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Modeling of the Optimal Distribution Scheme of Transport Flows in the Street-Road Network Considering the Influence of a Pedestrian Traffic

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Abstract*: The introduction of intelligent transport systems involves the automated selection of optimal solutions for all road users in real time. The aim of the research is to develop methods for modeling the optimal scheme of traffic flow distribution along the road network, taking into account the influence of pedestrian traffic. The paper covers the mathematical models of intersections in various schemes for organizing pedestrian traffic, develops an analytical apparatus for estimating vehicle delays in terms of pedestrian flows. For choosing the scheme of the movement organization under the conditions of uncertainty, the apparatus of mathematical game theory is used. Methods for calculating the delays of vehicles taking into account pedestrian traffic are made within the framework of hypotheses and attitudes of the author's model of the distribution of traffic flows over the network, which is aimed at solving optimization of transport problems in the online mode. Considering the pedestrian flows allows more accurately simulating the real traffic situation and fast responding to its changes.

Index Terms*: mathematical model, optimization, pedestrian traffic, traffic signal control, transport flows.

I. INTRODUCTION

Currently, the development of information and communication technologies in all areas is urgent, including the transport sector. The introduction of intelligent transport systems involves the automated selection of optimal solutions for all road users in real time. The concept of intelligent transport systems fits into a more general concept of forming Smart City, including automated traffic control system (Smart Traffic system, Automatic traffic signal, Public transport facility) as one of the components [1].The study of the pedestrian flow dynamics is an important task in the formation and optimization of urban transport infrastructure. Without considering the intensity of the pedestrian flow in the centers of attraction, the model of vehicle distribution along the road network will not be adapted to current conditions, and will not reflect the real situation on the network. Accordingly, the calculations for this model will not be sufficiently adequate.Pedestrian traffic has a particularly significant impact on the traffic flow parameters at network nodes, sometimes causing traffic jams. Moreover, the mutual influence of pedestrian and traffic flows changes over time, and therefore it is useful to have a mutually beneficial traffic optimization, considered with regard to dynamic changes.The transport network of cities should provide comfortable movement for both vehicles and pedestrians. The increase in intensity and the change in the speeds of traffic flows impose ever more rigid requirements on the means of traffic control, on the level of their efficiency and safety for all conflicting parties. Studying the effect of various pedestrian flow schemes on the magnitude of vehicle delays at network hubs is urgent. Mathematical modeling of these schemes provides an opportunity to choose the most acceptable one in a given situation. Due to the above-mentioned, an urgent task is to develop an automated control system for the distribution of traffic flows over the network, taking into account the changes in pedestrian flow dynamics. For this purpose, a mathematical model is needed, which allows obtaining control parameters based on real-time changing data. The model should adequately reflect the road situation. The source data for the model should cover the main parameters characterizing the flows in the network.

The research objective is to develop real-time modeling methods for the optimal scheme of traffic flow distribution along the road network, considering the influence of pedestrian traffic.

II. METHODS

Currently, there are various models for pedestrian flows. They can be divided into three classes: macroscopic, mesoscopic and microscopic [2, 3].Macroscopic models take into account the average parameters of the pedestrian flow, considering it as a physical process, e.g. fluid flow. Similar to how it was made for transport flows. This approach was used by Mahmassani [4], Twarogowska[5].Microscopic models are developed based on the individual characteristics of pedestrian behavior. The characteristics of the pedestrian flow are calculated taking into account the individual behavior of road users. Microscopic models are Cellular Automata (CA), Social Force (SF), Behavioral Heuristics (BH).The representation of pedestrian flows with cellular automata was used by Helbing [3], Flotterod and Lammel[6]. In this case, the process is discrete in time and space. All space is divided into cells that can be occupied by individual pedestrians. The transition from cell to cell is carried out according to the rules determined by the behavior of individual pedestrians in the flow.

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SF-theory is modeling the movement of pedestrians as a special physical process, taking into account the following parameters: desired speed, comfortable space for individual pedestrians (desired distance from other pedestrians and objects),
the attractiveness of conditions (Helbing, Molnar [7]; Johansson [8]).BH-models consider the peculiarities of human behavior in the flow. They reflect crowd turbulence in case of high intensity (Paris,
Pettre, Donikian[9])Mesoscopic models, in contrast to macroscopic models, take into account the peculiarities of the behavior of individual pedestrians but do not put them at the forefront, as in the microscopic. This is a compromise between micro and macro modeling. These models include, for example, the Treuille[10] and Daamen[11] models.Pedestrian flow modeling is an important part of a larger task: assessing its impact on the workload of the entire transport network, choosing the best solutions for designing the road network and optimizing the organization of traffic on it. This problem was covered by, e.g. F. Camili, A. Festa and S. A. Tozza[2], M. Di Francesco, P.A. Marcovich[12], Y. Jiang, P. Zhang [13], T.A. Iksakov, V.G. Sidorenko [14], S.N. Pavlov, M.A. Nekrasova, Yu.V. Pavlova [15], B.T. Torobekov[16], A. Kurganov, I. Timofeev [17].Graph theory, methods of probability theory, a theory of random processes, differential and integral calculus are widely used in solving such problems.Currently, there is a widespread introduction of Intelligent Transport Systems, which allow tracking the real situation in the network. In a number of cities following a Smart City concept, transport infrastructure problems are decisive. Therefore, the urgent task is to develop a comprehensive model that considers both transport and pedestrian flows of various intensity. At the same time, it should provide an opportunity to provide prompt solutions, with a short-term delay in responding to a constantly changing traffic situation. This question is still open.

III. RESULTS

The purpose of the introduction of Intelligent Transport Systems (ITS) is to obtain the ability to quickly respond to changing conditions in the system and optimize its operation. The ability to control the distribution of correspondences over the network and changes in the intensity of traffic flows in real time allows predicting the occurrence of congestion, preventing undesirable consequences of a fast increase in intensity in certain sections of the network.We have developed and theoretically substantiated a model of traffic flow distribution along the road network based on the concept of mesoscopic detail of the source data and analytical methods for determining the quality parameters of the vehicle movement organization (TIMeR_Mod – Transportation Intelligent Mesoscopic Real-time Model) [18, 19]. The transport flow is characterized by a statistical distribution of time intervals between vehicles. At first, it was proposed to adopt a hypothesis about the distribution of time intervals between vehicles in each lane according to the generalized Erlang law, which allows approximating high-intensity flows. The graph representation of the street-road network considers the distribution of traffic flows in all lanes between two adjacent nodal points (intersections). It is proposed to classify the nodal points as nodal points of the first type (corresponds to unregulated intersections) and nodal points of the second type (corresponds to signalized intersections). To store all the necessary information, a new matrix representation of the street-road network has been developed – matrices $A_{STREETS}$ and $B_{INTERSECTION}$, the structure of which allows one to get quick access to the necessary information in an automated task solution within the ITS.In the TIMeR_Mod model, the nodes (intersections) and arcs (roads connecting two adjacent intersections) of the network are dynamic elements (unlike other existing mesoscopic models CONTRAM, DynaMIT, MEZZO). Thus, it is possible to change the traffic scheme without going beyond the model in real time. The function of transportation costs along the routes of the road network has been developed and theoretically justified. In an analytical form, the transportation cost function considers the distribution of traffic flows for each lane. This makes it possible to predict the level of transportation costs with a short-term delay in responding to changes in traffic patterns on the road network. In addition, we have developed and substantiated a method for determining the dynamic correspondence matrix, which was constructed taking into account the source data required for calculations using our TIMeR_Mod model [18].The TIMeR_Mod model described above was used as the basis for solving the task of modeling the movement of transport flows, considering pedestrian traffic.

A. *Simulation of a conflict situation at an intersection*

A model of a conflict situation at an intersection is an antagonistic two-player game in which the only one can consciously choose the strategies. Therefore, let us consider this conflict situation as "a game with nature". The second player, "nature", does not specifically oppose the first, but accepts one or another state in an indefinite manner.

Player A – a pedestrian flow through a particular intersection. Possible strategies of player A:

– A_1 – rigid regulation cycle with a separate phase for pedestrian flow;

– A_2 – rigid regulation cycle without a separate phase for pedestrian flow;

– A_3 – use of calling devices for pedestrian traffic.

Uncertainty in making decisions is caused by the states of objective reality (of player B) – the traffic and pedestrian traffic intensities that change over time. The player B counteracts the player A unconsciously. The strategy of a player B, "nature", is the intensity distribution of road and pedestrian flows at a given intersection.An important point is the task of the elements of the payment matrix. Their choice depends on the priorities set, on the dominant optimization criterion. When focusing on the organization of a pedestrian flow through an intersection, then the elements of the payment matrix can be the values, opposite of the average delay for an individual pedestrian at the intersection.

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Let us call it the intensity of pedestrian service.
Thus, the payment matrix has the form:

$$A = \left(\begin{array}{c|cccc} & Q_1 & Q_2 & \dots & Q_3 \\ A_1 & a_{11} & a_{12} & \dots & a_{1n} \\ A_2 & a_{21} & a_{22} & \dots & a_{2n} \\ A_3 & a_{31} & a_{32} & \dots & a_{3n} \end{array} \right), \quad (1)$$

Q_j, $j = 1,\dots,n$ –

possible states of "nature" in the considered period of time, calculated using regression dependencies (with a given time step from the current moment) of the intensity of all conflict flows, both traffic and pedestrian;

$a_{ij} = \dfrac{1}{T_{serv}}$ – the intensity of pedestrian service with strategies A_i and Q_j fairness;

T_{serv} – the average delay of a pedestrian when crossing the intersection in the chosen direction with strategies A_i and Q_j fairness.

Criterion options are also possible. It is proposed to use either the Wald's maximin criterion or the Savage's minimax risk criterion.

According to the Wald's criterion, the strategy is considered optimal when meets the following condition:

$$W = \max_i \min_j a_{ij}. \quad (2)$$

In this case, the optimal intersection traffic organization scheme is the one, which ensures the selection of the best (with the maximum intensity of service for pedestrians) situation among all possible and unsuccessful in the distribution of traffic flows in the considered period of time.

The largest element of the j-th column $\beta_j = \max_i a_{ij}$ ($j = 1,\dots,n$) is called the indicator of a favorable state Q_j.

To apply the Savage's minimax risk criterion, it is necessary to build a risk matrix:

$$R = \{r_{ij}\}_{m\times n}, \quad (3)$$

where $r_{ij} = \beta_j - a_{ij}$ – the risk of player A as the missed opportunity for maximum winnings (the non-won part of the maximum winnings).

The best option according to Savage's criterion meets the following condition:

$$S = \min_i \max_j r_{ij}, \quad (4)$$

B. *Definition of the payment matrix elements*

Consider the issue of determining the possible states of "nature" Q_j – the options for the distribution of intensities of transport and pedestrian flows. The source data for solving various transport problems using the TIMeR_Mod model is the current traffic scheme on the road network and the distribution of traffic flows by lanes, determined on the basis of monitoring data. All information necessary for calculations is contained in the matrices $A_{STREETS}$ and $B_{INTERSECTION}$. We have developed a method for predicting the elements of these matrices for the next period of time using the Kalman filter, as well as a method for their automated calculation. To determine the state of Q_j at intervals Δt it is proposed to use the above-mentioned method.

1) Strategy A_1

Consider an intersection with a rigid regulation mode and a separate phase for pedestrian flow. Suppose that the arrival of a pedestrian to an intersection is random and does not depend on the phases of the traffic light cycle. Then the time of arrival of a pedestrian can be considered distributed evenly with respect to the phases of the traffic light cycle. If at this moment the movement of a pedestrian is allowed, then the elapsed time is only $Tcross_p$, otherwise, the pedestrian expects the opportunity to continue moving. According to the laws of the probability theory, the average waiting time for a pedestrian to continue moving, if he arrives at the time of the prohibitive traffic light turned on, is equal to $\dfrac{T_c - T_p}{2}$ seconds, where

T_c – the length of the regulation cycle, s.,

T_p – a separate phase of a traffic light cycle, the time during which pedestrians are allowed to move in the selected direction, s.Then the average time required for a pedestrian to continue moving through the intersection T_{serv} is:

$$T_{serv} = \frac{T_c - T_p}{T_c} \cdot \left(\frac{T_c - T_p}{2} + Tcross_p \right) + \frac{T_p}{T_c} \cdot Tcross_p, \quad (5)$$

where $Tcross_p$ – the average time required for a pedestrian to cross the road, s.

The duration of the phases of the traffic light regulation is defined as a solution to the problem of mathematical programming:

$$Z = \frac{\sum_i W_i(T_c - T_{gIi}, \lambda) + \sum_j W_j(T_c - T_{gIIj}, \lambda)}{T_c} \to \mathbf{min} \quad (6)$$

$$H_i(T_c, \lambda) - \frac{T_{gIi}}{h} \le 0, \quad i = 1, 2, \dots, p; \quad (7)$$

$$H_j(T_c, \lambda) - \frac{T_{gIIj}}{h} \le 0, \quad j = 1, 2, \dots, q, \quad (8)$$

where $H(T_c, \lambda)$ – a renewal function that expresses the number of vehicles arriving at the intersection over time T_c,

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$$W(T_i,\lambda)=\left(\frac{1}{\mu}\frac{t^2}{2}+\frac{\sigma^2-\mu^2}{2\mu^2}t\right)\Bigg|_0^{T_i}+\int_0^{T_i}R(t)dt$$ – an average delay over a control cycle.

The numerical solution of the task (6-8) is found, for example, using the Maple or MatLab systems.

2) *Strategy* A_2

In the case of using a rigid regulation mode without isolating a separate phase for pedestrian flow, the parameter T_{serv} is calculated similarly to clause 2.2.1 using formula (5). However, when calculating the lengths of traffic light regulation cycles in the mathematical programming problem, the objective function will be the following:

$$Z=\frac{\sum_i W_i\left(T_c-Tstop_I-T_{gIi},\lambda\right)+\sum_j W_j\left(T_c-Tstop_{II}-T_{gIIj},\lambda\right)}{T_c}\rightarrow min \quad , (9)$$

$Tstop_I$ and $Tstop_{II}$ - the average time required for vehicles to pass pedestrians through roads I and II, respectively, s.,

T_{gIi} – traffic on road I is allowed in the i-th direction, s.,

T_{gIIi} – traffic on road II is allowed in the j-th direction, s.

λ – a set of parameters of the generalized Erlang law for traffic flows.

Note that in strategies A_1 and A_2 for each state Q_i the optimal cycle of traffic light regulation is calculated, which depends on the intensity of all conflicting flows at a crossroad.

3) *Strategy* A_3

In the case of using traffic light calling devices, the average time required for a pedestrian to cross an intersection can be calculated as follows:

$$T_{serv}=\left(\frac{t_m+t_{pr1}}{2}+t_a+t_{pr2}\right)\cdot\lambda t_2e^{-\lambda t2}+\frac{1}{2}\left(t_a+t_{pr2}\right)\left(1-\left(\lambda t_1e^{-\lambda t1}+\lambda t_2e^{-\lambda t2}\right)\right) \quad , (10)$$

where t_{pr1} – the transitional interval between signals, s.,

t_{pr2} – green light, s.,

t_a – green light for transport of minimum duration, s.,

t_m – the period of the flashing green light in the pedestrian phase, s.,

λ – the parameter of an exponential distribution of the density of pedestrian flow.Formula (10) is derived under the assumption that the pedestrian flow arriving at the intersection is distributed according to the exponential law. It is assumed that pedestrians are fully subject to traffic lights. The properties of numerical characteristics of random variables were used in the calculations.Three main options were considered for the organization of traffic at the intersection. However, it is possible to expand the number of strategies A_i. For example, for individual intersections, it makes sense to add an option of a non-signalized intersection.

In addition, the number of phases of the traffic light control cycle can also be varied, which in turn will increase the number of possible player A strategies.The strategies of "nature" in the work are the distribution of the intensity of motor vehicles by lanes and the intensity of the pedestrian flow arriving at the intersection at certain intervals for a short period of time. In this case, predictive models are used to form the baseline data for TIMeR_Mod. However, it is possible to create a payment matrix for a period of time, for example, per day. Then the statistics for the compilation of matrices $A_{STREETS}$ and $B_{INTERSECTION}$ should be taken.

C. *Calculation of vehicle delays considering pedestrian traffic when using a rigid regulation mode and allocating a separate phase for pedestrian flow*

Consider the case when a separate phase of the traffic light cycle in each direction is allocated for pedestrian traffic. In the TIMeR_Mod model, a vehicle delay at an intersection means downtime, while driving in this direction is prohibited. Under these conditions, the analytical apparatus for estimating the average total delay of all vehicles at the intersection is similar to the method considered in the work. One should only consider changes in the phases of the traffic light cycle.

The following notations are introduced:

T_{gI} – the time during which the movement of vehicles on the road I (in various directions) is allowed;

T_{pI} – the time during which pedestrians are allowed to cross the road I;

T_{gII} – the time during which the movement of vehicles on the road II (in various directions);

T_{pII} – the time during which pedestrians are allowed to cross the road II.

$T_c=T_{gI}+T_{gI}+T_{pI}+T_{pII}$ – the length of the regulation cycle; (11)

$$T_{gI}=T_{gI1}+T_{gI2}+\ldots+T_{gIp}, \quad (12)$$

where T_{gIi} – traffic on road I in the i-th direction is allowed (then for a period of time $T-T_{gIi}$ traffic on road I in the i-th direction is prohibited);

$$T_{gII}=T_{gI1}+T_{gII2}+\ldots+T_{gIIq}, \quad (13)$$

where T_{gIIi} – traffic on road II in the j-th direction is allowed (then for a period of time $T-T_{gIIj}$ traffic on road I in the j-th direction is prohibited).The function $W_i(T,\lambda)$ determines the total delay of all the requirements of the flow № i with the parameters of the generalized Erlang law, specified by the set λ, over time T.

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In the adopted notations, the average total time loss by all vehicles at a given intersection in one hour is equal to:

$$Z = \frac{\sum_i W_i(T_c - T_{gIi}, \lambda) + \sum_j W_j(T_c - T_{gIIj}, \lambda)}{T_c} \quad (14)$$

The condition of elimination of the queue for one regulation cycle in all directions:

$$H_i(T_c, \lambda) - \frac{T_{gIi}}{h} \le 0, \quad i = 1, 2, \ldots, p; \quad (15)$$

$$H_j(T_c, \lambda) - \frac{T_{gIIj}}{h} \le 0, \quad j = 1, 2, \ldots, q. \quad (16)$$

Here $H(T_c, \lambda)$ is the renewal function, which expresses the number of vehicles arriving at the intersection for the time T_c in the i-th transport flow with parameters of the generalized Erlang law specified by the set λ.

D. *Calculation of vehicle delays taking into account pedestrian traffic when using a rigid regulation mode without selecting a separate phase for pedestrian flow*

Consider a cross-shaped intersection of roads I and II with hard traffic lights, but without a separate phase for pedestrian traffic. Assume that all road users strictly follow the regulations. Pedestrian flows approaching the intersection to cross the road i in two opposite directions have an exponential distribution with parameters λ_{pir} and λ_{pil}. Their distribution densities equal respectively $f_{pir} = \lambda_{pir} \cdot e^{-\lambda_{pir} t}$ and $f_{pil} = \lambda_{pil} \cdot e^{-\lambda_{pil} t}$, where t - is the time (in seconds) between two consecutive occurrences of events ($t > 0$).Let us operate with average values of random variables using the properties of their numerical characteristics. The average number of pedestrians who approached the intersection during the time when its crossing in a given direction is allowed equals:

– for the road I in two opposite directions of pedestrian traffic

$$\lambda_{p1r} \cdot (T_c - T_{gI}) \quad (17)$$

and $\lambda_{p1l} \cdot (T_c - T_{gI})$; (18)

– for road II in two opposite directions of pedestrian traffic

$$\lambda_{p2r} \cdot (T_c - T_{gII}) \quad (19)$$

and $\lambda_{p2l} \cdot (T_c - T_{gII})$. (20)

Then the average time required for vehicles to pass pedestrians is as follows:

– for road I

$$Tstop_{I\,r} = \lambda_{p1r}(T_c - T_{gI}) \cdot Tcross_{pI} \quad (21)$$

and $Tstop_{I\,l} = \lambda_{p1l}(T_c - T_{gI}) \cdot Tcross_{pI}$; (22)

– for road II

$$Tstop_{II\,r} = \lambda_{p2r} \cdot (T_c - T_{gII}) \cdot Tcross_{pII} \quad (23)$$

and $Tstop_{II\,l} = \lambda_{p2l} \cdot (T_c - T_{gII}) \cdot Tcross_{pII}$. (24)

Here $Tcross_{pI}$ and $Tcross_{pII}$ – the average time required for a pedestrian to cross roads I and II, respectively.Given the need to cross the intersection of a group of pedestrians who approached during the time of the prohibitory signal of the traffic light, the following average downtime is deduced:

– for the road I:

$$Tstop_I = (\lambda max_{p1} \cdot (T_c - T_{gI}) + 1) \cdot Tcross_{pI}, \quad (25)$$

where $\lambda max_{p1} = max\{\lambda_{p1r}, \lambda_{p1l}\}$;

– for road II:

$$Tstop_{II} = (\lambda max_{p2} \cdot (T_c - T_{gII}) + 1) \cdot Tcross_{pII}, \quad (26)$$

where $\lambda max_{p2} = max\{\lambda_{p2r}, \lambda_{p2l}\}$.

Given the above mentioned, the average total hourly delay for all vehicles at a given intersection is:

$$Z = \frac{\sum_i W_i(T_c - Tstop_I - T_{gIi}, \lambda) + \sum_j W_j(T_c - Tstop_{II} - T_{gIIj}, \lambda)}{T_c}. \quad (27)$$

The regulation mode described in this clause is appropriate under the following conditions:

$$(\lambda max_{p1} \cdot (T_c - T_{gI}) + 1) \cdot Tcross_{pI} << T_{gI}; \quad (28)$$

$$(\lambda max_{p2} \cdot (T_c - T_{gII}) + 1) \cdot Tcross_{pII} << T_{gII}. \quad (29)$$

Otherwise, a jam occurs at this nodal point of the transport network.

E. *Calculation of vehicle delays based on pedestrian traffic when using calling devices*

Pedestrian traffic during the day is uneven. Peak periods often exist in the morning and evening. And the use of hard traffic light regulation leads to unnecessarily high delays in road transport.Pedestrian crossings with traffic light calling devices (PUFFIN and Pelican) are widely used in the United Kingdom. This technology allows optimizing the loss of time for both vehicles and pedestrians.Consider a cross-shaped intersection of roads I and II, on which there is only a calling device for passing pedestrians on the main road. Without loss of generality, it can be assumed that I is the main road, II is a secondary one.In this case, the average delay of vehicles depends on the random process of arrival of pedestrians to this crosswalk. Introduce the following notations:

t_w – waiting time for the green signal for pedestrians after pressing the call button, s.;

$t_{gp\,I}$ – the duration of the permissive signal for pedestrians crossing the road I, s.;

t_{over} – the duration of the intermediate tact for freeing the carriageway from pedestrians, s.;

$T_{pI} = t_w + t_{gp\,I} + t_{over}$ – the duration of the permissive signal flashing for pedestrians crossing the road I, s.;

t_{ga} – the minimum duration of the permissive signal for vehicles, s.Vehicles on the main road I should pass only pedestrians. The arrival of vehicles of the road I to the intersection does not depend on the arrival of pedestrians to this intersection, therefore the average hourly delay of one vehicle traveling on the main road can be estimated as follows:

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$$W_I = \left(\lambda_{p1r} + \lambda_{p1l}\right) \cdot \frac{T_{pI}}{2}. \quad (30)$$

Consider the delays of vehicles of secondary road II. The time intervals in the mainstream traffic flows are distributed according to the generalized Erlang law with parameters $\lambda_{0j}, \lambda_{1j}, \ldots, \lambda_{k_j-1,j}, k_j$ – the parameters of the generalized Erlang law for the j -th intersected flow ($j = 1, 2, \ldots, L$), respectively, and in the pedestrian - with parameters λ_{pir} and λ_{pil} (the exponential law is a special case of the generalized Erlang law for $k = 1$). T_0– an acceptable interval for continuing movement (in seconds).

For vehicles of secondary road II, the average waiting time (in seconds) for an opportunity to cross L main road traffic flows and skip pedestrian flows crossing road II is:

$$m_z = \frac{\left(\sum_i a_{i1}\lambda_{i1} \cdot \frac{2}{\lambda_{i1}^{\ 3}}\right)}{2 \cdot \left(\sum_{i=0}^{k_1-1} \frac{1}{\lambda_{i1}}\right)} \cdot \Phi_0(T_0) + \left(\sum_{i=0}^{k_1-1} \frac{1}{\lambda_{i1}}\right) \cdot \Phi_0(T_0) \cdot \Phi(T_0) \cdot \frac{1}{(1 - \Phi(T_0))}, \quad (31)$$

where

$$\Phi_0(T_0) = 1 - \prod_{j=1}^{L+2} \left(\frac{1}{\sum_{i=0}^{k_j-1} \frac{1}{\lambda_{ij}}} \cdot \sum_{i=0}^{k_j-1} a_{ij} \cdot \frac{1}{\lambda_{ij}} e^{-\lambda_{ij} T_0} \right),$$

$$\Phi(T_0) = 1 - \left(\sum_{i=0}^{k_1-1} a_{i1} e^{-\lambda_{i1} T_0} \right) \prod_{j=2}^{L+2} \left(\frac{1}{\sum_{i=0}^{k_j-1} \frac{1}{\lambda_{ij}}} \cdot \sum_{i=0}^{k_j-1} a_{ij} \cdot \frac{1}{\lambda_{ij}} e^{-\lambda_{ij} T_0} \right).$$

Then for a non-signalized intersection the average delay (in seconds) of a single request of this secondary direction, taking into account the queue, is as follows:

$$W_{II} = m_Z \cdot (M(l) + 1), \quad (32)$$

where $M(l) = bp_0 \frac{\alpha^2}{(1-\alpha)^2}$, (33)

$$p_0 = \frac{1}{b\left(\sum_{j=0}^{n} \frac{\alpha^j}{j!} + \frac{\alpha^2}{1-\alpha}\right)}, \quad (34)$$

$$b = \sum_{i=0}^{k} \frac{\lambda_0}{\lambda_i}, \ \alpha = \frac{\lambda_0}{\mu}, \ \mu = k/(m_z). \quad (35)$$

The average total hourly delay of all vehicles at the intersection with a calling station for pedestrians is as follows:

$$Z = \sum_{i \in \Omega II} N_i \cdot W_{IIi} + W_I \cdot \sum_{j \in \Omega I} N_j, \quad (36)$$

where N_i, N_j - intensity (amount/hour) of transport and pedestrian flows in all directions,

ΩI – the set of all directions of the road I,

ΩII – the set of all directions of the road II.

IV. RESULTS ANALYSIS

As a result of the study, a new mathematical model for the distribution of traffic and pedestrian flows along the road network has been obtained. The TIMeR_Mod model developed earlier for the distribution of traffic flows over a network, taken as a basis in this study, is applicable to traffic flows of varying intensity and gives adequate results. As a hypothesis about the distribution of time intervals in each of the requirement flows in this model, is accepted that it is subject to the generalized Erlang law. This is a multiparameter law that allows describing traffic flows of sufficiently high density. Pedestrian flows are taken as distributed according to the exponential law. The exponential law is a special case of the generalized Erlang law. Therefore, a single approach was applied to all conflicting flows (both pedestrian and traffic) in analytical reasoning.The methods for calculating vehicle delays described in the paper, considering pedestrian traffic, are made within the framework of hypotheses and attitudes of our model TIMeR_Mod for the distribution of traffic flows over the network, which is aimed at solving optimization of transport problems in online mode. Considering of pedestrian flows will allow more accurately simulating the real traffic situation and fast responding to its changes. A new analytical apparatus for assessing the effectiveness of the organization of traffic and pedestrians at the nodal points of the network is derived from the general network assumptions. This makes it possible to take into account the peculiarities of the distribution of pedestrian flows throughout the network, their influence on the efficiency of the organization of traffic.It is rather difficult to optimize the traffic organization scheme at a separate intersection because it is necessary to take into account the interests of all the road participants. Depending on the choice of the optimization criterion, various results of solving the problem can be obtained. In addition, these decisions often have to be made in the face of uncertainty on individual factors. In such situations, mathematical methods of game theory are applied (in this case, games with "nature"). The result is still somewhat subjective, and depends on the choice of the criterion, but allows one to systematize the data and contributes to improving the quality of decisions made under conditions of uncertainty.The above-mentioned characteristics of the developed model make it possible to use it in software for Intelligent Transport Systems.

V. CONCLUSION

The developed mathematical model of the distribution of transport and pedestrian flows will allow one to automatically solve actual transport problems in real time. It gives the possibility of predicting possible transport problems caused by the concentration of pedestrian traffic, and therefore the possibility of their prompt solution. Methods and algorithms for automated optimization problems solution, based on this model, can be used in Intelligent Transport Systems to improve traffic management in the street-road network.

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