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Separation Axioms in N-ARY Topology

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Abstract: ***In the year 2011, Nithyanantha Jothi and Thangavelu introduced the concept of binary topology. Recently the authors extended the notion of binary topology to n-ary topology where n>1 an integer and extended some concepts in general topology to n-ary topology. In this paper separation axioms are introduced in n-ary spaces and their basic properties are discussed.***

Keywords: ***n-ary topology, n-ary open, n-ary closed and n-ary continuity.***

I. INTRODUCTION

Nithyanantha Jothi and Thangavelu[5-10] studied the notion of a binary topology and investigated the corresponding closure and interior operators in binary topological spaces. Following this topologists extended this concept to soft topological, generalized topological and supra topological spaces[1-4]. The second and the first authors[11,12,13] discussed nearly open sets in binary topology and n-ary topology. They also discussed n-ary closed sets[14], n-ary subspaces [15] and n-ary continuity[16]. In general topology, separation axioms play a vital role to classify topological spaces and will be used to study some topological properties. In this paper, by introducing new axioms in n-ary topological spaces, we will investigate the properties of n-ary topologies.

II. PRELIMINARIES

Let $X_1, X_2,\ldots, X_n$ be the nonempty sets and $P(X_1)\times P(X_2)\times\ldots\times P(X_n)$ be the Cartesian product of the their power sets. Examples can be constructed to show that the two notions 'product of power sets' and 'power set of the products' are different. Any typical element in $P(X_1)\times P(X_2)\times\ldots\times P(X_n)$ is of the form $A=(A_1,A_2,\ldots,A_n)$ where $A_i\subseteq X_i$ for $i\in\{1,2,3,\ldots,n\}$.
Throughout this paper $A=(A_1,A_2,\ldots,A_n)$ and $B=(B_1,B_2,\ldots,B_n)$ are members in $P(X_1)\times P(X_2)\times\ldots\times P(X_n)$. The notations and terminologies that are used here are taken from [11-16]. The union, intersection and complementation of n-ary elements are defined component wise. Let $T\subseteq P(X_1)\times P(X_2)\times\ldots\times P(X_n)$.

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Definition 2.1:

T is an n-ary topology on $(X_1, X_2 \ldots, X_n)$ if the following axioms are satisfied.
(i). $(\varnothing,\varnothing,\ldots,\varnothing)=\Phi\in T$
(ii). $(X_1, X_2,\ldots, X_n)=X\in T$
(iii). T is closed under finite intersections
(iv). If $(A_{1\alpha},A_{2\alpha},\ldots,A_{n\alpha})\in T$ for each $\alpha\in\Omega$ then $\bigcup_{\alpha\in\Omega}(A_{1\alpha},A_{2\alpha},\ldots,A_{n\alpha})\in T$.

If T is an n-ary topology then the pair (X, T) is an n-ary topological space. The element $x=(x_1,x_2,\ldots,x_n)\in X$ is an n-ary point of (X, T) and the members A of $P(X_1)\times P(X_2)\times\ldots\times P(X_n)$ are called the n-ary sets. The members of T are called the n-ary open sets in (X, T). It is noteworthy to see that product topology on $X_1\times X_2\times\ldots\times X_n$ and n-ary topology on $(X_1,X_2,\ldots,X_n)$ are independent concepts as any open set in product topology is a subset of $X_1\times X_2\times\ldots\times X_n$ and an open set in an n-ary topology is a member of $P(X_1)\times P(X_2)\times\ldots\times P(X_n)$.

Lemma 2.2:
$T_i=\{A_i: (A_1, A_2,\ldots,A_2)\in T\}$ is a topology on X_i.

Lemma 2.3:
Let τ_i be a topology on X_i. Then $T=\tau_1\times\tau_2\times\ldots\times\tau_n=\{(A_1, A_2,\ldots,A_2):A_i\in\tau_i\}$is an n-ary topology on X. Moreover $T_i=\tau_i$.

Definition 2.4:

$(A_1,A_2,\ldots,A_n)$ is n-ary closed if

$(A_1, A_2,\ldots,A_n)^c=(X\backslash A_1, X\backslash A_2,\ldots,X\backslash A_n)=X\oslash A$ is

n-ary open in (X, T).

For the notations and terminologies that are used here the reader may consult [11 -16].

III. HAUSDORFF AXIOM IN n-ary TOPOLOGY

Definition 3.1:

An n-ary space (X,T) satisfies an n-ary T_2-axiom if for any two absolutely distinct n-ary elements x, y of X, there are n-ary disjoint open sets U and V with $x\in U$ and $y\in V$ that is if $x \# y \Rightarrow$there are n-ary open sets U and V such that $U\cap V=\Phi$, $x\in U$, $y\in V$.
An n-ary space satisfying n-ary T_2-axiom is called an n-ary Hausdorff space.

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Theorem 3.2:

Let $x,y \in X$. The following are equivalent.

(i). X is n-ary Hausdorff

(ii).If $y \# x$, there is an n-ary nbd U(x) such that $y \notin$ n-*Cl*(U(x))).

(iii). If $y \# x$ then $y \notin \cap\{$ n-*Cl*(U): U is an n-ary nbd of x$\}$

Proof:

Suppose X is n-ary Hausdorff and $y \# x$, Then, there are n-ary disjoint open sets U(x) and V(y) such that $x \in U$ and $y \in V$. Now $U \cap V = \Phi \Rightarrow U \subseteq X \setminus V \Rightarrow$ n-*Cl* $U \subseteq X \setminus V$. Since $y \in V$, $y \notin X \setminus V$ that implies $y \notin$ n-*Cl*(U(x))). This proves (i) $\Rightarrow$(ii). Suppose (ii) holds. For each $y \# x$, $y \notin \cap\{$n-*Cl*U: U is an n-ary nbd of x$\}$. This proves (ii) $\Rightarrow$(iii). Suppose (iii) holds. Let $y \# x$. Choose an n-ary open set U with $x \in U$ and $y \notin$ n-*Cl*(U). Then $y \in X$ and let $A \setminus$ (n-*Cl*(U) =V is an n-ary open set with $y \in V$ and $U \cap V = \Phi$. This proves (iii) $\Rightarrow$(i).

Theorem 3.3:

(i).The n-ary Hausdorff topologies are invariant under n-ary closed bijections.

(ii). An n-ary subspace of the n-ary Hausdorff space is n-ary Hausdorff.

(iii). If $f:X \to Y$ is an n-ary continuous injection from (X, T_X) to (Y,T_Y) and if (Y,T_Y) is n-ary Hausdorff then (X, T_X) is n-ary Hausdorff.

(iv). (X, T) is n-ary Hausdorff iff each (X_i, T_i) is Hausdorff.

Proof:

Suppose $f:X \to Y$ is an n-ary closed bijection and (X, T_X) is n-ary Hausdorff. Let $y \# q$. Since f is a bijection $f(x)=y$ and $f(p)=q$ for some $x \# p$. Then there are disjoint n-ary open sets U and V with $x \in U$ and $p \in V$. Since every n-ary closed bijection is n-ary open, (i) follows easily. Now let (Y,T_Y) be an n-ary subspace of X. Let $y \# q$. Since (X, T_X) is n-ary Hausdorff, there are disjoint n-ary open sets U and V in X with $y \in U$ and $q \in V$. Clearly $U \cap Y$ and $V \cap Y$ have the required properties in Y. This shows (ii). Let $f:X \to Y$ be an n-ary continuous injection and (Y,T_Y) be n-ary Hausdorff. Let $x \# p$ in X. Since f is an injection with $y=f(x) \# f(p)=q$ in Y, there are disjoint n-ary open sets U and V in Y such that $y \in U$ and $q \in V$ that implies $f^{-1}(U)$ and $f^{-1}(V)$ are the desired disjoint n-ary open sets in X. This proves (iii).

Now it remains to establish (iv). Let (X, T) be n-ary Hausdorff. Fix j and let $p_j \neq q_j$ in X_j. Choose the n-ary points x and y in X with $x \# y$, $x_j = p_j$ and $y_j = q_j$. Since (X, T) is n-ary Hausdorff, there are disjoint n-ary open sets U and V with $x \in U$ and $y \in V$ so that $p_j \in U_j$ and $q_j \in V_j$ that implies (X_j, T_j) is Hausdorff. The converse part is obvious. This proves (iv).

IV. REGULAR AXIOM IN n-ary TOPOLOGY

Definition 4.1:

An n-ary Hausdorff space (X,T) is said to satisfy an n-ary T_3-axiom if for any n-ary element x of X and for every n-ary closed set F with $x \notin F$, there are n-ary disjoint open sets U and V such that $x \in U$ and $F \subseteq V$.

An n-ary space satisfying n-ary T_3-axiom is called an n-ary regular space.

Theorem 4.2:

Let (X, T) be n-ary Hausdorff and $x \in X$. The following are equivalent.

(i). X is n-ary T_3

(ii). For an n-ary open nbd U of x, there is an n-ary nbd V(x) with $x \in V \subseteq$ n-*Cl*$V \subseteq U$.

(iii). For each n-ary closed set A with $x \notin A$, there is an n-ary open nbd V of x such that n-*Cl*(V)$\cap A = \Phi$.

Proof:

Suppose (X, T) is n-ary T_3. Let U be an n-ary open nbd of x. Then $x \notin B$. Here $B = X \setminus U$. Since (X, T) is n-ary T_3, there are n-ary disjoint open sets V and W such that $x \in V$ and $B \subseteq W$. Since $W \cap V = \Phi$, it follows that $x \in V \subseteq$ n-*Cl* $V \subseteq X \setminus W \subseteq U$. This proves (i)$\Rightarrow$(ii). Now suppose (ii) holds. Let A be an n-ary closed set with $x \notin A$ that implies $B = X \setminus A$ is n-ary open such that $x \in B$. Now using (ii), there is an n-ary nbd V(x) such that $x \in V \subseteq$ n-*Cl* $V \subseteq B = X \setminus A$ that implies (n-*Cl* V) $\cap$ A = Φ. This proves (ii) $\Rightarrow$(iii). Now suppose (iii) holds. Let F be an n-ary closed set with $x \notin F$. By using (iii), there is an n-ary open nbd V of x such that (n-*Cl* V)$\cap F = \Phi$ that implies $X \setminus$(n-*Cl* V) and V are the desired n-ary open sets. This proves (iii)$\Rightarrow$(i).

Theorem 4.3:

(i). Any n-ary subspace of the n-ary T_3 space is n-ary T_3.

(ii). (X, T) is n-ary T_3 iff each (X_i, T_i) is T_3 .

Proof:

Let (X, T_X) be n-ary T_3 and (Y,T_Y) be its n-ary subspace.. Let $y \in Y$ and let K be an n-ary closed set in Y with $y \notin K$. Since (X, T_X) is n-ary T_3, there are disjoint n-ary open sets G and H in X with $y \in G$ and $K \subseteq H$. Clearly $G \cap Y$ and $H \cap Y$ are the desired disjoint n-ary open sets in Y. This shows (i).

Now let (X, T) be n-ary regular. Fix j and let $p_j \in X_j$ and F_j be a closed set in X_j such that $p_j \notin F_j$ Choose an n-ary points x and an n-ary closed set A with $x_j = p_j$ and $A_j = F_j$. Since (X, T) is n-ary regular there are disjoint n-ary open sets U and V with $x \in U$ and $A \subseteq V$ so that $p_j \in U_j$ and $F_j \subseteq V_j$. Therefore U_j and V_j are the desired open sets in (X_j, T_j) that implies (X_j, T_j) is regular. The converse part is obvious. This proves (ii).

V. NORMAL AXIOM IN n-ary TOPOLOGY

Definition 5.1:

An n-ary Hausdorff space (X,T) is known to satisfy an n-ary T_4-axiom if for given disjoint n-ary closed sets A and B in (X,T), there are n-ary disjoint open sets G and H such that $A \subseteq G$ and $B \subseteq H$.An n-ary space satisfying n-ary T_4-axiom is called an n-ary normal space.

Theorem 5.2:

. The following are equivalent.

(i). X is n-ary T_4

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(ii). If A is an n-ary closed set in X and U is an n-ary open set $U \supseteq A$, there is an n-ary open set V such that $A \subseteq V \subseteq$ n-*Cl* $V \subseteq U$.
(iii). If A and B are disjoint n-ary closed sets there is an n-ary open set U with $A \subseteq U$ and
(n-*Cl* U) ⋒ B = Φ.

Proof:

Suppose (X, T) is n-ary T_4. Let $A \subseteq U$ be an n-ary set and U be an n-ary open set in X so that $X \setminus U$ and A are disjoint n-ary closed sets in X that implies there are n-ary disjoint open sets V and W such that $V \subseteq X \setminus W \subseteq X \setminus B$ = U that shows that $A \subseteq V \subseteq$ n-*Cl*V $\subseteq X \setminus W \subseteq$ U. This proves (i)⇒(ii). Now suppose (ii) holds. Let A and B be any two disjoint n-ary closed sets in X. Then by applying (ii), there is an n-ary open set U such that $A \subseteq U \subseteq$ n-*Cl* U $\subseteq W$ that implies $B \subseteq X \setminus$ (n-*Cl* U) so that
(n-*Cl* U) ⋒ B =Φ. This proves (ii) ⇒(iii).
If (iii) holds and if t A and B be the two disjoint n-ary closed sets in X then
(n-*Cl* U) ⋒ B =Φ that shows that $X \setminus$ (n-*Cl* U) V and U are the expected n-ary open sets. This proves (iii) ⇒(i).

Theorem 5.3:

(i).The n-ary normal topologies are invariant under n-ary continuous open injection
(ii).The n-ary normal topologies are invariant under n-ary continuous closed surjections.
(iii). An n-ary closed subspace of the n-ary normal space is n-ary normal
(iv). (X, T) is n-ary normal iff each (X_i, T_i) is normal .

Proof:

Let $f:X \to Y$ be an n-ary continuous. Let A and B be any two disjoint n-ary closed sets in Y. If f is a surjection and (X, T_X) is n-ary normal then $f^{-1}(A)$ and $f^{-1}(B)$ are the disjoint n-ary closed sets in X that implies there are disjoint n-ary open sets U and V in X such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$ so that f(U) and f(V) are the desired n-ary open sets in Y. This proves (i). If further $f:X \to Y$ is n-ary closed also and if (Y, T_Y) is n-ary normal then f(A) and f(B) are disjoint n-ary closed sets in Y so that there are disjoint n-ary open sets U and V in Y with $f(A) \subseteq U$ and $f(B) \subseteq V$ that implies, $f^{-1}(U)$ and $f^{-1}(V)$ are the desired disjoint n-ary open sets in X. This proves (ii). Let (Y, T_Y) be an n-ary closed subspace of an n-ary normal space (X, T_X). If A⋒Y and B⋒Y are any two disjoint n-ary closed sets in Y then A⋒Y and B⋒Y are n-ary closed in X so that there are disjoint n-ary open sets U and V in X with A⋒Y⊆U and B⋒Y⊆V that implies U⋒Y and V⋒Y are the desired disjoint n-ary open sets in Y. This shows (iii). Now let (X, T) be n-ary normal Fix j. Let P_j and Q_j be any two disjoint closed sets in X_j. Choose n-ary closed sets A and B with $A_j = P_j$ and $B_j = Q_j$. Since (X, T) is n-ary normal there are disjoint n-ary open sets U and V with $A \subseteq U$ and $B \subseteq V$ so that $P_j \subseteq U_j$ and $Q_j \subseteq V_j$ Therefore U_j and V_j are the desired open sets in (X_j, T_j) that implies (X_j, T_j) is normal . The converse part is obvious.
This proves (iv).

VI. CONCLUSION

The notions of Hausdorff, regular and normal separation axioms are extended to n-ary topological spaces. These axioms are investigated in n-ary topological settings. Further it has been established that an n-ary space is n-ary Hausdorff, n-ary regular and n-ary normal iff the corresponding topologies on the coordinate spaces are Hausdorff, regular and normal respectively.

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