International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249-8958 (Online), Volume-8 Issue-5, June 2019

Finite Field Discrete Cosine Transform for Image Processing Applications

Salila Hegde, Rohini Nagapadma

Abstract**:** ***Discrete transforms such as DCT are found useful in several image processing applications. There exist discrete transforms for finite fields which use integer arithmetic. Discrete cosine transform over finite field (FFCT) is one such transform which uses k-cosine trigonometry. As all the arithmetic is carried out on integers the image processing algorithms based on FFCT do not suffer from round off errors as in DCT based algorithms. In this paper we investigate properties of FFCT and discuss parameters used for two dimensional FFCT pair. The transformation kernel over GF (p) where p is prime is calculated and FFCT of 8x8 regular test images are obtained. From the study of FFCT of the test images we suggest the application of this transform to detect one bit error that occurs in photo mask image of integrated circuits. An image compression algorithm is also implemented to show that FFCT based compression algorithms are lossless and yield higher compression ratio.***

Index Terms: DCT, Error detection,FFCT, Finite field,Image compression.

I. INTRODUCTION

In the previous years there have been numerous applications of discrete transforms in the field of image and signal processing. Discrete cosine transform(DCT) is an attractive choice among discrete transforms as it reduces blocking artifacts in images and high energy compaction compared to DFT. DCT is used as a standard in image compression.JPEG is the most widely used image compression standard, which uses DCT. DCT converts integer valued signal samples into real valued coefficients. After taking DCT of blocks of image the coefficients are quantized and rounded off. This introduces an error in algorithms based on DCT.

Pollard introduced DFT over finite fields in 1971 [1] and showed discrete convolutions using integer arithmetic. Since then transforms over finite fields like finite field Hartley transform (FFHT) [2], finite field wavelet transform (FFWT)[3]-[4],and finite field cosine transform(FFCT) [5] have been introduced and applied in signal and image processing applications, cryptography, error control coding and detection etc. These transforms gained significance as they employ modulo arithmetic and work with only integers and overcome round off errors.

Encouraged by these aspects we have carried out investigations on finite field cosine transform (FFCT) which was originally introduced by M.M. Campello de Souza ,H.M.de Oliveira, R.M. Campello de Souza, M.M. Vasconcelos [6]. We also carried out study and implementation of characteristics of FFCT. Section 2 of this paper gives the definition of finite field trigonometry and FFCT. Section 3 lists FFCT parameters and gives details of transformation matrices used for our algorithm. In section 4 we investigate characteristics FFCT of 8x8 regular images which is useful for image compression. In section 5 we suggest two different applications namely 1-bit error detection and image compression. In section 6 we discuss implementation and results. Finally in section 7 conclusion and future scope are presented.

II. FINITE FIELD TRIGONOMETRY AND DEFINITION OF FFCT

Finite field trigonometry concepts are explained in detail in the papers [2,5,810] by Campello de Souza and co-authors. We referred those papers and give here brief review of definition of FFCT.

General forward and inverse transform equations are given by

$$V_k = \sum_{i=0}^{N-1} u_i K(i,k,\zeta) \quad k = 0,1,2,\ldots.N-1) \tag{1}$$

$$u_i = \sum_{k=0}^{N-1} V_k K^{-1}(i,k,\zeta) \quad i = 0,1,\ldots.N-1) \tag{2}$$

Where $u = (u_0, u_1 \ \ldots. u_N - 1)$ and $V = (V_0, V_1 \ \ldots. V_N - 1)$ are two sequences of length N. Their elements belong to a Galois field GF. $K\ (i,k,\zeta\ )$ and $K^{-1}(i,k,\zeta)$ are kernels respectively and ζ is the fixed element of given GF.

Let GF (p) be a Galois field with p as odd prime.

If $p \equiv 3$ (mod 4) then extension GF (p^2) can be constructed and it is isomorphic to the Gaussian integer field

GI$(p)\ = (x + jy;\ x, y\epsilon GF\ (p))$, where

$j^2 = -1(mod\ p)$. [7].

$\zeta\ \epsilon\ GI(p)$ is an entity with multiplicative order N, then trigonometric cosine function is defined by

Manuscript published on 30 June 2019.
* Correspondence Author (s)
Salila Hegde*, Department of ECE, NIE Institute of Technology, Mysore, INDIA..
Rohini Nagapadma, Department of ECE, National Institute of Engineering, Mysore, INDIA.

$$Cos\zeta(i) = (\zeta_i + \zeta_{-i})/2 \quad i = 0, 1 \dots N-1 \tag{3}$$

Definition I: (Uni modularity; an element $x + jy \ \epsilon GI\ (p)$ is uni modular if $x^2 + y^2 = 1 (mod\ p)$. This property accomplishes two things

1. Finite field trigonometric function induces no complex arithmetic.

2. All computations performed by means of modulo arithmetic and $cos\zeta\ (i)\ = Real\ (\zeta i)$.

Definition II: Finite field cosine transform (FFCT) maps an N-dimensional sequence with elements $ui\epsilon GF\ (p);\ i\ = 0, 1, N-1$ into another sequence $V_k$as,

$$V_k = \sum_{i=0}^{N-1} 2u_i \cos\zeta\big((2i+1)k\big) \tag{4}$$

The inverse FFCT is given by,

$$u_i = \frac{1}{N\ mod\ \ p} \sum_{k=0}^{N-1} \alpha_k V_k \cos\zeta((2i+1)k) \tag{5}$$

Where $k\ \epsilon\ GI\ (p)$ with order $2N$ and

$$\alpha_k = \begin{cases} 2^{-1}(mod\ \ p) & if\ i = 0 \\ 1 \quad otherwise \end{cases} \tag{6}$$

Campello De Souza gives list of ζ element in the paper [11].

A 2D FFCT can be implemented in matrix form as,

$$V\ = F.U.F^T \tag{7}$$

Where V and U are MxN size data and F is $N \times N$ transformation matrix defined as,

$$Fi,k\ = \ 2cos\ \zeta \quad (2i+1)/k, \quad k = 0, 1, N-1 \tag{8}$$

Inverse two dimensional FFCT is given by

$$U = F^{-1}.V.F^{-1T} \tag{9}$$

Elements of inverse transformation matrix F^{-1} are given by

$$\left(\frac{1}{N\ \ mod\ \ p}\right)\alpha_k cos\ \zeta\ \big((2i+1)k\big)\ \ i, k\ = 0, 1, .N-1 \tag{10}$$

III. FFCT PARAMETERS AND TRANSFORMATION MATRICES

The computation of FFCT for a 1-dimensional vector x is given as

$$X = F.x \tag{11}$$

Where F corresponds to transformation matrix, and X is transformed vector. The coefficients of matrix F are obtained by equation (8). FFCT is extended to 2-dimensional signal x as

$$X = F.x.F^{-1}, \tag{12}$$

For an orthogonal transform

$$F^{-1} = F^T \quad Hence\ X = F.x.F^T \tag{13}$$

We find an 8x8 transformation matrix by choosing parameters as $p = 31$, and $\zeta\ = (7 + j13)\ \epsilon\ GF\ (31)$ has order *(p+1)/2 i.e. 16* [11].

F and F^{-1} are calculated using equations 8 and 10 as shown below [12].

2	2	2	2	2	2	2	2
27	10	20	22	9	11	21	4
14	5	26	17	17	26	5	14
10	9	4	11	20	27	22	21
8	23	23	8	8	23	23	8
20	4	22	10	21	9	27	11
5	17	14	26	26	14	17	5
22	11	10	4	27	21	20	9

Transformation matrix F

2	23	28	20	16	9	10	13
2	20	10	10	15	8	3	22
2	9	21	21	15	13	28	20
2	13	3	3	16	20	21	8
2	18	3	3	16	11	21	23
2	22	21	21	15	18	28	11
2	11	10	10	15	23	3	9
2	8	28	28	16	22	10	18

Inverse transformation matrix F^{-1}

IV. FFCT OF REGULAR IMAGES

We consider test images of size 8x8 and with different row and column period P_r and P_c respectively. FFCT forward and inverse transforms are implemented using MATLAB13 and result images are presented here. Fig. 1a, 2a and 3a are the test images and Fig. 1b up to 3b are their forward FFCT.

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Fig 1a.Test image 1 with Pr x Pc= 1x1 **Fig 1b: FFCT of 1a.**

5	6	7	8	5	6	7	8
1	2	3	4	1	2	3	4
5	6	7	8	5	6	7	8
1	2	3	4	1	2	3	4
5	6	7	8	5	6	7	8
1	2	3	4	1	2	3	4
5	6	7	8	5	6	7	8
1	2	3	4	1	2	3	4

5	13	0	30	0	25	0	19
1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0

Fig 2a: test image 2 with Pr x Pc= 2x4 **Fig 2b: FFCT of 2a.**

1	2	3	4	1	2	3	4
1	2	3	4	1	2	3	4
1	2	3	4	1	2	3	4
1	2	3	4	1	2	3	4
1	2	3	4	1	2	3	4
1	2	3	4	1	2	3	4
1	2	3	4	1	2	3	4
1	2	3	4	1	2	3	4

20	13	0	30	0	25	0	19
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Fig 3a: test image 3 with Pr x Pc= 1x4 **Fig 3b: FFCT of 3a.**

In a gray scale image each pixel depth is 8 bits and the magnitude is in the range 0-255. In order to avoid data loss due to modular arithmetic we choose p=256 to define FFCT, Using ζ =128 with order 2N=16. An 8x8 transform matrix is defined as in Fig.4a[13].

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication (BEIESP)

15	15	15	15	15	15	15	15
137	163	98	106	151	159	94	120
160	6	251	97	97	251	6	160
163	6	120	159	98	137	106	94
242	15	15	242	242	15	15	242
98	120	106	163	94	151	137	159
6	97	160	251	251	160	97	6
106	159	163	120	137	94	98	151

Fig 4a: $8x8$ transformation matrix for gray images

Test images 1a and 1b are transformed using the matrix in Fig. 4a and the result is as shown in Fig. 4b and 4c.

20	225	0	156	0	228	0	17
193	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
55	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
199	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
87	0	0	0	0	0	0	0

Fig 4b: FFCT of fig 2a

28	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Fig 4c: FFCT of fig 1a

V. APPLICATIONS

A. 1 Bit error detection

The regular structure of FFCT can be made use to detect 1 pixel error in the photo masks used for fabrication of integrated circuits. Before using the mask it must be thoroughly inspected for any possible defects as that will lead to fabrication of faulty circuits. Take a $M \times N$ size section of photo mask where error is suspected and take FFCT. If there is a micro level fault in mask then that particular block will have irregular structure due to 1-bit error and its FFCT also will be irregular. By visual inspection of FFCT faulty masks can be identified.

This is illustrated in Fig. 5a and 5b. Fig. 5a is small 8x8 section of photo mask having regular structure and Fig. 5b shows that its FFCT is also regular. In fig 5c single bit error which represents fault in mask is introduced at first row 7^{th} column position. Fig. 5d shows that its FFCT is drastically different than Fig.5b.

1	1	1	0	1	1	1	0
1	1	1	0	1	1	1	0
1	1	1	0	1	1	1	0
1	1	1	0	1	1	1	0
1	1	1	0	1	1	1	0
1	1	1	0	1	1	1	0
1	1	1	0	1	1	1	0
1	1	1	0	1	1	1	0

Fig: 5a: Regular structure of photo Mask image segment

6	18	0	15	23	5	0	9
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Fig 5b: FFCT of fig 7a

1	1	1	0	1	1	0	0
1	1	1	0	1	1	1	0
1	1	1	0	1	1	1	0
1	1	1	0	1	1	1	0
1	1	1	0	1	1	1	0
1	1	1	0	1	1	1	0
1	1	1	0	1	1	1	0
1	1	1	0	1	1	1	0

Fig 5c: 1 bit in error

2	7	21	2	8	13	28	0
8	22	20	26	30	15	6	18
3	16	23	2	19	25	10	30
11	7	12	28	18	9	16	17
15	18	22	10	2	1	19	26
22	14	24	25	5	18	1	3
21	19	6	14	9	20	8	24
18	3	14	12	21	26	26	25

Fig 5d: FFCT of fig 7c

B. Image compression

In section 4 we obtained FFCT of 8x8 regular images and observed that many coefficients become zero. And also FFCT of the test images are also regular and there is a relationship with row and column nonzero coefficients with row and column periodicity of input image. Inverse FFCT on the transformed images is also found reversible. As these transforms employ modulo arithmetic all the entities are integers and there is no round of error.

This forms the foundation for the application of FFCT to compress regular data effectively.

Real images are not regular in structure and the coefficients in FFCT of the input image don't have any relationship as in DCT. So the images are to be converted into regular images and then FFCT may be applied which will have many zero coefficients.

From the observations made in section IV we suggest an image encoding and decoding algorithm which is described below.

Step1: Find the 2D FFCT of given 8x8 image using transformation matrix F

Step2: From the transformed image it is observed that if $P_r = 1$ and $P_c > 1$ only first row has 5 nonzero coefficients and all other zero coefficients. And vice versa if $P_r > 1$ and $P_c = 1$.

Step3: If $P_r > 1$and $P_c > 1$there are 5 nonzero coefficients in first row and first column only, rest are zero.

Step4: If $P_r = P_c = 1$as in fig 1a, we get only very first coefficient is nonzero.

Step5: So the encoding algorithm need to transmit only rows and columns with nonzero coefficients and Pr and Pc.

Step6: Decoder uses Pr and Pc to reconstruct the original transformed matrix from nonzero coefficients.

Step7: Inverse transform is applied on reconstructed coefficients to get back the original image

VI. RESULTS AND DISCUSSIONS

All implementations are done using MATLAB13 on 64 bit processor. A gray scale image each pixel depth is 8 bits and the magnitude is in the range 0-255**.** In order to avoid data loss due to modular arithmetic we choose *p*=256 to define FFCT, Using ζ =128 with order 2N=16. An 8x8 transform matrix is defined as in Fig. 6 [13].

15	15	15	15	15	15	15	15
137	163	98	106	151	159	94	120
160	6	251	97	97	251	6	160
163	6	120	159	98	137	106	94
242	15	15	242	242	15	15	242
98	120	106	163	94	151	137	159
6	97	160	251	251	160	97	6
106	159	163	120	137	94	98	151

Fig 6: $8x8$ transformation matrix for gray images

We find FFDCT and DCT of grayscale image in Fig. 7a using transformation kernel as in Fig.6. Figures 7b and 7c are the transformed images respectively.

255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255

Fig. 7a: Gray scale image

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication (BEIESP)

241	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Fig 7b: FFCT of image in fig 6a

2040	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Fig 7c: DCT of image in fig 6a

Fig. 7a is a gray scale image of size 8x8 which has all pixels valued 255. From the results of Fig. 7b and 7c it is clear that to store DCT coefficients more bits are required and hence FFCT has advantages of better compression than DCT.

FFCT is 100 percent lossless as it involves only integers due to modulo arithmetic and hence it is free of round off error. This we verified by carrying out forward and inverse transforms using DCT and FFDCT on grayscale image in Fig.8. Table I lists the comparison between DCT and FFDCT based algorithms.

242	120	97	94	15	159	251	151
120	246	136	47	64	208	136	227
97	136	181	195	96	32	16	137
94	47	195	2	1	114	2	139
15	64	96	1	15	241	10	253
159	208	32	114	241	46	36	8
251	136	16	2	10	36	213	137
151	227	137	139	253	8	137	100

Fig.8. gray scale image of size 8x8

Table I: Comparison between DCT and FFDCT

	DCT	FFDCT
Compression ratio For image in figure 7a	46.54	56.88
No of bits in error after inverse transform for image in figure 8	5	0

VII. Conclusion And Future Scope

In this paper review of definition of FFCT is done and characteristics of 2DFFCT are investigated using MATLAB implementation. Some new applications namely image compression and 1-bit error detection are suggested. FFCT of 8x8 test images which are regular in structure are analyzed and compression algorithm for the same is implemented Based on this investigation, it is very clear that further study of FFCT is beneficial to explore further applications.

It is forecasted that compression scheme using FFCT is supposed to give higher compression ratio. Algorithms using DCT as in JPEG image compression have round off errors due to quantization process. As these transforms are carried out using modulo arithmetic it involves only integers and hence the algorithms using FFCT are lossless. In this study FFCT is carried out using 8x8 test images. In future this algorithm may be extended for real images.

REFERENCES

1. POLLARD, J.M, 'The fast Fourier transform in a finite field', 1971,Math.Comput., 25, pp. 365-374.
2. R.M.Campello de Souza, H.M. de Oliveira,A.Kauffman,A.J.A. Paschoal, Trigonometry infinite fields and a new Hartley transform,in: Proceedings of the IEEE International Symposium Information Theory, 1998, pp. 293
3. F. Fekri, S.W.McLaughlin, R.M. Mersereau, R.W. Schafer, Block error correcting codes using finite-fieldwavelet transforms, IEEE Trans. Signal Process. 54 (2006) 991–1004.
4. K.S. Chan, F. Fekri, A block cipher cryptosystem usingwavelet transforms over finite fields, IEEE Trans. Signal Process. 52 (2004) 2975–2991.
5. J.B. Lima, R.M. Campello de Souza, New trigonometric transforms over prime finite fields for image filtering, in: Proceedings
6. M.M. Campello de Souza,H.M.de Oliveira,R.M.Campello de Souza, M.M. Vasconcelos, The discrete cosine transform over prime finite fields, in:J.N.deSouza,P.Dini,P.Lorenz(Eds.),International Conference on Telecommunications,LectureNotes in Computer Science, Springer,Berlin,2004,pp.482–487.
7. R.E. Blahut, Fast Algorithms for Digital Signal Processing, Addison-Wesley Reading, 1985.
8. R.M.CampellodeSouza,H.M.deOliveira,L.B.Espinola-Palma,M.M Campello de Souza,Transform adasnume´ ricas de Hartley,in:XVIII Brazilian Symposium on Telecommunications, Gramado, Brazil, September 2000
9. R.M.CampellodeSouza,H.M.deOliveira,L.B.Espinola-Palma,M.M Campello de Souza,Hartley number theoretic transforms,in:IEEE International Symposium on InformationTheory,Washington,DC, USA, June2001.
10. R.M.Campellode Souza,H.M.deOliveira,A.N.Kauffman,The Hartley transform in a finite field, Journal of the Brazilian Telecommunications Society 14(1) (1999) 46–54.
11. R.J.Cintra,V.S.Dimitrov,H.M. de Oliveira, R.M.Campello de Souza, Fragile watermarking using finite field trigonometrical tramnsforms, in: Signal Processing: Image Communication,2009,24,587–597.
12. R.M.C.,Vasconcelos M.M. (2004) The Discrete Cosine Transform over Prime Finite Fields. In: de Souza J.N., Dini P., Lorenz P. (eds) Telecommunications and Networking - ICT 2004. ICT 2004. Lecture Notes in Computer Science, vol 3124. Springer, Berlin, Heidelberg.
13. J.B.Lima,E.A.O.Lima,F.Madeiro, Image encryption based on the finite field cosine transform, 2013,Signal processing:Image communication,vol.28, issue 10,pp.1537-1547.

AUTHORS PROFILE

Salila Hegde is Associate Professor & Head of the Department of Electronics & Communication Engineering. She is pursuing her Ph.D. from VTU, M.Tech from VTU, Belgavi and B.E. from Karnataka University, INDIA.

Her research interests are in the areas of Image Processing, Multimedia, VLSI and Embedded systems. Special Interest has been shown in the area of image compression when working towards PhD. She is teaching undergraduate and handles subjects like Analog electronic circuits, Linear integrated circuits, Microcontrollers/Microprocessors, Image processing, CMOS VLSI design, Programming in C++, ARM microcontrollers and embedded systems. She has 15 years of academic and 8 years of industrial experience. She has designed and developed microcontroller based systems and digital communication systems for satellite.

She has published papers in international journals and conferences. Many student projects in the UG level have been guided in the area of Image processing,

VLSI circuits, embedded systems. These projects have resulted in few paper publications in National and International conferences.

Dr. Rohini Nagapadma is Professor & Head of the Department of Electronics & Communication Engineering. She has received her Ph.D. from VTU, M.Tech from IIT, Madras and M.E.M. and B.E. from University of Mysore.

Her research interests are in the areas of Image Processing and Multimedia. Special Interest has been shown in the area of image compression when working towards PhD.

She is teaching both undergraduate and postgraduate students and handles subjects like Telecommunication Switching, Signals and Systems, Wireless Communication, DSP, Image processing and Multimedia.
She has published papers in international journals and conferences. Many student projects in the UG level have been guided in the area of Image Compression and Image Enhancement. These projects have resulted in few paper publications in National and International conferences. Presently two candidates are working towards Ph.D under her.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication (BEIESP)