equivalence relations \( \theta_1 \) and \( \theta_2 \) on \( U \), \( E_\delta(\theta_1 \theta_2) = E_\delta(\theta_1) - E_\delta(\theta_2) \).

Proof:
We have, \( E_\delta(\theta_1 \theta_2) = E_\delta(\theta_2) = -\frac{1}{|U|} \sum_{i=1}^n \frac{1}{|U_i|} \log_2 \left(1 - \frac{\delta([u_i]_{\theta_1 \theta_2}([u_i]_{\theta_1 \theta_2})}{\delta(\theta_2)}\right)
\]

Therefore, \( E_\delta(\theta_1 \theta_2) = -\frac{1}{|U|} \sum_{i=1}^n \frac{1}{|U_i|} \log_2 \left(1 - \frac{\delta([u_i]_{\theta_1 \theta_2}([u_i]_{\theta_1 \theta_2})}{\delta(\theta_2)}\right) = 0 \)

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Definition 3.9 Let $\Theta_1$ and $\Theta_2$ be fuzzy equivalence relations on $\mathcal{U}$. Then the $\delta$-mutual information of $\Theta_1$ and $\Theta_2$ is defined as
\[
\delta_{IJ}(\Theta_1, \Theta_2) = - \frac{1}{|\mathcal{U}|} \sum_{u \in \mathcal{U}} \log_2 \left( \frac{\delta(\Theta_1, u) \delta(\Theta_2, u)}{\delta(\Theta_1 \cup \Theta_2, u)} \right).
\]

Theorem 3.10 If $\Theta_1$ and $\Theta_2$ are fuzzy equivalence relations on $\mathcal{U}$, then $I_{\delta}(\Theta_1; \Theta_2) = \delta_{IJ}(\Theta_1, \Theta_2)$.

Proof: We have,
\[
I_{\delta}(\Theta_1; \Theta_2) = \frac{1}{|\mathcal{U}|} \sum_{u \in \mathcal{U}} \log_2 \left( \frac{\delta(\Theta_1, u) \delta(\Theta_2, u)}{\delta(\Theta_1 \cup \Theta_2, u)} \right).
\]

IV. IMAGE SEGMENTATION USING THE FUZZY ROUGHNESS MEASURE

Image segmentation is a major area of research in digital image processing. Object background segmentation falls under the category of region based image segmentation. In this section, the proposed fuzzy rough uncertainty measure is applied in object background segmentation using grey level thresholding.

Consider a grey level image of order $m \times n$. Let $G$ denote the collection of all pixel values in it. Then, $G = \{0,1,\ldots,255\}$. First, we divide the set of grey level values $G$ into $N$ parts and obtain the most frequent grey level value (say $v_k$) in each part by using the histogram of the given image. Corresponding to each $v_k$, for $k = 1,2,\ldots,N$, we define a fuzzy set $A_k$ on $G$ as
\[
A_k(i) = \begin{cases} 1 & \text{if } i = v_k, \\ 0 & \text{otherwise}. \end{cases}
\]

Remark: Definition 3.11 is also applicable to generalized fuzzy rough sets as it depends only on the fuzzy rough approximations.

Theorem 3.12 Let $F$ be a fuzzy approximation space. Then, a fuzzy set $X \neq \emptyset$ on $F$ is defined as
\[
E_{\Theta}(X) = \frac{\delta(\Theta(X), \Theta(X))}{\delta(\Theta(X), \emptyset)}.
\]

Proof: We have, $X = \cup_{u \in \mathcal{U}} \{u \in [u]_\theta \}$, otherwise.

Hence,
\[
\delta(\Theta(X), \Theta(X)) = \sum_{u \in \mathcal{U}} \delta(\Theta(X), u) - \Theta(X(u)) \leq \emptyset(X(u)) \leq \emptyset(X).
\]

Also,
\[
\delta(\Theta(X), \emptyset) = 0.
\]

Therefore,
\[
E_{\Theta}(X) = \frac{\delta(\Theta(X), \Theta(X))}{\delta(\Theta(X), \emptyset)} = \frac{|\Theta(X)| - |\Theta(X)|}{|\Theta(X)|}.
\]

Definition 3.12 Let $(U, \Theta)$ be a fuzzy approximation space. Then, a fuzzy set $X$ on $U$ is said to be

i. $\Theta$-definable if $X(\Theta)$ is an $\Theta$-definable

ii. roughly $\Theta$-definable if $X(\Theta) \neq \emptyset$ and $\Theta(X) \neq \emptyset$

iii. internally $\Theta$-definable if $X(\Theta) = \emptyset$ and $\Theta(X) = \emptyset$

iv. externally $\Theta$-definable if $X(\Theta) = \emptyset$ and $\Theta(X) \neq \emptyset$

v. totally $\Theta$-definable if $X(\Theta) = \emptyset$ and $\Theta(X) = \emptyset$

Theorem 3.13 Let $(U, \Theta)$ be a fuzzy approximation space. Then, $\forall X \in F(U)$,

i. $0 \leq E_{\Theta}(X) \leq 1$

ii. $E_{\Theta}(X) = 0$, if $X$ is $\Theta$-definable

iii. $E_{\Theta}(X) = 1$, if $X$ is totally $\Theta$-definable
h represents the Kullback Leibler symmetrical function [13]. The value of $s$ that makes the fuzzy roughness measure the minimum is selected as the threshold $c$. If the minimum is obtained for more than one value of $s$, then the largest of such values of $s$ is selected as $c$. Then the image is segmented into a binary image.

**Algorithm**

Step1: Input the grey level image $G$
Step2: Find the weak fuzzy partition $P$
Step3: For $s = 1:256$, find the fuzzy set $O_s$ (eqn 8)
Step4: Find the fuzzy rough approximations of $O_s$ (eqns 9 and 10)
Step5: Find the fuzzy roughness measure $G(s)$ of $O_s$ (eqn 11)
Step6: Find the value of $s$ for which $G$ is the minimum
Step7: Segment the image into a binary one using $c$

**Experimental Results**

The histogram based image thresholding algorithm for object background segmentation using the fuzzy roughness measure is implemented using twelve standard test images namely, baboon, barbara, boat, bridge, cameraman, house, lena, mountain, peppers, rice, sailboat and zelda. The original images and the segmented ones using the proposed method, OTSU method and FCM method are presented in figures 1-12.

It is observed that the proposed method produces clear binary images in all the twelve instances. It is clear from the figures that the overlapping of the foreground background pixels in the images segmented using the proposed method is lesser than those of the other two methods in most of the cases. Moreover, the RMSE values of the segmented images using the proposed
algorithm are found to be the least in comparison with those segmented by OTSU and FCM methods. The values are presented in table 1.

<table>
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<th>Sl.No</th>
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<th>Proposed</th>
<th>FCM</th>
<th>OTSU</th>
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<td>House</td>
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</table>

V. CONCLUSION

Measures of uncertainty are proved to be effective tools for image segmentation. In this paper, new fuzzy rough uncertainty measures have been defined on fuzzy approximation spaces. A histogram based image thresholding algorithm using the fuzzy roughness measure is developed and experimented with different images. The segmented images of twelve test images using the proposed algorithm have been compared with those of OTSU method and FCM method. It is observed that the overlapping of the foreground background pixels in the images segmented using the proposed method is lesser than those of the other two methods. Further, the proposed algorithm produces images having the least root mean square error values in ten out of twelve cases. Thus, the proposed image segmentation algorithm can be used as an alternative for OTSU and FCM methods which are two popular methods used for binary segmentation.

REFERENCES


AUTHORS PROFILE

Mrs. Sheeja T. K. was awarded PhD degree by Mahatma Gandhi University, Kottayam in 2019. Her research interests include fuzzy set theory and rough set theory. She has published 5 research papers in international journals. She has fifteen years of teaching experience. She is presently working as an Assistant professor under the Collegiate Education Department of Govt. of Kerala.

Dr. Sunny Kuriakose A. was awarded PhD degree by Cochin University of Science and Technology, Kerala, India in 1995. He has more than three decades of teaching experience. He is currently serving as the Dean and a Professor at Federal by Institute of Science and technology, Angamaly, Kerala, India. Sixteen scholars have been awarded Ph.D degree under his supervision. He has published more than sixty research papers in various national and international journals. He authored two books and edited a number of volumes. His research interest includes Fuzzy Logic, Graph Theory, Decision Theory etc. He served the Kerala Mathematical Association as its General Secretary for eight years. Presently, he is the Academic Secretary of the Association.