New Relation of Fluid Flow in Fractal Porous Medium: I. Theoretical Analysis

E.J. Suarez-Dominguez, A. Palacio-Perez, Y.G. Aranda-Jimenez, E. Izquierdo-Kulich

Abstract*: *The study in porous media fluid flow is of great interest by its practical application mainly in reservoirs oil production. Although there are different models in this regard, not all of them can reproduce the results in the field, mainly due to the complexity of the system and the difficulty of properly characterizing of the solid medium. In the present work, a theoretical model is proposed, based on fractional differential equations, which allows to relate the porosity of a porous media with its fractal dimension and then estimate the velocity profile of the fluid flow inside and its respective pressure drop. The obtained model, analytically solved, allows a quick evaluation of pressure according to friction losses in porous media and can be transferred to two phase flow.

.

Index Terms: fractional differential equations, porous media fluid Flow model, fractal dimension in fluid flow.

I. INTRODUCTION

The porous beds are characterized by a complex morphology, there being a non-trivial relationship between the particle size distribution, the spatial arrangement of these and the porosity of the bed (Kundu, et al. 2016). This is the fundamental reason why the phenomenological description of the flow through porous beds is a complicated task, which leads to the need for the use of numerical techniques and a high computational cost (Muljadi, B. P. 2016). From the practical point of view, relatively simple models are usually used, which relate the velocity of the fluid to the steady state pressure gradient, and which involve general aspects such as the size and shape of the particles, the porosity and the viscosity of the fluid, as well as the need to adjust empirical parameters from the experimental results observed (Vafai, K. 2015; Wang, Y. et al. 2014). These models are based on two fundamental approaches (Rong, L. 2015). In the first, the bed formed by a set of particles is visualized, around which the fluid phase circulates, estimating the pressure losses from the sum of the individual resistances of the particles that make up the bed. In the second approach, which for reasons of simplicity is the most used, the bed is visualized as a bundle of tubes of random section and shape, and the basic equations related to the flow of pipes are applied. They have the following disadvantages: 1) they are not adequate to describe behavior in a non-stationary state (Cirpka, O. A. et al. 2015); 2) since the behavior of the velocity profile in the bed can not be predicted is inappropriate for treating non-Newtonian fluids, in which the apparent viscosity is a function of the velocity profile, nor the case of biphasic flows with total separation of the phases, where the composition and relative arrangement of the phases have a significant influence on pressure losses (Shoghi, M. R. & Norouzi, M. 2015) and 3) the use of empirical parameters may eventually limit the validity of the model when the operating conditions of the bed differ significantly from those under which they were estimated these parameters (Heider, Y. & Markert, B. 2017). These limitations are important in some practical situations, such as those that occur when trying to describe the effect of steam injection or less viscous fluids to facilitate the dragging and extraction of oil from porous deposits through the generation of a two-phase flow. Actually, is possible to see new analytical solution to transport phenomena processes like empirical evaluation of heat or mass diffusivity (Vargas-González, S. et al. 2017). To increase these kind of solutions to other areas it is interesting to analyze if the use of non-conventional tools, such as fractal geometry and fractional calculation, make it possible to express the phenomenological equations of continuity and momentum in such a way that, when resolved, allow obtaining the velocity profile and the Average speed as a function of porosity and particle size. Fractal geometry is applied to characterize systems that exhibit complex spatial patterns (Strogatz, S. H. 2018), which, ultimately, reflect the complexity and self-organization associated with the dynamics of their formation. On the other hand, the fractional differential calculation is an extension of the ordinary calculation, in the sense that the order of the derivatives and integrals is a non-integer (Baleanu, D. et al. 2016; Richard, H. 2014). Although the existence of derivatives and integrals of fractional order is something that is known almost since the beginning of the development of differential calculus, its use has been relatively limited to model physical systems (Li, C. & Zeng, F. 2015). Whereas in the ordinary calculation the relation between the whole order of the spatial and temporal derivatives and the physical processes that occur in space and time is well established, this is not the case in the case of a fractional order (Machado, J. T. 2015).When a system of fractional differential equations is proposed to describe a real system, the question that almost remains is: what is the relationship between the order of the derivatives and the spatial - temporal properties of the system under study?

Manuscript published on 30 June 2019.
* Correspondence Author (s)
Edgardo J. Suarez-Dominguez, FADU Universidad Autonoma de Tamaulipas. Tampico, Tam. Mexico.
Arturo Palacio-Pérez, Instituto de Ingeniería. UNAM. Ciudad de México. México.
Yolanda G. Aranda-Jimenez, FADU Universidad Autonoma de Tamaulipas. Tampico, Tam. Mexico.
Elena Izquierdo-Kulich. FQ Universidad de la Habana. La Habana, Cuba.

Attempting to answer this question, some authors defend the idea that the order of the fractional derivative is related to the fractal dimension, there being a close connection between both formalisms that can be used to describe underlying facts in nature, while others have shown that There is no logical proof that demonstrates this connection, pointing out that the only thing in common that both formalisms have is the use of fractional orders. The present work is based on the hypothesis that it is possible to use these tools together to describe the behavior of stationary laminar flow in a porous bed, the objective being to obtain a model that describes the relationship between flow velocity and pressure losses through friction. For this, two basic ideas were taken into account. The first was to apply a fractional integral to obtain a relationship between the fractal dimension that characterizes the particle distribution in the bed and the porosity, while the second idea was based on rewriting the differential equation that describes the velocity profile in a tube. as a fractional differential equation, where the order one of each derivative involved equals the fractal dimension of the particle distribution, and solve that equation. Once both results were obtained, the fractal dimension is appropriately replaced in the velocity profile equation, so that it is expressed as a function of porosity. From the velocity profile, we obtain the relationship between the average velocity and friction pressure losses predicted by the proposed model, theoretical predictions that were compared by those obtained using the Blake and Kozeny model, which has been validated experimentally

A. Model development

This section presents the methodology applied to obtain the proposed model, and it is divided into three subsections: in the first one, the meaning of porosity is briefly described, and an equation is obtained that correlates this property with the fractal dimension of the distribution of particles that form the bed. In the second, a phenomenological equation is proposed to describe the velocity profile in porous beds, from which the behavior of the profile with respect to the porosity and the size of the particles is obtained. The third one presents the obtaining of the proposed model to describe the relationship between average speed and frictional pressure losses.

B. Relationship between the porosity and the fractal dimension of the particle distribution

The system under study is visualized as a tube with radius $R(m)$ and total length $L(m)$, through which a fluid of density ρ(kg.m-3) and viscosity moves μ (Pa.s) constants The tube is not an empty space, but inside it there is a set of immobile particles, which are those that form the porous bed, as illustrated in Figure 1.1.

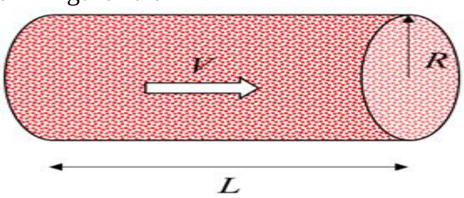

Figure 1.1. Visualization of the flow through a porous bed of cylindrical shape, through which a fluid is transported at an average speed *V* (m.s-1) in the direction indicated by the arrow The porosity ε is a property that represents the empty fraction of the bed, and that is calculated through the relationship:

$$\varepsilon = \frac{V - V_p}{V} \tag{1.1}$$

where *V* is the total volume of the bed (m3) and *Vp* is the total volume occupied by the particles (m3). Because to describe the velocity profile it is necessary to define an area perpendicular to the flow, and the porosity is a property that is defined in three dimensions, this flow area will be replaced by a cylindrical section perpendicular to the flow, of radius R (m) and infinitesimal length dz (in m), which in this work is identified as an ideal dividing area. If the particles that make up the bed are spherical and of the same size, and dz is equal to the radius δ of the particles, then the porosity of the bed, which in this case is related to the fraction of the dividing area through which the fluid is transported, estimated through the relationship:

$$\varepsilon = \frac{\pi R^2 \delta - N\frac{4}{3}\pi\delta^3}{\pi R^2 \delta} = 1 - \frac{4}{3}\frac{N}{R^2}\delta^2$$

(1.2)

where N is the total number of particles that fit within the cross section that is identified with the dividing area. Another way to calculate the porosity is to divide the cross section into N cubic cells of volume 4δ3 and to assume that in each of these is a particle of radius of radius δ, from which a value of Theoretical porosity:

$$\varepsilon = \frac{N(2\delta)^3 - N\frac{4}{3}\pi\delta^3}{N(2\delta)^3} = 0.4764 \tag{1.3}$$

Although equations (1.2) and (1.3) are very simple ways to estimate porosity, in reality the situation is much more complex. In many systems the particles do not have regular shapes, nor are they all of the same size (Rao, G. et al. 2016), where the standard deviation of the probability function describing the size distribution can be of the same order of magnitude as the average value (Tan, X. H. et al. 2017).

In this case the smallest particles can be located between the empty spaces that are available between the larger particles, which allows to increase the total number of particles in the bed with the consequent decrease in porosity. On the other hand, the empty spaces between the particles depend on how they are distributed spatially, as well as on the specific form of these, aspects that often have a high degree of randomness (Regelink, I. C. et al. 2015). There is another way to characterize the morphology of a porous bed, which while not solving all problems, may be more appropriate to describe its complexity. If the cross section is divided perpendicular to the flow in Nt
(l) cells of size l3 and the number of cells Nt (l) that are occupied by the particles are counted, and this process is repeated progressively decreasing the size3, then the fractal dimension α that characterizes the morphology of the bed is calculated through the relationship:

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication (BEIESP)

$$\alpha = \lim_{l \to \delta} \frac{\ln N(l)}{\ln N_t(l)} \quad (1.4)$$

where the number of particles contained within a section of radius r <R inside the cross section to the flow or divider area is determined as:

$$n \sim n_0 \left(\frac{r}{\delta}\right)^{\alpha}$$
$$1 < \alpha; \delta < r \quad (1.5)$$

where n0 is the number of particles that are at a distance r> δ. The value of n0 depends on the size distribution, being equal to 1 when all the particles are of equal size and increasing above this value when the particles are of different sizes, situation in which δ is identified with the average radius of particle. For equal values of α and r the total number of particles increases with the decrease of δ, which corresponds to the fact that the number of particles that are found in the bed increases when their average size decreases. On the other hand, for a constant / δ relation, the number of particles increases with the fractal dimension α, which, when describing the spatial pattern that the particles form as a whole, implicitly considers the average and the standard deviation of the size of the particles, as well as the shape and spatial arrangement of these. The value of α cannot be greater than 3, because the particles are within a three-dimensional space. On the other hand, the value of α can be less than 2 for low values of porosity, for which the cross-sectional flow can be visualized as a real area, where a value of α equal to 1 indicates that within the cross-sectional area flow there is no particle. To estimate the porosity, we will consider an arbitrary value of r (0 <r <R) and calculate a non-dimensional divider area Ω through a fractional integral in polar coordinates, whose order is assumed equal to the fractal dimension α:

$$\Omega(\gamma,\alpha) = \int_0^{2\pi} \kappa^{\alpha-1} \left({}_0\mathbf{D}_{\gamma}^{-\alpha}\gamma\right) d\theta = \kappa^{\alpha-1} \int_0^{2\pi} \int_0^{\gamma} \frac{\gamma^{\alpha}}{\alpha\Gamma(\alpha)} d\gamma d\theta$$
$$\kappa = \frac{\delta}{R}; \gamma = \frac{r}{R} \quad (1.6)$$

where ${}_0\mathbf{D}_{\gamma}^{-\alpha}$ represents the integral of order α with respect to the variable γ, κ is a scaling parameter that represents the relationship between the size of the particles and the size of the bed and Γ (α) is the gamma function, defined as:

$$\Gamma(\alpha) = \int_0^{\infty} e^{-x} x^{\alpha-1} dx$$

(1.7)

Note that the exponent α - 1 associated to the scaling parameter κ when added with -α, which is the order of the fractional integral, is equal to -1, in such a way that the dimensional consistency is guaranteed, since the area has always units of length squared. The exact solution of the integral (1.6) is:

$$\Omega(\gamma,\alpha) = \kappa^{\alpha-1} \frac{2\pi\gamma^{\alpha+1}}{\alpha(\alpha+1)\Gamma(\alpha)} \quad (1.8)$$

If it is taken into account that the divisor area for α = 1 and γ = 1 is equivalent to the total area of the cross section of the flow (sum of the free area plus that occupied by the particles, equal in turn to the area of the tube within which there is no particle), and that the infinitesimal length dz of the cross section to the flow area is not affected by the porosity, then the quotient between the estimated area for α> 1 and γ = 1 and the area of the cross section total is equal to the volume fraction not occupied by the particles, which is just the porosity corresponding to this cross section. On the other hand, if it is considered that ε is also equivalent to the quotient between the area of flow af and the total area at when the system is displayed at the scale of a single individual particle in the sense that γ→κ→ 1 to be able to estimate the fractal dimension, then:

$$\varepsilon = \left(\lim_{\kappa \to 1}\left(\lim_{\gamma \to \kappa} \kappa^{\alpha-1} \frac{2\pi\gamma^{\alpha+1}}{\Gamma(\alpha+2)} \times \left(\frac{2\pi\gamma^{1+1}}{\Gamma(1+2)}\right)^{-1}\right)\right)$$

That simplifying gives:

$$\varepsilon = \frac{2}{\alpha(\alpha+1)\Gamma(\alpha)} \quad (1.9)$$

Equation (1.9) represents the relationship between the fractal dimension α and the porosity ε of the bed, which, by involving the function Γ (α), makes it impossible to determine α analytically for a specified value of ε requiring methods numerical for this determination. It is noted that the porosity tends to 1 when the fractal dimension tends to 1, which corresponds to the case of a tube free of particles in which the porosity is equal to 1, whereas when the fractal dimension tends to infinite the porosity tends to zero Since the fractal dimension increases with the number of particles and the decrease in the spaces between them, this result corresponds to the qualitatively expected.

The experimental determination of the fractal dimension of a porous bed in three dimensions is practically impossible, while the porosity is an observable property that can be determined experimentally. For this reason, it is more convenient to have a relationship that allows to estimate in a simple way the fractal dimension from the observed porosity, for which statistical methods of non-linear regression were applied. From the application of these methods, we obtained:

$$\varepsilon = \left(\frac{5}{4} - \frac{1}{4}\alpha\right)^4 \quad (1.10)$$

with a maximum relative error of 5% and 95% reliability. From the equation (1.10) we can then clear the value of α:

$$\alpha = 5 - 4\varepsilon^{0.25} \quad (1.11)$$

II. Estimation of the Flow Velocity Profile in Porous Medium

The partial differential equation that describes the laminar velocity profile in cylindrical coordinates for a Newtonian fluid, incompressible and in steady state in a tube of radius R, was modified to describe the flow through a tube within which solid particles are distributed motionless, which represents the porous bed.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication (BEIESP)

The modification consisted in replacing the spatial derivatives of order one with spatial derivatives of fractional order and equal to the fractal dimension, in such a way that:

$$-\frac{\partial p}{\partial z}+\mu R^{2\alpha-2}\left(\frac{\delta}{R}\right)^{1-2\alpha}\frac{1}{r}\frac{\partial^{\alpha}}{\partial r^{\alpha}}\left(r\frac{\partial^{\alpha}v}{\partial r^{\alpha}}\right)=0 \quad (2.1)$$

To obtain the solution of equation (2.1) this is expressed as a system of non-dimensional fractional differential equations with respect to the spatial coordinate:

$$\Phi\kappa\gamma+\kappa^{1-\alpha}\frac{\partial^{\alpha}\sigma}{\partial\gamma^{\alpha}}=0$$

$$\mu\gamma\kappa^{1-\alpha}\frac{\partial^{\alpha}v}{\partial\gamma^{\alpha}}=\sigma \quad (2.2)$$

where γ = r / R is the non-dimensional spatial coordinate v (ms-1) is the velocity of the fluid in the direction parallel to the coordinate z and Φ (Pa.m) is a parameter that involves frictional pressure losses $\left(-\frac{\partial P}{\partial z}\right)$ (Pa.m-1) and that is defined as:

$$\Phi=-\frac{\partial P}{\partial z}R^{2} \quad (2.3)$$

The boundary conditions established to solve the problem are given by:

$$\sigma(0)=0 \quad (2.4)$$

$$v(1)=0 \quad (2.5)$$

and they express that in the wall of the bed the speed is equal to zero (condition 2.5), while in the center the velocity exhibits a maximum value (condition 2.4). The exact solution of the system of fractional differential equations (2.2) allows obtaining the velocity profile:

$$\frac{v(\gamma,\alpha)}{v_{\max}}=\left(1-\gamma^{2\alpha}\right) \quad (2.6)$$

where the maximum velocity vmax (m.s-1) is given by:

$$v_{\max}=\frac{\Phi\kappa^{2\alpha-1}}{\mu(1+\alpha)2\alpha\Gamma(2\alpha)} \quad (2.7)$$

It is now appropriate to express the velocity profile as a function of porosityε, which is the parameter that is measured experimentally. Properly replacing equation (1.9) in equation (2.7):

$$v_{\max}=\frac{1}{4}\Phi\frac{\kappa^{2\alpha-1}}{\mu}\varepsilon\frac{\Gamma(\alpha)}{\Gamma(2\alpha)} \quad (2.8)$$

y la ecuación (1.11) en (2.6) y (2.8), y dimensionando la expresión resultante con respecto a la coordenada espacial se obtiene el perfil de velocidad del lecho poroso:

$$\frac{v(r)}{v_{\max}}=\left(1-\left(\frac{r}{R}\right)^{10-8\varepsilon^{0.25}}\right)$$

(2.9)

where the maximum speed is determined through the relationship:

$$v_{\max}=-\frac{\partial P}{\partial z}\frac{R^{2}}{4\mu}\varepsilon\frac{\Gamma\left(5-4\varepsilon^{0.25}\right)}{\Gamma\left(10-8\varepsilon^{0.25}\right)}\left(\frac{\delta}{R}\right)^{8\left(1-\varepsilon^{0.25}\right)+1} \quad (2.10)$$

Equation 2.10 let us contribute and can be use with those showed in (Carpinteri, A. & Mainardi, F. 2014)

III. ESTIMATION OF THE BEHAVIOR OF THE VELOCITY AND THE PRESSURE GRADIENT

The average speed, defined as the flow per unit of total area, is determined through the integral:

$$V=\frac{\int_0^{2\pi}\int_0^1 v_{\max}\left(\left(1-\gamma^{10-8\varepsilon^{0.25}}\right)\right)\gamma d\gamma d\theta}{\int_0^{2\pi}\int_0^1\gamma d\gamma d\theta} \quad (3.1)$$

whose solution is given by:

$$V=v_{\max}\frac{\alpha}{\alpha+1}$$

$$=v_{\max}\frac{5-4\varepsilon^{0.25}}{6-4\varepsilon^{0.25}}$$

(3.2)

Substituting (2.9) in (3.2) finally gets:

$$V=\left(-\frac{\partial P}{\partial z}\right)\frac{R^{2}}{4\mu}\frac{\varepsilon}{\left(6-4\varepsilon^{0.25}\right)}\frac{\Gamma\left(6-4\varepsilon^{0.25}\right)}{\Gamma\left(10-8\varepsilon^{0.25}\right)}\left(\frac{\delta}{R}\right)^{8\left(1-\varepsilon^{0.25}\right)+1} \quad (3.3)$$

On the other hand, the average speed vc (ms-1), defined as the flow per unit of actual flow area, and which allows to estimate the speed of the flow through the channels that are formed between the particles of the bed, is determined finding the ratio between V and the porosity:

$$v_{c}=\frac{\partial P}{\partial z}\frac{R^{2}}{4\mu}\frac{1}{\left(6-4\varepsilon^{0.25}\right)}\frac{\Gamma\left(6-4\varepsilon^{0.25}\right)}{\Gamma\left(10-8\varepsilon^{0.25}\right)}\left(\frac{\delta}{R}\right)^{8\left(1-\varepsilon^{0.25}\right)+1} \quad (3.3)$$

From equation (3.2) the pressure gradient is cleared:

$$\frac{\partial P}{\partial z}=-\frac{4\mu}{R^{2}}V\frac{\left(6-4\varepsilon^{0.25}\right)}{\varepsilon}\frac{\Gamma\left(10-8\varepsilon^{0.25}\right)}{\Gamma\left(6-4\varepsilon^{0.25}\right)}\left(\frac{R}{\delta}\right)^{8\left(1-\varepsilon^{0.25}\right)+1} \quad (3.4)$$

According to this equation, the frictional pressure losses increase linearly with the product of the speed and viscosity of the fluid, which is a characteristic flow behavior in a laminar regime, which is also reflected in the simple models of these systems, as well as as potentially with respect to the parameter R / δ, which is related to the average relative size of the particles and the number of particles in the cross section to the flow

IV. PREDICTED THEORETICAL RESULTS AND DISCUSSION

If we consider δ / R = 1 and α = 1, which correspond in this model to the case of a tube free of particles and is substituted in equations (2.8) and (3.3), we obtain:

$$v=\left(-\frac{\partial P}{\partial z}\right)\frac{R^{2}}{4\mu}\left(1-\left(\frac{r}{R}\right)^{2}\right) \quad (4.1)$$

$$V_{0}=\left(-\frac{\partial P}{\partial z}\right)\frac{R^{2}}{8\mu}$$

(4.2)

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication (BEIESP)

Equations (4.1) and (4.2) represent the speed profile and the average velocityV0, respectively, for isothermal, stationary, Newtonian and incompressible flows that are transported in a tube under a laminar regime. Therefore, this result shows that the proposed model for porous bed flow is reduced to the tube flow model for porosity equal to 1 and number of particles equal 1.

In order to analyze the predictions of the model based on the effect of bed morphology on speed, to obtain these we will define the non-dimensional speed ϕ:

$$\phi = \frac{v}{V_0} \qquad (4.3)$$

and the velocity profile is written non-dimensionally:

$$\phi = \kappa^{9-8\varepsilon^{0.25}} \frac{\varepsilon\Gamma\left(5-4\varepsilon^{0.25}\right)}{\Gamma\left(10-8\varepsilon^{0.25}\right)}\left(1-|\gamma|^{10-8\varepsilon^{0.25}}\right)$$

$$\kappa = \frac{\delta}{R};\gamma = \frac{r}{R} \qquad (4.4)$$

Consequently, the average non-dimensional velocityη is defined:

$$\eta = \frac{V}{V_0} \qquad (4.5)$$

$$\eta = \frac{\Gamma\left(6-4\varepsilon^{0.25}\right)}{\Gamma\left(10-8\varepsilon^{0.25}\right)}\frac{2\varepsilon}{\left(6-4\varepsilon^{0.25}\right)}\kappa^{9-8\varepsilon^{0.25}} \qquad (4.6)$$

The non-dimensional velocity profiles obtained for different values of the parametersκ and ε are shown in Figures 4.1 and 4.2. As can be seen, the model predicts a decrease in the maximum velocity with the decrease in porosity and particle size, which is an expected result, since the decrease of both parameters implies an increase in the number of particles, which In turn, it implies a greater loss of friction pressure. It is to be expected, then, that, for the same pressure gradient, lower speeds will be obtained. On the other hand, it is appreciated that the velocity profile is distorted with respect to the parabolic profile corresponding to the laminar flow in tubes, showing a flattening in the areas near the center, which increases with the number of particles and the decrease in porosity of the bed. This behavior is explained considering that the proposed model takes into account, simultaneously, the effects of the wall (in the boundary conditions) and the particles (in the fractional order of the spatial derivatives). In areas far from the wall, the influence of this decreases and becomes practically negligible, since the effects that predominate correspond to the movement of the fluid through the channels that are formed between the particles, which is manifested in the flattening of the velocity profile.

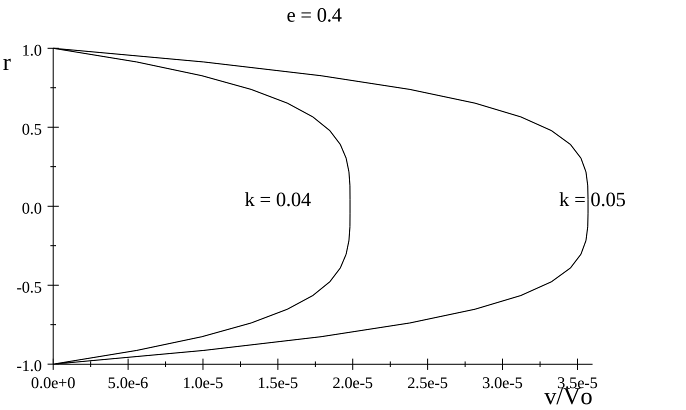

Figure 4.1. Behavior of the non-dimensional profile of the velocity for a porosity value of e = 0.4 considering as a parameter k = δ / R

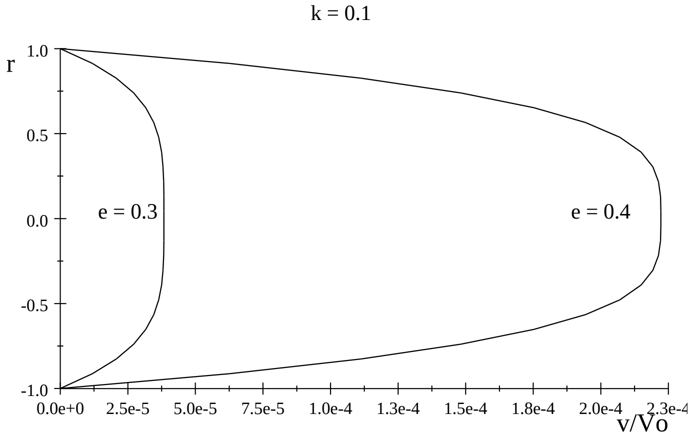

Figure 4.2. Behavior of the non-dimensional velocity profile for a ratio of size = δ / R, considering porosity e as a parameter. To analyze the predicted behavior of the average speed, it is convenient to compare this with that predicted by the model proposed by Blake and Kozeny for laminar regime and spherical particles, and that is given by:

$$-\frac{\partial P}{\partial z} = \lambda \frac{V}{(2\delta)^2}\mu\frac{(\varepsilon-1)^2}{\varepsilon^3}$$

$$\lambda = 150 \qquad (4.6)$$

in which the value of the parameter λ was calculated by statistical techniques from the experimental results observed. From the equation (4.6) the average speed V is cleared:

$$V = -\frac{\partial P}{\partial z}R^2\frac{2}{75}\frac{\Phi}{\mu}\left(\frac{\delta}{R}\right)^2\frac{\varepsilon^3}{(\varepsilon-1)^2} \qquad (4.7)$$

Expressing the average speed predicted by the model of Blake and Kozeny (Santoso, R. K. et al. 2018; MacDonald, M. J. et al. 1999) (equation (4.7)) in a non-dimensional way we obtain:

$$m = \frac{V}{V_0} = 0.21333\kappa^2 \frac{\varepsilon^3}{(\varepsilon - 1)^2} \tag{4.8}$$

Before presenting this comparative analysis, it is convenient to discuss some issues related to the limitations common to both models and the differences between the two. The formalism adopted by Blake and Kozeny is based on visualizing the flow through n channels between the particles, where n scales with the size of these, and in applying the equations for flow in tubes to predict the flow velocity in each channel. This visualization is inappropriate to establish the velocity profile, so this model does not consider the effect of the bed wall (different models have been proposed to correct this limitation (MacDonald, M. J. et al. 1999) or new ways of studying when chemical species are involved (Abbas, Z. et al. 2016)). On the other hand, according to this model, for a porosity value equal to 1, which corresponds to the flow in tubes, the speed becomes infinite for a specified pressure drop (or the pressure gradient becomes equal to zero, for any speed value), predictions that are not correct, which can significantly limit the scope of the model for porosity values greater than 0.5. The model proposed in this paper is based on a phenomenological equation that includes as parameters the porosity and the particle size, in addition to the dimensions of the system and the viscosity of the fluid, in such a way that the model obtained from the solution of this equation takes into account the effect of the wall of the tube, allows to predict the velocity profile and is reducible to the flow in tubes for values of porosity and extreme particle size. Another relevant difference between both models is that, while the Blake and Kozeny model involves an empirical parameter whose value should be determined from the experimental observations, the proposed model does not need to involve a priori any empirical parameter, although a future experimental test it can lead to the need to involve this type of parameter to allow an appropriate adjustment of the model to the experimental data observed. In both models it is considered that all the free volume of particles of the bed is occupied by the fluid in motion. However, for very small particle sizes, the phenomena related to the surface tension of the fluid and its affinity for the surface of the particles, which depend on the chemical composition, can have appreciable effects on the fluid velocity and can even cause that the fluid is retained in some sections of the bed. Another limitation common to both models is that they consider the size of the particles, the bed radius and the porosity as parameters that can be treated independently, when in fact both are closely related to each other. In the formalism adopted in this paper, an equation is obtained to describe the flow divider area (equation 1.8) through which it is predicted that it will increase with the particle size for a constant fractal dimension, just as shown in Figure 4.3. This result indicates the close relationship that exists between these parameters, but unfortunately equation (1.8) cannot be equated to porosity, since the fractal dimension can only be determined when the system is observed at the scale of the particle size, it is say, under the condition $\delta / r \to 1$, where r is the radius of the observed system, and what is interesting is to model the behavior of the system in the macroscopic scale. It is important to consider that when the results predicted by a model are presented, they are interpreted observing in a graph in rectangular coordinates how the output variable (in this case the velocity) changes with respect to one of the parameters, which is taken as an independent variable, while the other parameter or the others are considered constant. In the case in question, for example, we will analyze how the velocity varies with respect to the particle size, considering porosity as a parameter, or how the velocity changes with respect to porosity, considering the particle size as a parameter. But since these two parameters are related, as shown in Figure (4.3), there may be combinations of values of both that are not physically possible. However, this is not of great importance from the practical point of view, since, due to all the limitations posed, it is always necessary to determine experimentally the particle size distribution and the porosity of the bed taking into account the specific fluid that is moves through the bed, since experimental evidence shows that the observed porosity depends on the chemical nature of this.

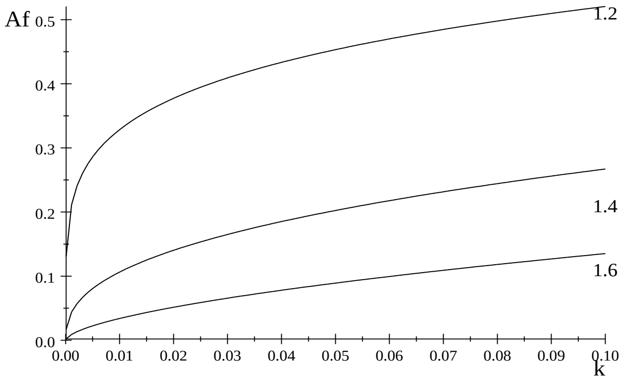

Figure 4.3. Behavior of the predicted flow divider area as a function of particle size considering the fractal dimension as a parameter Figure 4.4 shows the average speed behavior predicted by the proposed model (MP) and by the Blake and Kozeny (ME) model with respect to bed porosity considering the ratio δ / R as a parameter. Both models predict very similar qualitative behaviors, where velocity values have the same order of magnitude, and where for porosity values less than 0.5 the proposed model predicts a lower velocity. Although the prediction of a lower velocity could be explained because this model includes the effect of the wall, this can also be a result of the limitations of the model, while for higher values of porosity, questions could be raised about the validity of the model. Blake and Kozeny for the reduction aspects discussed above. The smallest differences are observed for porosity values between 0.4 and 0.5, which include the theoretical value for an ideal bed occupied by spherical particles of the same size ($\varepsilon = 0.47$, equation (1.3), obtaining that for this theoretical value there is an exact correspondence between both models for a value of $\kappa = 0.1$, as shown in Figure 4.5, taking into account the limitations of both models, and the significant differences that exist between them, this correspondence turns out to be surprising (Figure 4.6), and can only be be explained if i) it is considered that the mathematical formalism used is valid for the description of this system, and therefore, could in principle be used for the description of physical systems in general, although there is in fact no relationship between the fractal dimension and the order of the fractional derivatives; ii) if there is a relationship between

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication (BEIESP)

fractal geometry and fractional calculus, even when this cannot be demonstrated logically, which allows describing underlying physical facts in complex heterogeneous systems, in such a way that it is possible to obtain more general equations to describe the dynamics of fluids, being the Navier Stokes equation a particular case of these yiii) in fact there is no relationship between fractal geometry and fractional calculation, and the correspondence obtained between the predicted results and those observed in this work (the Blake model) and Kozeny was corroborated experimentally) is entirely the result of chance. We leave this answer to the reader's consideration.

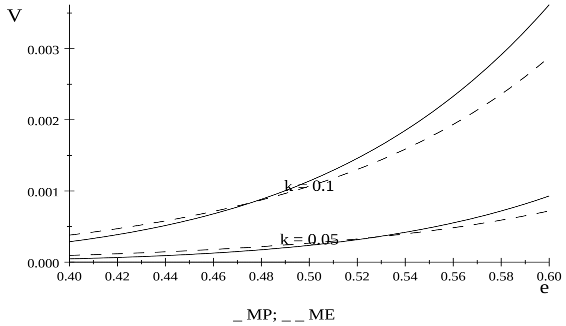

Figure 4.4. Prediction of the behavior of the average speed by the proposed model (MP) and by the model of Blake and Kozeny (ME) with respect to porosity and considering as a parameter the relation k = δ / R

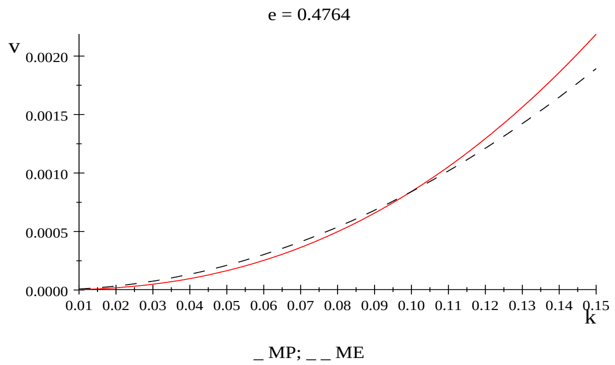

Figure 4.5. Prediction of the behavior of the average speed by the proposed model (MP) and by the model of Blake and Kozeny (ME) with respect to the relation k = δ / R for the theoretical value of porosity equal to 0.4764

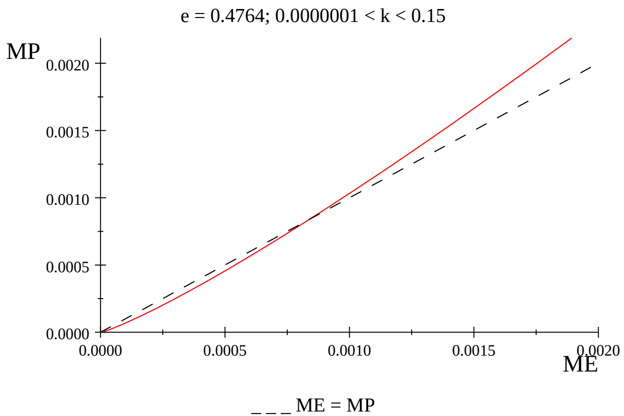

Figure 4.6. Comparison between the values predicted by the proposed model (MP) and by the model of Blake and Kozeny (ME) considering as a parameter the relation k = δ / R for the theoretical value of porosity equal to 0.4764

In a later article the experimental reproducibility will be analyzed, at the moment the proposed model presents the following advantages with respect to the precedents: 1) since it is an analytical model, it does not require the use of numerical techniques or a high computational cost for the prediction of its results, which is one of the advantages of the practical models that are commonly used for the design and evaluation of this type of systems; 2) when predicting the speed profile behavior, it is appropriate to predict the speed behavior in biphasic systems with total separation of the phases, for which the same methodology used for this type of flow in tubes is applied; 3) considering the temporal derivative in the fractional differential equation that describes the velocity profile, one can predict the non-stationary state behavior in these systems and 4) it could, in principle, be appropriate to describe the non-Newtonian flow behavior in porous beds , although it remains to be investigated if the obtained equations can be solved in an analytical way.

ACKNOWLEDGMENT

EJSD Appreciates the PRODEP oficio 511-06/18-9249support received for new PTCs and project 282278. APP appreciates the support received by the SENER-CONACYT-HYDROCARBONS fund; project number 282278. YGAJ Appreciates the PROFOCIE support received in the UAT FADU.

REFERENCES

1. Abbas, Z., Sheikh, M., & Sajid, M. (2016). Diffusion of chemically reactive species in stagnation point flow of a micropolar viscoelastic fluid in a porous medium over a stretching/shrinking sheet. Revista Mexicana de Física, 62(4).
2. Baleanu, D. Diethelm, K. Scalas, E. & Trujillo, J. J. (2016). Fractional Calculus. World Scientific Publishing Company.
3. Carpinteri, A., & Mainardi, F. (Eds.). (2014). Fractals and fractional calculus in continuum mechanics (Vol. 378). Springer.
4. Cirpka, O. A. Chiogna, G. Rolle, M. & Bellin, A. (2015). Transverse mixing in three-dimensional nonstationary anisotropic heterogeneous porous media. Water Resources Research, 51(1), 241-260.
5. Heider, Y. & Markert, B. (2017). A phase-field modeling approach of hydraulic fracture in saturated porous media. Mechanics Research Communications, 80, 38-46.
6. Kundu, P. Kumar, V. & Mishra, I. M. (2016). Experimental and numerical investigation of fluid flow hydrodynamics in porous media: Characterization of pre-Darcy, Darcy and non-Darcy flow regimes. Powder Technology, 303, 278-291.
7. Li, C. & Zeng, F. (2015). Numerical methods for fractional calculus. Chapman and Hall/CRC Press.
8. MacDonald, M. J. Chu, C. F. Guilloit, P. P. & Ng, K. M. (1991). A generalized Blake‐Kozeny equation for multisized spherical particles. AIChE Journal, 37(10), 1583-1588.
9. Machado, J. T. Mainardi, F. & Kiryakova, V. (2015). Fractional calculus: Quo vadimus? (Where are we going?). Fractional Calculus and Applied Analysis, 18(2), 495-526.
10. Muljadi, B. P. Blunt, M. J. Raeini, A. Q. & Bijeljic, B. (2016). The impact of porous media heterogeneity on non-Darcy flow behaviour from pore-scale simulation. Advances in water resources, 95, 329-340.
11. Rao, G. Plank, B. & Kastner, J. (2016). Comparison of different segmentation methods for porosity evaluation in CFRP-reference samples with real porosity samples. In Conference on Industrial Computed Tomography, Wels/Österreich.
12. Regelink, I. C. Stoof, C. R. Rousseva, S. Weng, L. Lair, G. J. Kram, P. ... & Comans, R. N. (2015). Linkages between aggregate formation, porosity and soil chemical properties. Geoderma, 247, 24-37.
13. Richard, H. (2014). Fractional calculus: an introduction for physicists. World Scientific.
14. Rong, L. W. Zhou, Z. Y. Yu, A. B. (2015). Lattice–Boltzmann simulation of fluid flow through packed beds of uniform ellipsoids. Powder Technology, 285: 146-156.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication (BEIESP)

15. Santoso, R. K. Fauzi, I. Hidayat, M. Swadesi, B. Aslam, B. M. y Marhaendrajana, T. (2018). Study of Non-Newtonian fluid flow in porous media at core scale using analytical approach. Geosystem Engineering, 21(1), 21-30.
16. Shoghi, M. R. & Norouzi, M. (2015). Linear stability analysis and nonlinear simulation of non-Newtonian viscous fingering instability in heterogeneous porous media. Rheologica Acta, 54(11-12), 973-991.
17. Strogatz, S. H. (2018). Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering. 2nd Ed. CRC Press.
18. Tan, X. H. Jiang, L. Li, X. P. Li, Y. Y. & Zhang, K. (2017). A complex model for the permeability and porosity of porous media. Chemical Engineering Science, 172, 230-238.
19. Vafai, K. (2015). Handbook of porous media. 3rd Ed. CRC Press.
20. Vargas-González, S. Pacheco-Aguirre, F. M. López-Sánchez, E. Carrillo-Ahumada, J. Ruiz-López, I. I. & García-Alvarado, M. A. (2017). Analytical solutions of conduction/diffusion equation in media contacting a finite volume solution and their topological relations. Revista Mexicana de Ingeniería Química, 16(3).
21. Wang, Y. Hajibeygi, H. & Tchelepi, H. A. (2014). Algebraic multiscale solver for flow in heterogeneous porous media. Journal of Computational Physics, 259, 284-303
22.details, their publications, research work, membership, achievements, with photo that will be maximum 200-400 words.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication (BEIESP)