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T-Odd Sequential Harmonious Labeling of Cycle C_N with Parallel Chords

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Abstract: ***Labeling is an active area of research in graph theory with its applications in communication network, coding theory, channel assignment problems and data base management. Graceful Labeling, Harmonious Labeling and variations of these Labelings are the majorlabeling methodologies used in the theory of graphs. Some odd harmonious graphs are paths, cyclesC_{4n}, caterpillars, shadow graph of a path and a star. Odd sequential harmonious labeling of trees, double quadrilateral snakes,twigs and middle graph of a path have been established.In this paper, a generalisation of odd sequential harmonious labeling on cycle related graphs was considered and shown that cycles C_n with parallel chords and new families of graphs based on cycles C_n with parallel chords are t-odd sequential harmonious for all positive integers $t \geq 1$ with suitable illustrations.***

Keywords: Cycles with parallel chords, chain, harmonious labeling, path union, t-odd sequential harmonious labeling.
Subject classification: 05C78

I. INTRODUCTION

In graph theory when the vertices or edges of a graph are assigned integer values under certain conditions labeling is obtained. Labeling of graph was initiated by Rosa [10] in 1967 as β – valuations which was renamed by Golomb [4] as graceful labeling. Gallian [1] is referred for an extensive survey of graph labeling. Harmonious labeling was introduced by Graham and Sloane [6]. Researchers have introduced several variations of harmonious labeling.Odd harmonious labeling was introduced by Liang and Bai [8]. Gayathri and Muthuramakrishnan [2] introduced k-even sequential harmonious labeling and shown that some cycle related graphs admit k-even sequential harmonious labeling. Another variation of harmonious labeling namely k-odd sequential harmonious labeling was introduced by Muthuramakrishnan [3]and [9]. It is shown in [3] that $K_{1,n}$($n \geq 3$), Triangular snake T_n ($n \geq 2$), splitting graphs Spl ($K_{1,n}$) for $n \geq 3$, one point union of cycle with one chord and $K_{1,m}$ are k-odd sequential harmonious. Many labelings on cycles C_n with parallel chords and cycles C_n with parallel P_3 chords can be referred in [5], [12], [13], [14] and [15].

It is proved in this paper that cycles C_n with parallel chords, two copies of even cycle C_n with parallel chords joined by a path of even order, path union of cycles C_n with parallel chords and chain of even cycles C_n with parallel chords are t-odd sequential harmonious for all positive integers $t \geq 1$.

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Throughout this paper C_n denotes a cycle of length n and a chord is an edge which connects two non-adjacent vertices of the cycle. The basic definitions that are required for this work are given below.

Definition 1.1 [3]
Consider a graph G having p vertices and q edges. A labeling is a t-odd sequential harmonious labeling of G when there is an injective function $f : V(G) \rightarrow \{t\text{-}1, t, t+1,., t+2q\text{-}1\}$such that f induces a bijection $f^* : E(G) \rightarrow \{2t\text{-}1, 2t+1, 2t+3, \ldots 2t+2q\text{-}3\}$ given by

$$f^*(xy) = \begin{cases} f(x) + f(y), & if\ f(x) + f(y)\ is\ odd \\ f(x) + f(y) + 1, & if\ f(x) + f(y)\ is\ even \end{cases}$$

for every edge xy. Hence graph G is a t-odd sequential harmonious graph.

Definition 1.2 [7]
Let $G_1, G_2, \ldots, G_s$ be s copies of a graph G with $s \geq 2$. If G_j to G_{j+1} is joined by an edge for j=1,2,…,s-1, path union of G is obtained.

Definition 1.3 [11]
Let $v_{0,j}, v_{1,j}, v_{2,j}, \ldots, v_{n-1,j}$ be the n vertices of j^{th} copy of even cycle C_n with parallel chords for $1 \leq j \leq s$. Chain of cycles $C_{n,s}$ is obtained from s copies of cycle by identifying $v_{n/2,j}$ with $v_{0,j+1}$ for j = 1,2,….s-1.

Definition 1.4 [12]
A cycle with parallel chords is defined as a graph G obtained by adding the chords between the pair of vertices v_1v_{n-1}, v_2v_{n-2}, …,$v_\alpha v_\beta$ of the cycle C_n:$v_0v_1 \ldots v_{n-1}v_0$ ($n \geq 6$) where $\alpha = \left\lfloor \frac{n}{2} \right\rfloor - 1$, $\beta = \left\lfloor \frac{n}{2} \right\rfloor + 1$ if n is even and $\beta = \left\lfloor \frac{n}{2} \right\rfloor + 2$ if n is odd as shown in Fig.1(a) and Fig.1(b). Then $|V(G)| = n$ and $|E(G)| = M = \frac{3n-3}{2}$if n is odd, $M = \frac{3n-2}{2}$ if n is even.

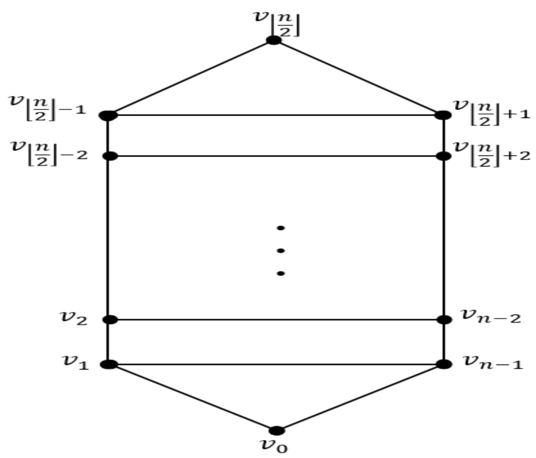

Fig.1(a): Cycle C_n with parallel chords (n-even)

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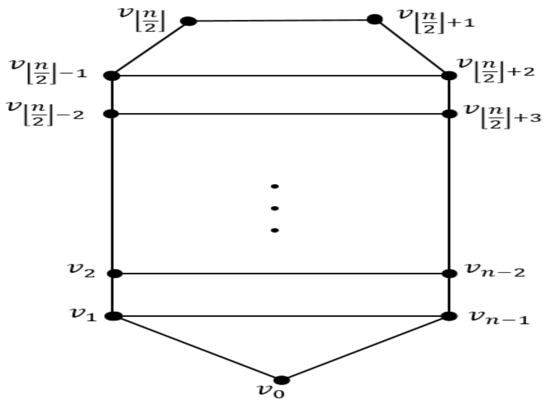

Fig.1(b): Cycle C_n with parallel chords (n-odd)

II. MAIN RESULTS

Theorem 2.1: *Every cycle C_n ($n \geq 6$) with parallel chords is t-odd sequential harmonious for all $t \geq 1$.*

Proof: A cycle C_n with parallel chords is denoted as G having n vertices and M edges using definition 1.4. Let v_0, v_1, … v_{n-1} be the n vertices of G. The labeling$f$ for the vertices and the labeling$f$ * for the edges are given respectively in the following two cases depending on n being even and n being odd.

Case 1: Let n be even

The vertices and edges of G are n and M respectively where $M = (3n-2) / 2$

The vertex labeling$f$: $V(G) \rightarrow \{t-1, t, t+1, \ldots t + 3n -3\}$ is shown below.

$f(v_0) = t - 1$

$$f(v_{2i-1}) = \begin{cases} t + 6i - 6, 1 \leq i \leq \left\lfloor \frac{n}{4} \right\rfloor \quad and \\ t + 6i - 5, \quad i = \left\lfloor \frac{n}{4} \right\rfloor + 1 \ \ if\ n \equiv 2\ mod\ 4 \end{cases}$$

$$f(v_{2i}) = \begin{cases} t + 6i - 1, 1 \leq i \leq \left\lfloor \frac{n}{4} \right\rfloor \ if\ n \equiv 0\ mod\ 4 \\ t + 6i - 2, 1 \leq i \leq \left\lfloor \frac{n}{4} \right\rfloor \ if\ n \equiv 2\ mod\ 4 \end{cases}$$

$$f(v_{n-2i}) = \begin{cases} t + 6i - 2, 1 \leq i \leq \left\lfloor \frac{n-2}{4} \right\rfloor \ if\ n \equiv 0\ mod\ 4 \\ t + 6i - 1, 1 \leq i \leq \left\lfloor \frac{n-2}{4} \right\rfloor \ if\ n \equiv 2\ mod\ 4 \end{cases}$$

$f(v_{n-(2i-1)}) = t + 6i - 4$, $1 \leq i \leq \left\lfloor \frac{n}{4} \right\rfloor$

The Edge set$E(G) = E_1 \cup E_2 \cup E_3$ where

$E_1=\{v_i v_{i+1}, 0 \leq i \leq \frac{n-2}{2}\}$,

$E_2 = \{v_i v_{i+1}, \frac{n}{2} \leq i \leq n-1\}$,

$E_3 = \{v_i v_{n-i}, 1 \leq i \leq \frac{n-2}{2}\}$

The bijective function f^*: $E(G) \rightarrow \{2t-1, 2t+1, 2t+3, \ldots 2t+3n-5\}$is shown below.

$f^*(v_i v_{i+1}) = 2t + 6i - 1$, $\quad 0 \leq i \leq \frac{n-2}{2}$

$f^*(v_i v_{i+1}) = 2t + 6n - 6i - 5$, $\frac{n}{2} \leq i \leq n - 1$

$f^*(v_i v_{n-i}) = 2t + 6i - 3$, $\quad 1 \leq i \leq \frac{n-2}{2}$

Hence the graph under consideration is t-odd sequential harmonious for all $t \geq 1$.

Example 1: An illustration for case 1 of theorem 2.1 is given in Fig. 2(a).

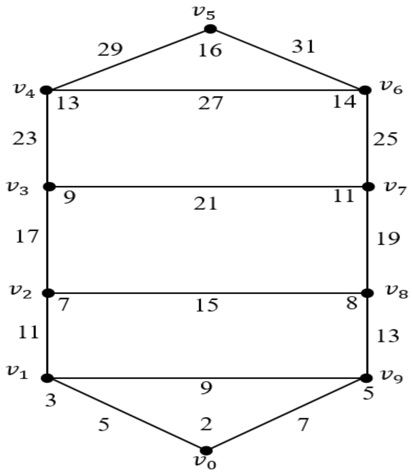

Fig.2(a): 3-odd sequential harmonious labeling of C_{10} with parallel chords

Case2: Let n be odd

The vertices and edges of G are n and M respectively where $M = (3n-3) / 2$

The vertex labeling$f$: $V(G) \rightarrow \{t-1, t, t+1, \ldots t + 3n - 4\}$ is shown below.

$f(v_0) = t-1$

$f(v_{2i-1}) = t + 6i - 6$, $\quad 1 \leq i \leq \left\lfloor \frac{n+1}{4} \right\rfloor$

$f(v_{2i}) = t + 6i - 1$, $\quad 1 \leq i \leq \left\lfloor \frac{n-1}{4} \right\rfloor$

$f(v_{n-2i}) = t + 6i - 2$, $\quad 1 \leq i \leq \left\lfloor \frac{n-1}{4} \right\rfloor$

$f(v_{n-(2i-1)}) = t + 6i - 4$, $\quad 1 \leq i \leq \left\lfloor \frac{n+1}{4} \right\rfloor$

$E(G) = E_1 \cup E_2 \cup E_3$ is the edge set where

$E_1 = \{v_i v_{i+1}, 0 \leq i \leq \frac{n-3}{2}\}$,

$E_2 = \{v_i v_{i+1}, \frac{n+1}{2} \leq i \leq n-1\}$,

$E_3 = \{v_i v_{n-i}, \ 1 \leq i \leq \frac{n-1}{2}\}$

The bijective function f^*: $E(G) \rightarrow \{2t-1, 2t+1, 2t+3, \ldots 2t+3n-6\}$ is shown below.

$f^*(v_i v_{i+1}) = 2t + 6i - 1$, $\quad 0 \leq i \leq \frac{n-3}{2}$

$f^*(v_i v_{i+1}) = 2t + 6n - 6i - 5$, $\frac{n+1}{2} \leq i \leq n-1$

$f^*(v_i v_{n-i}) = 2t + 6i - 3$, $\quad 1 \leq i \leq \frac{n-1}{2}$

Hence the graph G is t-odd sequential harmonious for all $t \geq 1$.

Example 2. An illustration for case 2 of theorem 2.1 is given in Fig. 2(b).

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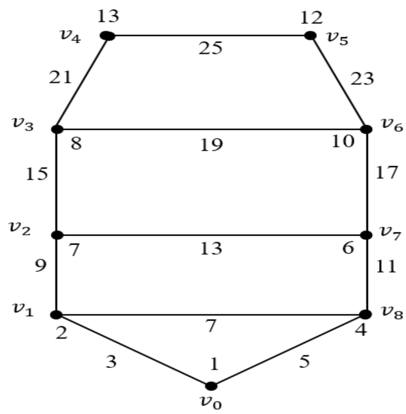

Fig.2(b): 2-odd sequential harmonious labeling of C_9 with parallel chords

Theorem 2.2: *Two copies of even cycle C_n $(n \geq 6)$ with parallel chords joined by a path H_g of even order g is t-odd sequential harmonious for all $t \geq 1$.*

Proof: Two copies of even cycle $C_n$with parallel chords having vertices $v_0, v_1,\ldots,v_{n-1}$ for the first copy and $w_0, w_1, \ldots,w_{n-1}$ for the second copy are considered. Let G be the graph obtained by joining them by a path H_g of even order g. The vertices of H_g are $h_1, h_2,\ldots h_g$ with $h_1 = v_{\frac{n}{2}}$ and $h_g = w_{\frac{n}{2}}$.

Then $|V(G)| = 2n + g - 2$, $|E(G)| = 3n + g - 3$.

The vertex labeling $f : V(G) \rightarrow \{t-1, t, t+1,\ldots,t+6n+2g-7\}$ is shown below.

$f(v_0) = t - 1$

$$f(v_{2i-1}) = \begin{cases} t + 6i - 6, 1 \leq i \leq \left\lfloor \frac{n}{4} \right\rfloor \text{and} \\ t + 6i - 5, \quad i = \left\lfloor \frac{n}{4} \right\rfloor + 1 \ if\ n \equiv 2 \bmod 4 \end{cases}$$

$$f(v_{2i}) = \begin{cases} t + 6i - 1, 1 \leq i \leq \left\lfloor \frac{n}{4} \right\rfloor \ if\ n \equiv 0 \bmod 4 \\ t + 6i - 2, 1 \leq i \leq \left\lfloor \frac{n}{4} \right\rfloor \ if\ n \equiv 2 \bmod 4 \end{cases}$$

$$f(v_{n-2i}) = \begin{cases} t + 6i - 2, 1 \leq i \leq \left\lfloor \frac{n-2}{4} \right\rfloor \ if\ n \equiv 0 \bmod 4 \\ t + 6i - 1, 1 \leq i \leq \left\lfloor \frac{n-2}{4} \right\rfloor \ if\ n \equiv 2 \bmod 4 \end{cases}$$

$f(v_{n-(2i-1)}) = t + 6i - 4,\ 1 \leq i \leq \left\lfloor \frac{n}{4} \right\rfloor$

$f(w_0) = t + 3n + g - 4$

$$f(w_{2i-1}) = \begin{cases} t + 3n + g - 6i, 1 \leq i \leq \left\lfloor \frac{n}{4} \right\rfloor \ if\ n \equiv 0 \bmod 4 \\ t + 3n + g - 6i - 1, 1 \leq i \leq \left\lfloor \frac{n-2}{4} \right\rfloor \ if\ n \equiv 2 \bmod 4 \end{cases}$$

$f(w_{2i}) = t + 3n + g - 6i - 3,\ 1 \leq i \leq \left\lfloor \frac{n-2}{4} \right\rfloor$

$$f(w_{n-2i}) = \begin{cases} t + 3n + g - 6i - 4, 1 \leq i \leq \left\lfloor \frac{n}{4} \right\rfloor \ if\ n \equiv 0 \bmod 4 \\ t + 3n + g - 6i - 5, 1 \leq i \leq \left\lfloor \frac{n-2}{4} \right\rfloor \ if\ n \equiv 2 \bmod 4 \end{cases}$$

$$f(w_{n-(2i-1)}) = \begin{cases} t + 3n + g - 6i - 1, 1 \leq i \leq \left\lfloor \frac{n}{4} \right\rfloor \ if\ n \equiv 0 \bmod 4 \\ t + 3n + g - 6i, 1 \leq i \leq \left\lfloor \frac{n+2}{4} \right\rfloor \ if\ n \equiv 2 \bmod 4 \end{cases}$$

For the path H_g

$$f(h_{2i+1}) = \begin{cases} \frac{2t + 3n + 4i - 2}{2}, 1 \leq i \leq \frac{g-2}{2} \ if\ n \equiv 0 \bmod 4 \\ \frac{2t + 3n + 4i - 4}{2}, 1 \leq i \leq \frac{g-2}{2} \ if\ n \equiv 2 \bmod 4 \end{cases}$$

$$f(h_{g-2i}) = \begin{cases} \frac{2t + 3n + 2g - 4i - 8}{2}, 1 \leq i \leq \frac{g-2}{2} \ if\ n \equiv 0 \bmod 4 \\ \frac{2t + 3n + 2g - 4i - 6}{2}, 1 \leq i \leq \frac{g-2}{2} \ if\ n \equiv 2 \bmod 4 \end{cases}$$

The edge set of G is given by $E(G) = E_1 \cup E_2 \cup E_3 \cup E_1' \cup E_2' \cup E_3' \cup E_4' \cup E_5$ where

$E_1 = \{v_i v_{i+1},\ 0 \leq i \leq \frac{n-2}{2}\}$,

$E_2 = \{v_i v_{i+1},\ \frac{n}{2} \leq i \leq n - 1\}$,

$E_3 = \{v_i v_{n-i},\ 1 \leq i \leq \frac{n-2}{2}\}$,

$E_1' = \{ w_i w_{i+1},\ 0 \leq i \leq \frac{n-2}{2}$ if $n \equiv 0 \bmod 4$ and $1 \leq i \leq \frac{n-2}{2}$ if $n \equiv 2 \bmod 4\}$,

$E_2' = \{w_{n-i} w_{n-(i+1)},\ 0 \leq i \leq \frac{n-2}{2}$ if $n \equiv 0 \bmod 4$ and $1 \leq i \leq \frac{n-2}{2}$ if $n \equiv 2 \bmod 4\}$,

$E_3' = \{w_i w_{n-i},\ 1 \leq i \leq \frac{n-2}{2}\}$, $E_4' = \{w_0 w_1$ and $w_0 w_{n-1}$ if $n \equiv 2 \bmod 4\}$, $E_5 = \{h_1 h_2, h_2 h_3, \ldots h_{g-1} h_g\}$.

Define the bijective mapping $f^*: E(G) \rightarrow \{2t-1, 2t+1, 2t+3,\ldots 2t+6n+2g-9\}$ as follows.

$f^*(v_i v_{i+1}) = 2t + 6i - 1,\ 0 \leq i \leq \frac{n-2}{2}$

$f^*(v_i v_{i+1}) = 2t + 6n - 6i - 5,\ \frac{n}{2} \leq i \leq n-1$

$f^*(v_i v_{n-i}) = 2t + 6i - 3,\ 1 \leq i \leq \frac{n-2}{2}$

$f^*(w_i w_{i+1}) = 2t + 6n + 2g - 6i - 9,\ 0 \leq i \leq \frac{n-2}{2}$ if $n \equiv 0 \bmod 4$ and $1 \leq i \leq \frac{n-2}{2}$ if $n \equiv 2 \bmod 4$

$f^*(w_{n-i} w_{n-(i+1)}) = 2t + 6n + 2g - 6i - 11,\ 0 \leq i \leq \frac{n-2}{2}$ if $n \equiv 0 \bmod 4$ and $1 \leq i \leq \frac{n-2}{2}$ if $n \equiv 2 \bmod 4$

$f^*(w_i w_{n-i}) = 2t + 6n + 2g - 6i - 7,\ 1 \leq i \leq \frac{n-2}{2}$

$f^*(w_0 w_{n-1}) = 2t + 6n + 2g - 9$ if $n \equiv 2 \bmod 4$

$f^*(w_0 w_1) = 2t + 6n + 2g - 11$ if $n \equiv 2 \bmod 4$

$f^*(h_i h_{i+1}) = 2t + 3n + 2i - 5,\ 1 \leq i \leq g-1$

Hence the graph is t-odd sequential harmonious for all $t \geq 1$.

Example 3: An illustration of above theorem is shown in Fig 3

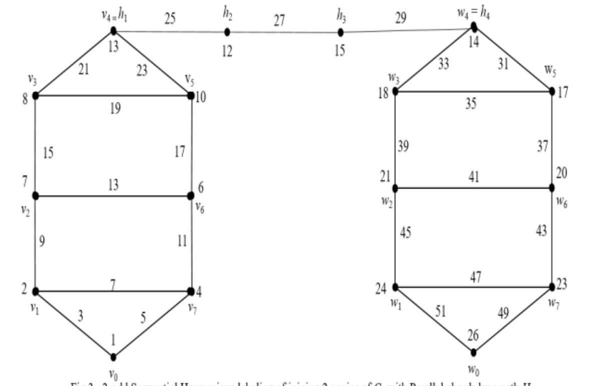

Fig.3: 2-odd Sequential Harmonious labeling of joining 2 copies of C_8 with Parallel chords by a path H_4

Theorem 2.3: *Path union of cycle C_n $(n \geq 6)$ with parallel chords is t-odd sequential harmonious for all $t \geq 1$*

Proof: Let s copies of cycle C_n with parallel chords be considered. $v_{0,j}, v_{1,j}, \ldots, v_{n-1,j}$ are the n vertices of j^{th} copy of cycle C_n with parallel chords for $1 \leq j \leq s$. The path union G is obtained by attaching $v_{0,j}$ with $v_{0,j+1}$ by an edgefor $1 \leq j \leq s-1$. The vertex labeling and the edge labeling for the two cases depending on n being even and n being odd are given below.

Case 1: If n is even

The vertices and edges of G are $sn$and $sM + s - 1$ respectively where $M = (3n - 2) / 2$

The vertex labeling $f : V(G) \rightarrow \{t-1, t, t+1, \ldots t+3ns - 3\}$is as defined below.

For $1 \leq j \leq s$ and j odd

$$f(v_{0,j}) = (t-1) + (j-1)\frac{3n}{2}$$

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$$f(v_{2i-1,j}) = \begin{cases} (t+6i-6)+(j-1)\frac{3n}{2}, 1 \le i \le \lfloor\frac{n}{4}\rfloor \text{ and} \\ t+6i-5+(j-1)\frac{3n}{2}, \; i = \lfloor\frac{n}{4}\rfloor+1 \text{ if } n \equiv 2 \bmod 4 \end{cases}$$

$$f(v_{2i,j}) = \begin{cases} (t+6i-1)+(j-1)\frac{3n}{2}, 1 \le i \le \lfloor\frac{n}{4}\rfloor \text{ if } n \equiv 0 \bmod 4 \\ (t+6i-2)+(j-1)\frac{3n}{2}, 1 \le i \le \lfloor\frac{n}{4}\rfloor \text{ if } n \equiv 2 \bmod 4 \end{cases}$$

$$f(v_{n-2i,j}) = \begin{cases} (t+6i-2)+(j-1)\frac{3n}{2}, 1 \le i \le \lfloor\frac{n-2}{4}\rfloor \text{ if } n \equiv 0 \bmod 4 \\ (t+6i-1)+(j-1)\frac{3n}{2}, 1 \le i \le \lfloor\frac{n}{4}\rfloor \text{ if } n \equiv 2 \bmod 4 \end{cases}$$

$$f(v_{n-(2i-1),j}) = (t+6i-4)+(j-1)\frac{3n}{2}, 1 \le i \le \lfloor\frac{n}{4}\rfloor$$

For $2 \le j \le s$ and j even

$$f(v_{0,j}) = (t-2)+\frac{3n}{2}j$$

$$f(v_{2i-1,j}) = \begin{cases} (t-6i+2)+j\frac{3n}{2}, 1 \le i \le \lfloor\frac{n}{4}\rfloor \text{ if } n \equiv 0 \bmod 4 \\ (t-6i+1)+j\frac{3n}{2}, \; 1 \le i \le \lfloor\frac{n-2}{4}\rfloor \text{ if } n \equiv 2 \bmod 4 \end{cases}$$

$$f(v_{2i,j}) = (t-6i-1)+j\frac{3n}{2}, 1 \le i \le \lfloor\frac{n-2}{4}\rfloor$$

$$f(v_{n-2i,j}) = (t-6i-3)+j\frac{3n}{2}, 1 \le i \le \lfloor\frac{n-2}{4}\rfloor$$

$$f(v_{n-(2i-1),j}) = \begin{cases} (t-6i+1)+j\frac{3n}{2}, 1 \le i \le \lfloor\frac{n}{4}\rfloor \text{ if } n \equiv 0 \bmod 4 \\ (t-6i+2)+j\frac{3n}{2}, \; 1 \le i \le \lfloor\frac{n+2}{4}\rfloor \text{ if } n \equiv 2 \bmod 4 \end{cases}$$

$$f(v_{n/2,j}) = \frac{2t+3n-4}{2}+\frac{3n}{2}(j-2) \text{ if } n \equiv 0 \bmod 4$$

$E(G)=\{v_{i,j}v_{i+1,j}, 0 \le i \le n-1, 1 \le j \le s\} \cup \{v_{i,j}v_{n-i,j}, 1 \le i \le \lfloor\frac{n-2}{2}\rfloor, 1 \le j \le s\} \cup \{v_{0,j}v_{0,j+1}, 1 \le j \le s-1\}$ is the edge set of G.

The bijective mapping $f^*: E(G) \to \{2t - 1, 2t + 1, \ldots 2t + 3ns - 5\}$ is as defined below.

For $1 \le j \le s$, j odd

$$f^*(v_{i,j}v_{i+1,j}) = (2t+6i-1)+3n(j-1), 0 \le i \le \frac{n-2}{2}$$

$$f^*(v_{i,j}v_{i+1,j}) = (2t+6n-6i-5)+3n(j-1), \frac{n}{2} \le i \le n-1$$

$$f^*(v_{i,j}v_{n-i,j}) = (2t+6i-3)+3n(j-1), 1 \le i \le \frac{n-2}{2}$$

For $2 \le j \le s$, j even

$$f^*(v_{i,j}v_{i+1,j}) = (2t+6n-6i-5)+3n(j-2),$$

$0 \le i \le \frac{n-2}{2}$ if $n \equiv 0 \bmod 4$ &

$1 \le i \le \frac{n-2}{2}$, if $n \equiv 2 \bmod 4$

$$f^*(v_{n-i,j}v_{n-(i+1),j}) = (2t+6n-6i-7)+3n(j-2),$$

$0 \le i \le \frac{n-2}{2}$ if $n \equiv 0 \bmod 4$ & $1 \le i \le \frac{n-2}{2}$ if $n \equiv 2 \bmod 4$

$$f^*(v_{i,j}v_{n-i,j}) = (2t+6n-6i-3)+3n(j-2), 1 \le i \le \frac{n-2}{2}$$

$$f^*(v_{0,j}v_{1,j}) = (2t+3nj-7), \text{ if } n \equiv 2 \bmod 4$$

$$f^*(v_{0,j}v_{n-1,j}) = (2t+3nj-5), \text{ if } n \equiv 2 \bmod 4$$

For $j = 1,2,3,\ldots,s$-1

$$f^*(v_{0,j}v_{0,j+1}) = (2t+3n-3)+(3n)(j-1)$$

Hence the graph is t-odd sequential harmonious for all $t \ge 1$.

Case 2: when *n* is odd

The vertices and edges of G are sn and $sM + s - 1$ respectively where $M = (3n - 3) / 2$

The vertex labeling $f: V(G) \to \{t\text{-}1, t, t\text{+}1, \ldots, t + 3ns - s - 3\}$ is shown below.

For $1 \le j \le s$, j odd

$$f(v_{0,j}) = t-1+(j-1)\frac{3n-1}{2}$$

$$f(v_{2i-1,j}) = t+6i-6+(j-1)\frac{3n-1}{2}, 1 \le i \le \lfloor\frac{n+1}{4}\rfloor$$

$$f(v_{2i,j}) = t+6i-1+(j-1)\frac{3n-1}{2}, 1 \le i \le \lfloor\frac{n-1}{4}\rfloor$$

$$f(v_{n-2i,j}) = t+6i-2+(j-1)\frac{3n-1}{2}, 1 \le i \le \lfloor\frac{n-1}{4}\rfloor$$

$$f(v_{n-(2i-1),j}) = t+6i-4+(j-1)\frac{3n-1}{2}, 1 \le i \le \lfloor\frac{n+1}{4}\rfloor$$

For $2 \le j \le s$, j even

$$f(v_{0,j}) = (t-2)+\frac{3n-1}{2}j$$

$$f(v_{2i-1,j}) = (t-6i+3)+j\frac{3n-1}{2}, 1 \le i \le \lfloor\frac{n+1}{4}\rfloor$$

$$f(v_{2i,j}) = (t-6i-2)+j\frac{3n-1}{2}, 1 \le i \le \lfloor\frac{n-1}{4}\rfloor$$

$$f(v_{n-2i,j}) = (t-6i-1)+j\frac{3n-1}{2}, 1 \le i \le \lfloor\frac{n-1}{4}\rfloor$$

$$f(v_{n-(2i-1),j}) = (t-6i)+j\frac{3n-1}{2},$$

$1 \le i \le \lfloor\frac{n+1}{4}\rfloor$ if $n \equiv 1 \bmod 4$ &

$1 \le i \le \lfloor\frac{n-1}{4}\rfloor$ if $n \equiv 3 \bmod 4$

$$f(v_{(n+1)/2,j}) = \frac{2t+3n-7}{2}+\frac{3n-1}{2}(j-2), \text{ if } n \equiv 3 \bmod 4$$

$E(G)=\{v_{i,j}v_{i+1,j}, \; 0 \le i \le n-1, 1 \le j \le s\} \cup \{v_{i,j}v_{n-i,j}, 1 \le i \le \lfloor\frac{n-3}{2}\rfloor, 1 \le j \le s\} \cup \{v_{0,j}v_{0,j+1}, 1 \le j \le s-1\}$ is the edge set of G.

Define the induced edge labeling $f^*: E(G) \to \{2t - 1, 2t + 1, \ldots 2t + 3ns - s - 5\}$ as follows

For $1 \le j \le s$, j odd

$$f^*(v_{i,j}v_{i+1,j}) = (2t+6i-1)+(3n-1)(j-1), 0 \le i \le \frac{n-3}{2}$$

$$f^*(v_{i,j}v_{i+1,j}) = (2t+6n-6i-5)+(3n-1)(j-1), \frac{n+1}{2} \le i \le n-1$$

$$f^*(v_{i,j}v_{n-i,j}) = (2t+6i-3)+(3n-1)(j-1), 1 \le i \le \frac{n-1}{2}$$

For $2 \le j \le s$, j even

$$f^*(v_{i,j}v_{i+1,j}) = (2t-6i-5)+j(3n-1), 0 \le i \le \frac{n-3}{2}$$

$$f^*(v_{n-i,j}v_{n-(i+1),j}) = (2t-6i-7)+j(3n-1), 0 \le i \le \frac{n-3}{2}$$

$$f^*(v_{i,j}v_{n-i,j}) = (2t+6i-3)+j(3n-1), 1 \le i \le \frac{n-1}{2}$$

For $j = 1,2,3,\ldots,s$-1

$$f^*(v_{0,j}v_{0,j+1}) = (2t+3n-4)+(3n-1)(j-1)$$

Hence G is t-odd sequential harmonious graph for all $t \ge 1$.

Example 4: An illustration of case 1 of theorem 2.4 is shown in Fig.4

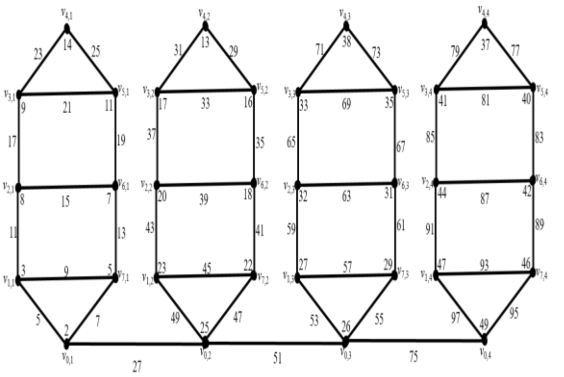

Fig.4: 3 odd sequential harmonious labeling of Path Union of 4 copies of Cycle C_8 with parallel chords

Theorem 2.4: *Chainof even cycle C_n $(n \ge 6)$ with parallel chords is t-odd sequential harmonious for all $t \ge 1$*

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Proof: Let s copies of even cycle C_n with parallel chords be considered. $v_{0,j}$, $v_{1,j}$, … $v_{n-1,j}$ are the n vertices of j^{th} copy of cycle C_n with parallel chords for $1 \le j \le s$.By identifying $v_{n/2,\, j}$ with $v_{0,\, j+1}$ for $1 \le j \le s-1$, chain of cycles $C_{n,s}$ is obtained from s copies of C_n with parallel chords. This graph is denoted as G. Then the vertices and edges of G are $s(n-1) + 1$ and sM respectively.

The vertex labeling $f : V(G) \rightarrow \{t-1, t, t+1, \ldots t + 3ns - 2s - 1\}$is shown below.

For $1 \le j \le s$

$$f(v_{0,j}) = (t-1) + (j-1)\frac{3n-2}{2}$$

$$f(v_{2i-1,j}) = \begin{cases} (t+6i-6) + (j-1)\frac{3n-2}{2}, 1 \le i \le \left\lfloor \frac{n}{4} \right\rfloor \ and \\ (t+6i-5) + (j-1)\frac{3n-2}{2}\ ,\ i = \left\lfloor \frac{n}{4} \right\rfloor + 1 \ if\ n \equiv 2 \ mod\ 4 \end{cases}$$

$$f(v_{2i,j}) = \begin{cases} (t+6i-1) + (j-1)\frac{3n-2}{2}, 1 \le i \le \left\lceil \frac{n-2}{4} \right\rceil \ if\ n \equiv 0 \ mod\ 4 \\ (t+6i-2) + (j-1)\frac{3n-2}{2}\ , 1 \le i \le \left\lceil \frac{n-2}{4} \right\rceil \ if\ n \equiv 2 \ mod\ 4 \end{cases}$$

$$f(v_{n-2i,j}) = \begin{cases} (t+6i-2) + (j-1)\frac{3n-2}{2}, 1 \le i \le \left\lfloor \frac{n}{4} \right\rfloor \ if\ n \equiv 0 \ mod\ 4 \\ (t+6i-1) + (j-1)\frac{3n-2}{2}\ , 1 \le i \le \left\lfloor \frac{n}{4} \right\rfloor \ if\ n \equiv 2 \ mod\ 4 \end{cases}$$

$$f(v_{n-(2i-1),j}) = (t+6i-4) + (j-1)\frac{3n-2}{2}, 1 \le i \le \left\lfloor \frac{n}{4} \right\rfloor$$

$E(G)=\{v_{i,j}v_{i+1,j}, 0 \le i \le n-1, 1 \le j \le s\} \cup \{v_{i,j}v_{n-i,j}, 1 \le i \le \left\lceil \frac{n-2}{2} \right\rceil, 1 \le j \le s\}$ is the edge set of G.

The bijective mapping $f^* : E(G) \rightarrow \{2t - 1, 2t + 1, \ldots 2t + 3ns - 2s - 3\}$is shown below.

For $1 \le j \le s$

$$f^*(v_{i,j}v_{i+1,j}) = (2t+6i-1) + (3n-2)(j-1), 0 \le i \le \frac{n-2}{2}$$

$$f^*(v_{i,j}v_{i+1,j}) = (2t+6n-6i-5) + (3n-2)(j-1), \frac{n}{2} \le i \le n-1$$

$$f^*(v_{i,j}v_{n-i,j}) = (2t+6i-3) + (3n-2)(j-1), 1 \le i \le \frac{n-2}{2}$$

Hence G admits t-odd sequential harmoniouslabeling for all $t \ge 1$.

Example 5: An illustration for theorem 2.5 is given in Figure 5.

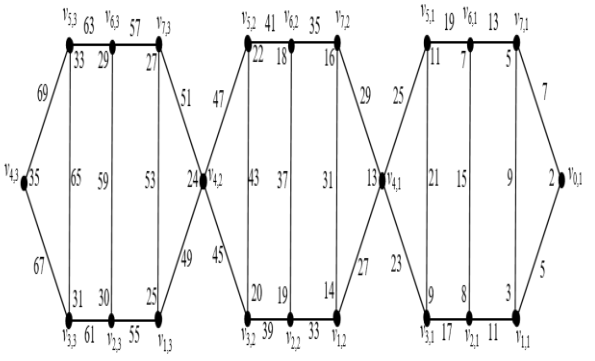

Fig.5: 3 Odd Sequential Harmonious Labeling of $C_{8,3}$ with Parallel Chords

III. CONCLUSION

In this present work, new families of graphs are obtained from cycles with parallel chords that admit t-odd sequential harmonious labeling for all $t \ge 1$. Suitable illustrations are given to graphically explain the results.Since odd cycles are not odd harmonious, variation of odd harmonious labeling is applied on odd & even cycles with parallel chords to show that they admit t-odd sequential harmonious labeling for all $t \ge 1$. The major focus of attention for future work will be on labeling of similar cycle related graph families.

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