

A Cyclotomic Lattice Based Closed Loop QOSTBC for Four Transmit Antennas



K. Senthil Kumar, M. Palanivelan, M. Sivaram, P. Shanmugapriya, V. Bakyalakshmi

Abstract: Drawback of the Space Time Block Code (STBC) is, for complex constellations, full rate and full diversity design exist only for two transmit antennas. In this paper, we propose a novel closed loop Quasi Orthogonal Space Time Block Code (QOSTBC) system based on cyclotomic lattices for four transmit antennas. It is a full rate and full diversity design where the information bits are mapped into four dimensional (4-D) lattice points. The bit error and symbol error performance of the system are evaluated by simulation.

Index Terms: MIMO, QOSTBC, cyclotomic, lattice, diversity product.

I. INTRODUCTION

4G and 5G are required to have improved quality and data rate services. But, the time varying multipath fading makes reliable wireless transmission as a difficult task. Various techniques like transmitter power control, Time diversity [1], Frequency diversity [2], Time interleaving have been adopted to overcome fading. A practical, more effective and widely adopted technique is antenna diversity [3] and [4]. Much attention is paid on transmitter diversity which involves many antennas for transmission. Space Time Trellis Coding (STTC) is appropriate to multiple transmit antennas [5][6][7] and [8]. However, when the number of transmit antennas is fixed, the decoding complexity of STTC increases exponentially with transmission rate.

A simple Transmit diversity was proposed by Alamouti [9] in 1998, regarded as the first Orthogonal Space Time Block Code (OSTBC). Because of its simplicity, STBCs became an attractive approach for transmit diversity. However, for complex constellations, the code rate falls from unity [10]-[15]. The diversity order can be increased by choosing signals from rotated constellations [16], [17]. A closed loop scenario is presented in [18] which increases the transmit diversity by feeding back channel information using one bit. A new QOSTBC design based on cyclotomic lattice was proposed in [19], which is an open loop system. The contribution of the proposed scheme is conversion of the open loop system proposed in [19] into a closed loop system based on the scheme proposed in [18].

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II. SYSTEM MODEL

The codeword S [20] is given by Eq (1)

$$S_J = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{bmatrix} \quad (1)$$

The received signal is given by

$$R = HS_J + N \quad (2)$$

where H represents the channel matrix and N represents the noise.

III. LATTICE QOSTBC

A four dimensional complex cyclotomic lattice $\Gamma_4(G_{4 \times 4})$ [19] and [21] over a ring of integers $Z[\zeta_m]$ is given by

$$\Gamma_4(G_{4 \times 4}) = \{ \mathbf{v} = G_{4 \times 4} \mathbf{a} | \mathbf{a} \in Z[\zeta_m]^4 \} \quad (3)$$

where $\mathbf{v} = [v_1 \ v_2 \ v_3 \ v_4]^T$, $\mathbf{a} = [a_1 \ a_2 \ a_3 \ a_4]^T$, $G_{4 \times 4}$ is a 4 x 4 complex matrix called the generating matrix, v is a point in the complex lattice, a is the point in the real lattice, m is a positive integer; $Z[\zeta_m] = \{Z | Z = Z_1 + Z_2 \zeta_m, \zeta_m = e^{j2\pi/m}, Z_1, Z_2 \in \omega Z\}$; The Minimum Product Distance (MPD) of the $\Gamma_4(G_{4 \times 4})$ is defined as

$$d_{MPD} = \min_{\mathbf{v} \neq \mathbf{v}' \in \Gamma_4(G)} (\|\Delta v_1\| \|\Delta v_2\| \|\Delta v_3\| \|\Delta v_4\|) \quad (4)$$

where $\mathbf{v}, \mathbf{v}' \in \Gamma_4(G_{4 \times 4})$, $\Delta \mathbf{v} = \mathbf{v} - \mathbf{v}' = [\Delta v_1 \ \dots \ \Delta v_4]^T$ and $\mathbf{v} \neq \mathbf{v}'$. The Lattice considered in this paper is a full diversity lattice i.e. $d_{MPD} > 0$.

The block diagram of the proposed system is shown in the fig.1. Four lattice points s_1, s_2, s_3 and s_4 that are closer to the zero in the lattice have been selected. The incoming bit stream is mapped to these lattice points. After multiplying the entries of S_J given by Eq.(1) by four phase factors, the proposed scheme is presented in Eq.(5).

The codeword S_p is given by

$$S_p = \begin{bmatrix} s_1 e^{j\alpha} & s_2 e^{j\beta} & s_3 e^{j\gamma} & s_4 e^{j\theta} \\ -s_2^* e^{-j\alpha} & s_1^* e^{-j\beta} & -s_4^* e^{-j\gamma} & s_3^* e^{-j\theta} \\ -s_3^* e^{-j\alpha} & -s_4^* e^{-j\beta} & s_1^* e^{-j\gamma} & s_2^* e^{-j\theta} \\ s_4 e^{j\alpha} & -s_3 e^{j\beta} & -s_2 e^{j\gamma} & s_1 e^{j\theta} \end{bmatrix} \quad (5)$$

The received signals are given by

$$R = H_p S + N \tag{6}$$

where, $R = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix}$ (7)

$$H_p = \begin{bmatrix} h_1 e^{j\alpha} & h_2 e^{j\beta} & h_3 e^{j\gamma} & h_4 e^{j\theta} \\ h_2^* e^{j\beta} & -h_1^* e^{j\alpha} & h_4^* e^{j\theta} & -h_3^* e^{j\gamma} \\ h_3^* e^{j\gamma} & h_4^* e^{j\theta} & -h_1^* e^{j\alpha} & -h_2^* e^{j\beta} \\ h_4 e^{j\theta} & -h_3 e^{j\gamma} & -h_2 e^{j\beta} & h_1 e^{j\alpha} \end{bmatrix} \tag{8}$$

$$S = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix}, \text{ and } N = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{bmatrix} \tag{9}$$

The Grammian matrix G is calculated as

$$G = H_p^H H_p = U_p + V_p \tag{10}$$

where $U_p = h^2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (11)

and $V_p = \omega \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$ (12)

The parameter ω is given by Eq.(13)

$$\omega = e^{j(\alpha-\beta)} \cdot 2Re(h_1^* h_4) - e^{j(\gamma-\theta)} \cdot 2Re(h_2^* h_3) \tag{13}$$

To get full diversity, ω should be minimized. The ω value can be minimized by knowing the channel information. Based on the channel information, the factors $e^{j(\alpha-\beta)}$ and $e^{j(\gamma-\theta)}$ are adjusted. When $Re(h_1^* h_4) \cdot Re(h_2^* h_3) \geq 0$, the product $e^{j(\alpha-\beta)} \cdot e^{j(\gamma-\theta)}$ is set equal to 1. Otherwise, the product is set equal to -1.

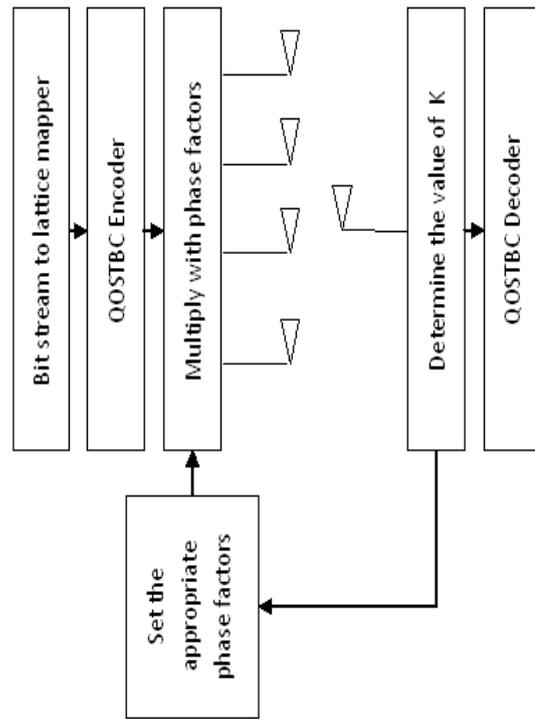


Fig.1 Cyclotomic lattice based closed loop System

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The channel information is fed back to the transmitter using one bit. The feedback bit k is set equal to zero, when $Re(h_1^* h_4) \cdot Re(h_2^* h_3) \geq 0$. Otherwise, set k equal to one. On examining the value of k, the phase factors are set as follows. The angles α and γ in Eq.(5) are set equal to 180°, the angles β and θ in Eq.(5) are set equal to zero, if k=1. The angles α , β and θ are set equal to zero, the angle γ is set equal to 180°, if k=0.

IV. SIMULATION RESULTS

The parameters considered for simulating the proposed system is given in the Table I.

Table I Simulation parameters

Channel	Quasi-static, Flat fading
Lattice points	S1 = [-0.1399 - 0.1016i 0.5000 + 0.3633i -0.0572 - 0.0416i 0.2045 + 0.1486i] S2 = - S1 S3 = [-0.3489 + 0.2605i 0.4563 - 0.0526i 0.2226 + 0.2692i -0.2045 + 0.0616i] S4 = -S3
Noise	AWGN
Channel State Information(CSI)	Known at both Transmitter and Receiver end
No. of Transmit antennas	4
No. of Receive antennas	1
Type of Decoding	Maximum Likelihood Decoding

As it is impossible to search the set of lattice points which are used here, $\Theta = \{-3, -1, 1, 3\}$ is chosen. The generating matrix $G_{4 \times 4}$ considered in this paper is given by Eq.(14)

$$G_{4 \times 4} = \begin{bmatrix} \lambda & \lambda^2 & \lambda^3 & \lambda^4 \\ \lambda^7 & \lambda^{14} & \lambda^{21} & \lambda^{28} \\ \lambda^{13} & \lambda^{26} & \lambda^{39} & \lambda^{52} \\ \lambda^{19} & \lambda^{38} & \lambda^{57} & \lambda^{76} \end{bmatrix} \quad (14)$$

where $\lambda = e^{j\pi/15}$ and the $\Gamma_4(G_{4 \times 4})$ is formed over a ring of integers $Z[\zeta_6]$ [19][20] and [21].

Fig.3 compares the Bit error performance of cyclotomic lattice based open loop system and the proposed cyclotomic lattice based closed loop system.

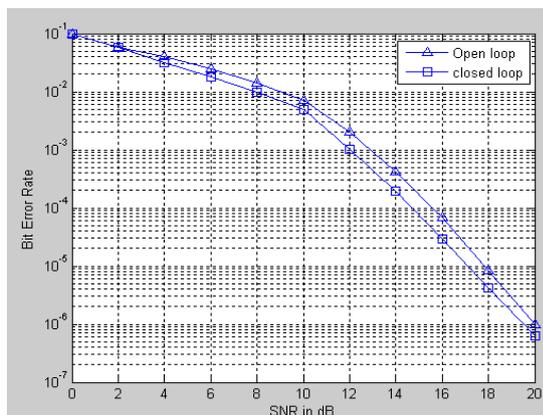


Fig. 3 Bit error rate versus Signal to noise ratio for lattice based open and closed loop QOSTBC

The proposed lattice based closed loop QOSTBC outperforms the open loop lattice based QOSTBC. Table II compares the Bit error rate of cyclotomic lattice based open loop system and the proposed cyclotomic lattice based

closed loop system at different values of SNR. It is observed that at 12 dB, the Bit error performance of the proposed closed loop system is 49.24% better than the open loop system.

Table II Signal to Noise Ratio Vs Bit Error Rate

SNR in dB	BER (Closed Loop)	BER (Open Loop)
0	1.0×10^{-1}	1.0×10^{-1}
2	5.7×10^{-2}	5.9×10^{-2}
4	3.2×10^{-2}	4.1×10^{-2}
6	1.8×10^{-2}	2.5×10^{-2}
8	1.0×10^{-2}	1.4×10^{-2}
10	5.0×10^{-3}	7.0×10^{-3}
12	10.1×10^{-4}	19.9×10^{-4}
14	1.98×10^{-4}	4.23×10^{-4}
16	29.13×10^{-5}	6.75×10^{-5}
18	4.15×10^{-6}	8.24×10^{-6}
20 dB	6.30×10^{-7}	1.0×10^{-6}

Table III compares the Bit error rate of cyclotomic lattice based open loop system and the proposed cyclotomic lattice based closed loop system at different values of Bit error ratio. It is observed that at BER of value 10^{-3} , the proposed closed loop system achieves a gain of 0.8 dB.

Table III Bit Error Rate Vs Signal to Noise Ratio

BER	SNR in dB (Closed Loop)	SNR in dB (Open Loop)
10^{-1}	0	0
10^{-2}	8.00	9.00
10^{-3}	11.99	12.79
10^{-4}	12.74	15.65
10^{-5}	17.14	17.82

Fig.4 compares the symbol error performance of cyclotomic lattice based open loop system and the proposed cyclotomic lattice based closed loop system. It has been observed that the proposed scheme outperforms the conventional scheme.

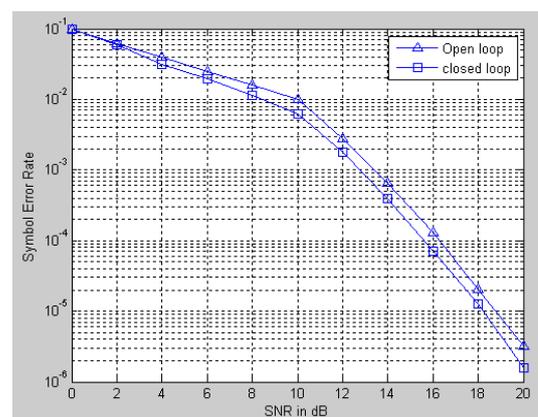


Fig. 4 Symbol error rate versus Signal to noise ratio for lattice based open and closed loop QOSTBC

Table IV compares the symbol error rate of cyclotomic lattice based open loop system and the proposed cyclotomic lattice based closed loop system at different values of SNR. It is observed that at 12 dB, the proposed system outperforms the conventional system by 35.71%.

Table IV. Signal to Noise Ratio Vs Symbol Error Rate

SNR in dB	SER (Closed Loop)	SER (Open Loop)
0	1×10^{-1}	1×10^{-1}
2	5.9×10^{-2}	6.12×10^{-2}
4	3.15×10^{-2}	4×10^{-2}
6	1.95×10^{-2}	2.51×10^{-2}
8	11.5×10^{-3}	1.6×10^{-2}
10	6.3×10^{-3}	1×10^{-2}
12	1.8×10^{-3}	2.8×10^{-3}
14	4.0×10^{-4}	6.5×10^{-4}
16	7.11×10^{-5}	13.12×10^{-5}
18	12.6×10^{-6}	20.2×10^{-6}
20	1.58×10^{-6}	3.16×10^{-6}

Table V compares the Symbol error rate of cyclotomic lattice based open loop system and the proposed cyclotomic lattice based closed loop system at different values of Symbol error ratio. It is observed that at SER of value 10^{-3} , the proposed closed loop system achieves a gain of 0.705 dB.

Table V Symbol Error Rate Vs Signal to Noise Ratio

SER	SNR in dB (Closed Loop)	SNR in dB (Open Loop)
10^{-1}	0	0
10^{-2}	8.5	10
10^{-3}	12.65	13.33
10^{-4}	15.71	16.25
10^{-5}	18.10	18.75

V.CONCLUSION

A novel closed loop QOSTBC system for four transmit antennas based on cyclotomic lattices is proposed in this paper. As every two bits of the input bit stream is mapped to a lattice point, the spectral efficiency of the proposed system is 2 bits per channel use. The channel state information is fed back to the transmitter using 1 bit, which further improves the system performance. From the simulation results, it can be observed that the proposed lattice based closed loop QOSTBC for four transmit antennas outperforms the lattice based open loop QOSTBC system.

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