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Intuitionistic Fuzzy Completely Β Generalized Α Continuous Mappings

DGomathi M, Geetha N K

Abstract: *we introduce intuitionistic fuzzy completely β generalized α continuous mappings. We investigate various properties and explore characterizations of intuitionistic fuzzy completely β generalized α continuous mappings.*

Keywords: *β generalized α $T_{1/2}$ space in intuitionistic fuzzy, intuitionistic fuzzy completely β generalized α continuous mappings.*

Subject classification code: 03F55, 54A40.

I. INTRODUCTION

The idea of intuitionistic fuzzy sets was introduced by Atanassov [1]. Coker [2] gave a definition of Intuitionistic fuzzy topological spaces. Gomathi, M and Jayanthi, D [5] introduced intuitionistic fuzzy β generalized α continuous mappings. Here in this paper we introduce the notion of intuitionistic fuzzy completely β generalized α continuous mappings and also provide characterizations, properties of intuitionistic fuzzy completely β generalized α continuous mappings.

II. PROPOSED METHODOLOGY

Definition 2.1: A mapping q: (R, τ) ⟶ (S, σ) is said to be an intuitionistic fuzzy completely β generalized α continuous (Int.fuz completely βGα continuous for short) mapping if pre image of every IFβGαCS in S is an IFRCS in R.

Theorem 2.2: Every int.fuz completely βGα continuous mapping is an int.fuz continuous mapping [7].

Proof: Take q: (R, τ) → (S, σ) is an int.fuz completely βGα continuous mapping . Let M be an IFCS in S. We know that every IFCS is an IFβGαCS[4], M is an IFβGαCS in S. Then pre image of M is an IFRCS in R. Since each IFRCS is an IFCS, q^{-1} (M) is an IFCS in R. Therefore q is an int.fuz continuous mapping.

Example 2.3: Every int.fuz continuous mapping is not an int.fuz completely βGα continuous mapping.

It is proved by example.

Let R = {c, d}, S = {i, j} and $k_1 = \langle r, (0.5_c, 0.3_d), (0.5_c, 0.7_d)\rangle$, $K_2 = \langle r, (0.4_c, 0.3_d), (0.6_c, 0.7_d)\rangle$, $k_3 = \langle s, (0.5_i, 0.3_j), (0.5_i, 0.7_j)\rangle$ and $K_4 = \langle s, (0.4_i, 0.3_j), (0.6_i, 0.7_j)\rangle$. Then $\tau = \{0_\sim, k_1, k_2\ 1_\sim\}$ and $\sigma = \{0_\sim, k_3, k_4, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping q: (R, τ) → (S, σ) by q(c) = i and q(d) = j.

$IF\alpha O(S) = \{0_\sim, K_3, K_4, 1_\sim\}$

$IF\beta C(S) = \{0_\sim, 1_\sim, \mu_i \in [0, 1], \mu_j \in [0, 1], \nu_i \in [0, 1], \nu_j \in [0, 1] / \ \mu_i \geq 0.5$ whenever $\mu_j \geq 0.3$, or $\mu_i < 0.4$ or $\mu_j < 0.3$, $0 \leq \mu_i + \nu_j \leq 1$ and $0 \leq \mu_i + \nu_j \leq 1\}$

Hence it is proved, since K_4^c is an IFβGαCS in S, but $q^{-1}(K_4^c)$ is not an IFRCS in R as $cl(int((q^{-1}(K_4^c)) = cl(K_1) = K_1^c \neq q^{-1}(K_4^c)$.

Theorem 2.4: Every int.fuz completely βGα continuous mapping is an int.fuz semi continuous mapping [11] but reverse process is not true in general.

Proof: Take q: (R, τ) → (S, σ) is an int.fuz completely βGα continuous mapping and also assume M is an IFCS in S then M is an IFβGαCS in S. Now q^{-1} (M) is an IFRCS in R then q^{-1} (M) is an IFSCS in R. Henceforth q is an int.fuz semi continuous mapping.

Example 2.5: Take Example 2.3, q is an int.fuz semi continuous mapping and q is not an int.fuz completely βGα continuous mapping.

Theorem 2.6: All int.fuz completely βGα continuous mapping is an int.fuz pre-continuous mapping [11] but reverse process is not true in general.

Proof: Assume q: (R, τ) → (S, σ) is an int.fuz completely βGα continuous mapping and also assume M be an IFCS in R then M is an IFβGαCS in S. Now q^{-1} (M) is an IFRCS in R. then q^{-1} (M) is an IFPCS in R. Hence f is an IF pre continuous mapping.

Example 2.7: Take Example 2.3, q satisfies the definition of int.fuz pre-continuous mapping and q is not an int.fuz completely βGα continuous mapping.

Theorem 2.8: All int.fuz completely βGα continuous mapping is an int.fuz alpha continuous mapping [11] on the other hand reverse process is not true in general.

Proof: Assume q: (R, τ) → (S, σ) is an int.fuz completely βGα continuous mapping and also assume M be an IFCS in S then M is an IFβGαCS in S. Now q^{-1} (M) is an IFRCS in R then q^{-1} (M) is an int.fuz alpha closed set in R. Henceforth f is an int.fuz alpha continuous mapping.

Example 2.9: Take Example 2.3, q satisfies the int.fuz alpha continuous mapping and also q is not satisfies int.fuz completely βGα continuous mapping.

Theorem 2.10: Every int.fuz completely βGα continuous mapping is an IFβ continuous mapping[11] but reverse process is not true in general.

Proof: Take q: (R, τ) → (S, σ) is an int.fuz completely βGα continuous mapping and assume M is an IFCS in S then M is an IFβGαCS in S. Now q^{-1} (M) is an IFRCS in R then q^{-1} (M) is an IFβCS in R. Henceforth q is an IFβ continuous mapping.

Example 2.11: Take Example 2.3, q satisfies the definition of IFβ continuous mapping and not satisfies int.fuz completely βGα continuous mapping.

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* Correspondence Author (s)

Dr. M. Gomathi, Department of Science and Humanities, Sri Krishna college of engineering and technology, Coimbatore, India.

N. K. Geetha, Department of Science and Humanities, Sri Krishna college of engineering and technology, Coimbatore, India.

Theorem 2.12: Every int.fuz completely βGα continuous mapping is an int.fuz γ continuous mapping [8] but reverse implication is not true.

Proof: Take q: $(R, \tau) \to (S, \sigma)$ is an int.fuz completely βGα continuous mapping and take M is an IFCS in S then M is an IFβGαCS in S. Now $q^{-1}(M)$ is an IFRCS in R then $q^{-1}(M)$ is an IFγCS in R. Henceforth q is an int.fuz γ continuous mapping.

Example 2.13: From Example 2.3, q satisfies int.fuz γ continuous mapping, and it does not satisfies int.fuz completely βGα continuous mapping.

Theorem 2.14: Every int.fuz completely βGα continuous mapping is an int.fuz semi pre continuous mapping[13] but reverse process is not true in general.

Proof: Assume q: $(R, \tau) \to (S, \sigma)$ is an int.fuz completely βGα continuous mapping and M is an IFCS in R then M is an IFβGαCS in S. Now $q^{-1}(M)$ is an IFRCS in R then $q^{-1}(M)$ is an IFSPCS in S. Henceforth q satisfies int.fuz semi pre continuous mapping.

Example 2.15: Take Example 2.3, q satisfies the definition of int.fuz semi pre continuous mapping and not an int.fuz completely βGα continuous mapping.

Theorem 2.16: All int.fuz completely βGα continuous mapping is an IFβGα continuous mapping [5] but reverse process is not true.

Proof: Assume q: $(R, \tau) \to (S, \sigma)$ is an int.fuz completely βGα continuous mapping and M is an IFCS in S then M is an IFβGαCS in S. Now $q^{-1}(M)$ is an IFRCS in R then $q^{-1}(M)$ is an IFβGαCS in R. Therefore q is an IFβGα continuous mapping.

Example 2.17: Let R = {c, d}, S = {i, j} and $K_1 = \langle r, (0.5_c, 0.3_d), (0.5_c, 0.7_d)\rangle$, $K_2 = \langle r, (0.4_c, 0.3_d), (0.6_c, 0.7_d)\rangle$, $K_3 = \langle s, (0.5_i, 0.3_j), (0.5_i, 0.7_j)\rangle$ and $K_4 = \langle s, (0.4_i, 0.3_j), (0.6_i, 0.7_j)\rangle$. Then $\tau = \{0_\sim, K_1, K_2, 1_\sim\}$ and $\sigma = \{0_\sim, K_3, K_4, 1_\sim\}$ are IFTs on R and S respectively. Define a mapping q: $(R, \tau) \to (S, \sigma)$ by q(c) = i and q(d) = j. Then

$$\text{IF}\alpha\text{O}(R) = \{0_\sim, K_1, K_2, 1_\sim\}$$

$\text{IF}\beta\text{C}(R) = \{0_\sim, 1_\sim, \mu_c \in [0,1], \mu_d \in [0,1], \nu_c \in [0,1], \nu_d \in [0,1] / \mu_c \geq 0.5$ whenever $\mu_d \geq 0.3$, or $\mu_c < 0.4$ or $\mu_d < 0.3$, $0 \leq \mu_c + \nu_c \leq 1$ and $0 \leq \mu_d + \nu_d \leq 1\}$

$$\text{IF}\alpha\text{O}(S) = \{0_\sim, K_3, K_4, 1_\sim\}$$

$\text{IF}\beta\text{C}(S) = \{0_\sim, 1_\sim, \mu_i \in [0,1], \mu_j \in [0,1], \nu_i \in [0,1], \nu_j \in [0,1] / \mu_i \geq 0.5$ whenever $\mu_j \geq 0.3$, or $\mu_i < 0.4$ or $\mu_j < 0.3$, $0 \leq \mu_i + \nu_i \leq 1$ and $0 \leq \mu_j + \nu_j \leq 1\}$

The IFS K_4^c is an IFCS in S. Then $q^{-1}(K_4^c) = \langle r, (0.6_c, 0.7_d), (0.4_c, 0.3_d)\rangle$ is an IFβGαCS in R. Therefore q is an IFβGα continuous mapping.

Now K_4^c is an IFβGαCS in R but $q^{-1}(K_4^c) = \langle r, (0.6_c, 0.7_d), (0.4_c, 0.3_d)\rangle$ is not an IFRCS in R, as $cl(int((q^{-1}(K_4^c)) = cl(K_1) = K_1^c \neq q^{-1}(K_4^c)$, q is not an int.fuz completely βGα continuous mapping.

Theorem 2.18: Every int.fuz completely βGα continuous mapping is an int.fuz almost βGα continuous mapping[6] but reverse procedure is not true.

Proof: Assume q: $(R, \tau) \to (S, \sigma)$ is an int.fuz completely βGα continuous mapping and M is an IFRCS in S. then M is an IFβGαCS in S. Then $q^{-1}(M)$ is an IFRCS and hence is an IFβGαCS in R. Thus q is an int.fuz almost βGα continuous mapping.

Example 2.19: Let R = {c, d}, S = {i, j} and $K_1 = \langle r, (0.5_c, 0.4_d), (0.5_c, 0.6_d)\rangle$, $K_2 = \langle s, (0.5_i, 0.3_j), (0.5_i, 0.7_j)\rangle$ and $K_3 = \langle s, (0.4_i, 0.3_j), (0.6_i, 0.7_j)\rangle$. Then $\tau = \{0_\sim, K_1, 1_\sim\}$ and $\sigma = \{0_\sim, K_2, K_3, 1_\sim\}$ are IFTs on R and S respectively. Define a mapping q: $(R, \tau) \to (S, \sigma)$ by q(c) = i and q(d) = j. Then

$$\text{IF}\alpha\text{O}(R) = \{K_1, 1_\sim, 0_\sim\}$$

$\text{IF}\beta\text{C}(R) = \{0_\sim, 1_\sim, \mu_c \in [0,1], \mu_d \in [0,1], \nu_c \in [0,1], \nu_d \in [0,1] / 0 \leq \mu_c + \nu_c \leq 1$ and $0 \leq \mu_d + \nu_d \leq 1\}$

$$\text{IF}\alpha\text{O}(S) = \{0_\sim, K_2, K_3, 1_\sim\}$$

$\text{IF}\beta\text{C}(S) = \{0_\sim, 1_\sim, \mu_i \in [0,1], \mu_j \in [0,1], \nu_i \in [0,1], \nu_j \in [0,1] / \mu_i \geq 0.5$ whenever $\mu_j \geq 0.3$, or $\mu_i < 0.4$ or $\mu_j < 0.3$, $0 \leq \mu_i + \nu_i \leq 1$ and $0 \leq \mu_j + \nu_j \leq 1\}$

The IFS $K_2^c = \langle s, (0.5_i, 0.7_j), (0.5_i, 0.3_j)\rangle$ is an IFRCS in S. Then $q^{-1}(K_2^c) = \langle r, (0.5_c, 0.7_d), (0.5_c, 0.3_d)\rangle$ is an IFβGαCS in R. Therefore q is an int.fuz almost βGα continuous mapping.

Now $K_2^c = \langle s, (0.5_i, 0.7_j), (0.5_i, 0.3_j)\rangle$ is an IFβGαCS in S but $q^{-1}(K_2^c) = \langle r, (0.5_c, 0.7_d), (0.5_c, 0.3_d)\rangle$ is not an IFRCS in R, as $cl(int((q^{-1}(K_2^c)) = cl(K_1) = K_1^c \neq q^{-1}(K_2^c)$. Then q is not an int.fuz completely βGα continuous mapping.

Theorem 2.20: If q: $(R, \tau) \to (S, \sigma)$ is an int.fuz completely βGα continuous mapping then $\beta cl(q^{-1}(W)) \subseteq q^{-1}(cl(W))$ for every FβOS A ⊆ Y.

Proof: Assume W is an IFβOS in S. Then cl(W) is an IFRCS in S. Hence cl(W) is an IFβGαCS in S. By premise, $q^{-1}(cl(W))$ IFRCS in R and so an IFβCS in W. Hence $\beta cl(q^{-1}(W)) \subseteq \beta cl(q^{-1}(cl(W))) = q^{-1}(cl(W))$.

Theorem 2.21: Assume q: $(R, \tau) \to (S, \sigma)$ is a mapping. Then the following are alike:

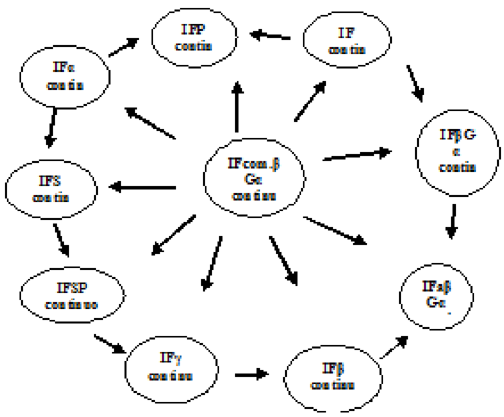
(i) q is an int.fuz completely βGα continuous mapping

(ii) $q^{-1}(W)$ is an IFROS in R for every IFβGαOS W in S

(iii) for every intuitionistic fuzzy point of $p_{(\alpha,\beta)}$ belongs to R and for every IFβGαOS U in S such that $q(p_{(\alpha,\beta)})$ belongs to U there exists an IFROS in R such that point of $(\alpha,\beta) \in W$ and $q(W) \subseteq U$

III. BLOCK DIAGRAM

Relations among various mappings with intuitionistic fuzzy completely β generalized α continuous mappings.



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Theorem 2.22: Assume q: (R, τ) → (S, σ) is an int.fuz completely βGα continuous mapping then for every int.fuz point $p_{(\alpha,\beta)}$ belongs to R and for every int.fuz neighbourhood [11] W of $q(p_{(\alpha,\beta)})$, there occurs an IFROS U ⊆ R such that point of (α,β) ∈ U ⊆ $q^{-1}(W)$.

Proof: Let as assume point of (α,β) belongs to R and W is an int.fuz neighbourhood of $q(p_{(\alpha,\beta)})$. Then there occurs an IFOS C in S such that $q(p_{(\alpha,\beta)}) \in V \subseteq W$. We know that each IFOS is an IFβGαOS, V is an IFβGαOS in S. Hence by hypothesis, $q^{-1}(V)$ is an IFROS in R and $p_{(\alpha,\beta)} \in q^{-1}(q(p_{(\alpha,\beta)})) \subseteq q^{-1}(V) \subseteq q^{-1}(W)$ and therefore $p_{(\alpha,\beta)} \in q^{-1}(V)$. Now let $q^{-1}(V) = U$. Therefore $p_{(\alpha,\beta)} \in U \subseteq q^{-1}(W)$.

Theorem 2.23: A mapping q: (R, τ) → (S, σ) is an int.fuz completely βGα continuous mapping then for every int.fuz point $p_{(\alpha,\beta)} \in R$ and for every int.fuz neighbourhood W of $q(p_{(\alpha,\beta)})$, there occurs an IFROS U ⊆ R such that $p_{(\alpha,\beta)} \in U$ and $q(U) \subseteq W$.

Proof: Let as assume intuitionistic fuzzy point of (α,β) ∈ R and W is an int.fuz neighbourhood of $q(p_{(\alpha,\beta)})$. Then there occurs an IFOS E in S such that $q(p_{(\alpha,\beta)}) \in E \subseteq W$. Since each IFOS remains an IFβGαOS, E is an IFβGαOS in S. Hence by assumption, $q^{-1}(E)$ is an IFROS in R and $p_{(\alpha,\beta)} \in q^{-1}(E)$. Now take $q^{-1}(E) = U$. Therefore int.fuz point of (α,β) ∈ U ⊆ $q^{-1}(W)$. So $q(U) \subseteq q(q^{-1}(W)) \subseteq W$. That is $q(U) \subseteq W$.

Theorem 2.24: For any two int.fuz completely βGα continuous functions q_1, q_2 : (R, τ) → (S, σ), the function (q_1, q_2) : (R, τ) → (S x S, σ x σ) is also an int.fuz $(q_1(r), q_2(r))$ for every r ∈ R.

Proof: Let M x N be an IFβGαOS in S x S. Then $(q_1, q_2)^{-1}$ (M x N) (r) = (M x N) $(q_1(r), q_2(r))$

$= \langle r, \min(\mu_m(q_1(r)), \mu_n(q_2(r))), \max(\nu_m(q_1(r)), \nu_n(q_2(r)))\rangle$

$= \langle r, \min(q_1^{-1}(\mu_m)(r), q_2^{-1}(\mu_n)(r)), \max(q_1^{-1}(\nu_m)(r), q_2^{-1}(\nu_n)(r))\rangle$

$= (q_1^{-1}(m) \cap q_2^{-1}(n))(r)$.

Then q_1 and q_2 are int.fuz completely βGα continuous functions, $q_1^{-1}(m)$ and $q_2^{-1}(n)$ are IFROSs in R. As we know that the intersection of any IFROSs is an IFROS, $q_1^{-1}(m) \cap q_2^{-1}(n)$ is an IFROS in R. Henceforth (q_1, q_2) remains int.fuz completely βGα continuous mappings.

Theorem 2.25: The composition of any two int.fuz completely β generalized α continuous mapping is an int.fuz completely β generalized α continuous mapping in common.

Proof: Let q: R → S and h: S → G be any two int.fuz completely βGα continuous mappings. Let as take U is an IFβGαOS in G. As h is an int.fuz completely βGα continuous mapping, $h^{-1}(U)$ is an IFROS in S. Since each IFROS is an IFβGαOS, $h^{-1}(U)$ is an IFβGαOS in S. Since q is an int.fuz completely βGα continuous mapping, $q^{-1}(h^{-1}(B)) = (h \circ q)^{-1}(U)$ is an IFROS in R. Hence h ∘ q is an int.fuz completely βGα continuous mapping.

Theorem 2.26: Let as take q: R → S and h: S → G be any two functions. Then

(i) h∘ q is an int.fuz completely βGα continuous mapping if q is an int.fuz completely βGα continuous mapping and h is an IFβGα irresolute mapping.

(ii) h∘ q is an IFβGα continuous mapping if q remains a IF completely βGα continuous mapping and h remains a IFβGα continuous mapping.

Proof:

(i) Take U is an IFβGαOS in G. AS h remains a IFβGα irresolute mapping, $h^{-1}(U)$ remains IFβGαOS in S. Also, as q remains a int.fuz completely βGα continuous mapping, $q^{-1}(h^{-1}(U))$ is an IFROS in R. Thus $(h \circ q)^{-1}(U) = q^{-1}(h^{-1}(U))$, h ∘ q remains IF completely βGα continuous mapping.

(ii) Take U is an IFOS in G. As h remains IFβGα continuous mapping, $h^{-1}(U)$ is an IFβGαOS in S. Also q remains int.fuz completely βGα continuous mapping, $q^{-1}(h^{-1}(U))$ remains IFROS in R. Henceforth $q^{-1}(h^{-1}(U))$ is an IFβGαOS in R. From the fact that $(h \circ q)^{-1}(U) = q^{-1}(h^{-1}(U))$, it follows that (h ∘ f) is an IFβGα continuous mapping.

IV. RESULT ANALYSIS

The observation of intuitionistic fuzzy completely β generalized α continuous mapping is very strong continuous mapping when comparing to other existing continuous mappings. The characterization and their properties are studied and produced examples wherever necessary.

V. CONCLUSION

The ideas and definitions of completely β generalized α continuous mappings in intuitionistic fuzzy topological spaces are introduced and studied. It is compared with the already existing intuitionistic fuzzy continuous mappings and also discussed the properties and composition of completely β generalized α continuous mapping. Finally conclude that it is more stronger than other sets. For future it may extend to nano topological spaces, supra topological spaces and it is applicable for real life problems whenever human decision plays a vital role.

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AUTHORS PROFILE

Dr. M. Gomathi , Assistant Professor, Department of Science and Humanities, Sri Krishna college of engineering and technology. Published 3 scopus paper . gomathimathaiyan@gmail.com

N. K. Geetha , Assistant Professor, Department of Science and Humanities, Sri Krishna college of engineering and technology. Published 14 scopus paper 16 wos paper. nkgeeth@gmail.com

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